Transactions on Ecology and the Environment vol 48, © 2001 WIT Press, www.witpress.com, ISSN 1743-3541

Optimal irrigated cropping pattern of a multicrop system under water scarcity constraints

E. C. Kipkorir[1], A. Sahli[2], D. Raes[1], & E. Tollens[1]
[1]*K.U. Leuven University, Belgium*
[2]*Institut National Agronomique de Tunis (INAT), Tunis, Tunisia*

Abstract

This paper develops a dynamic programming optimisation model, that consider the competition of crops in a season, both for irrigation water and cultivated area. Decision variables are the cultivated area and water allocated to each crop. The objective function of the model is based on crop-water production functions, production costs and crop prices. The model is solved using CSUDP generalised dynamic programming package for conditions existing in Northern Tunisia where a large fraction of the irrigation water is stored in the reservoir mainly during the winter period. The model gives the optimal distribution of area and water to each crop and profit. Results indicate that the model can be a valuable tool in determining seasonal cropping pattern for a region at the beginning of the season.

1 Introduction

During the 20th century the rate of water withdrawals in the world increased much faster than population, owing mostly to irrigation and industrial development. Although the current irrigated area of the world covers about 17 percent of the total cultivated area and contributes about one third of the world food production, it actually accounts for 73 percent of the world water consumption today. In the next few decades the world's population is expected to grow from 6 billion today to at least 8 billion by the year 2025, with about 90 percent of the increase being added to the developing world. It is therefore clear that achieving food security will continue to pose major challenges to decision-makers in the next few decades. At a time when industrial and municipal users are in competition with agriculture for the limited water, farmers have to grow more food crops with the same or smaller amount of water, to feed the increasing population. This can be achieved by increasing the irrigation efficiencies by using new irrigation methods or by better planning of the water application. If there is still a water shortage, the limited amount of water to the crops should be distributed optimally.

This paper briefly describes an optimisation model to aid decision making of optimal distribution of area and water in a region at the beginning of the season, followed by application of the model to an existing system.

2 Model development

The problem may be considered to be one of maximizing the utilisation of the available water supply when conflicts between supply and demand is expected in the season. Solving this problem in a single-level is very complex. To reduce complexity and making sure that an optimal solution is attained, the problem is solved in a multilevel approach. The multilevel approach comprises of (i) single crop seasonal water production function; (ii) seasonal allocation of both water and area to all crops and (iii) for each crop determine the intra-seasonal water allocation (irrigation intervals for specified irrigation depth).

2.1 System

Let the number of field groups in the system be NF. A field group is defined as the total area planned to be grown with a particular crop at approximate same date, on a particular soil type and to use the same irrigation method. The objective function is derived for maximising the profit from the NF field groups to be irrigated by the seasonal estimated water supplies.

2.2 Single crop seasonal water production function

Allocation of irrigation water becomes particularly sensitive under conditions of limited water resources, when water shortages require a refined timing of irrigation in order to minimise yield reductions. By allocating water to the most sensitive crop stages, yields can be optimised. The optimal distribution of a limited supply of irrigation water can be calculated according to Eqn(1), proposed by Doorenbos and Kassam [1], for each characteristic growth period.

$$\left(1-\frac{Ya}{Ym}\right)=Ky\left(1-\frac{ETa}{ETc}\right) \tag{1}$$

where Ya is actual crop yield, Ym is maximum crop yield under the given management conditions, Ky is crop yield response factor to water, ETa is actual crop evapotranspiration and ETc is crop evapotranspiration without water stress. Application of Eqn(1) to a crop with the following characteristic growth periods:

Transactions on Ecology and the Environment vol 48, © 2001 WIT Press, www.witpress.com, ISSN 1743-3541

establishment (1), vegetative (2), flowering (3), yield formation (4) and ripening (5) and ensuring that full supply is given during the establishment period, the following relationship can be derived [2]:

$$WS_F = \frac{\eta In_{season}\left(\frac{FIR_F}{Ky_F}\right)}{\sum_{j=2}^{5}\left(\frac{FIR_j}{Ky_j}\right)} \quad (3); \qquad WS_j = \left(\frac{Ky_F}{Ky_j}\right)\left(\frac{FIR_j}{FIR_F}\right)WS_F \quad (4)$$

$$\frac{WS_j}{FIR_j} \le 0.5 \quad (5); \qquad 0 \le WS_j \le In_j \quad (6)$$

where F is the subscription for the growth period with the lowest Ky and for growth period j, WS is water deficit, In is the net irrigation requirement to avoid water stress and FIR is crop water requirement when water is not limiting, In_{season} is seasonal net irrigation requirement needed to be applied to avoid water stress and η is seasonal water deficit level (fraction). In_{season} and In can be determined by solving water balance equation for the root zone.

By varying the seasonal water deficit (for η= 0.0 to 1.0) in Eqn(3) for a specified discretization interval, subject to the constraints Eqn(5) and Eqn(6), a set of water deficits for the four growth periods can be generated. Once the water deficit in each period have been calculated, the actual water that can be supplied in a particular period is calculated by subtracting the water deficit (WS_j) in that period from the full supply requirement, Eqn(7).

$$FISn_j = FIRn_j - WS_j \quad (7)$$

Then a dated water production function is used to calculate the relative yield for each set of water deficit. In this study two dated water productions functions are used. The multiplicative relation Eqn(8) proposed by Jensen [3] is used for grain yielding crops. The relation has been used in other earlier studies [4], [5], [6] dealing with effect of water deficit on grain yield.

$$\frac{Ya}{Ym} = \prod_{j=1}^{N}\left(\frac{FISn_j}{FIRn_j}\right)^{\lambda_j} \quad (8)$$

where λ_j is a sensitivity factor which can be determined graphically [7]. The minimum relation Eqn(9) suggested by Allen [8] is used for non grain yielding crops for example pasture, fruits and vegetables. The relation has been used in an earlier study [9].

Transactions on Ecology and the Environment vol 48, © 2001 WIT Press, www.witpress.com, ISSN 1743-3541

$$\frac{Ya}{Ym} = Min\left(Yr_1, Yr_2, ..., Yr_j, ..., Yr_N\right) \qquad (9)$$

where Yr_j is the relative yield for growth stage j, computed using Eqn(1).

The resulting set of data for each crop is used to derive a function relating allocated amount of water and relative yield due to water deficit, $Y^*(Q)$. The important point here is that several solutions are obtained by varying η, which then results in several different allocations, Q.

2.3 Seasonal allocation

After the optimal yield curves for each field group i, $Y^*_i(Q_i)$ as a function of seasonal allocations Q have been found, the seasonal allocations for each field group that will maximise total crop production can be determined. Dynamic programming (DP) is used. If the objective is to maximise the profit from the NF field groups, the objective function can be expressed as:

$$R = \max \sum_{i=1}^{NF} \left[A_i P_i Ym_i Y_i^*(Q_i) - A_i C_i\right] \qquad (10)$$

where for field group i; R is the profit (US$); A is the cultivated area (ha); P is crop price (US$/ton); Ym is maximum crop yield under the given management conditions (ton/ha); C is production cost per unit area (US$/ha). C can be subdivided into: the cost of irrigation water (US$/mm ha); the costs of irrigation system (US$/ha); the cost of labor (US$/ha); and other costs such as seed, fertiliser, pesticides, machinery, harvesting, unexpected costs, etc. (US$/ha).

2.3.1 DP Formulation

The DP formulation determines the optimal seasonal allocations of both water and area to the NF field groups. A field group constitutes a DP stage. The decision variables are the area and water to be allocated to each field group. State variables X_{i1} and X_{i2} are defined as amount of water and area left respectively for the remaining field groups after field groups 1 through i-1 have received water and area respectively. Therefore the system state is characterised by a vector $\mathbf{X}_i = (X_{i1}, X_{i2})$ at each DP stage, along with a decision vector $\mathbf{U}_i = (U_{i1}, U_{i2})$. The inverted form of state equation and the backward looking dynamic programming recursive algorithm are used.

2.3.2 Constraints

The constraints are based on availability of resources, and market considerations as follows:

(1) Water availability:

$$\sum_{i=1}^{NF}\left(\frac{A_i Q_i}{10}\right) \le V_{max} \quad (11); \qquad d_{i_{min}} \le Q_i \le d_{i_{max}} \quad (12)$$

where V_{max} is the total seasonal water available, for all the field groups (m^3), $d_{i,max}$ is maximum crop water demand and $d_{i,min}$ is the water supply to attain the minimum allowable yield in field group i. Some field groups in the system may be managed under full irrigation supply. This restriction can be achieved by setting $d_{i,min}$ very close to $d_{i,max}$, for example 99% of $d_{i,max}$.

(2) Area availability: Some social economic, management and market considerations restrict the model variables. For example market limitations and agronomic management limit the maximum and/or minimum areas cultivated with specific crops. Mathematically, this restriction can be expressed as:

$$\sum_{i=1}^{NF} A_i \le A_{max}, \qquad A_{i_{min}} \le A_i \le A_{i_{max}} \quad (13)$$

where A_{max} is the total available area, $A_{i,max}$ and $A_{i,min}$ are upper and lower area bounds of field group i.

2.4 Intra-seasonal water allocation

Once the seasonal water allocation and area for each field group have been found, the intra-seasonal allocations for each field group that will maximise profit can be determined from the single crop seasonal water production function. Then the irrigation schedule for each field group can be determined through soil water balance, calculation procedure.

3 Example application

3.1 Study site

The Ariana region in Northern Tunisia was used as an application example for the optimisation model. The main source of irrigation water in the region is from Laroussia dam with an estimated capacity of 179.41 million m^3. The total developed irrigated area, using stored water in the dam, in the region is 30,961 ha. Water availability in the dam at the start of irrigation season varies in function of the winter rain and determines the area that can be irrigated. The irrigation season starts in March and ends in October. The peak irrigation demand is expected in the months of May to August when the reference evapotranspiration is at the maximum and the expected rainfall is close to zero (Figure 1). The reference crop evapotranspiration is derived from 10-day climatic data of the region by means of the FAO Penman-Monteith equation [10]. Dependable rainfall for different levels where statistically derived from the

10-day rainfall records of the last 25 years with the help of RAINBOW software [11]. The 10-day ETo and rainfall dependable levels are presented in Figure 1. Three irrigation methods namely surface, sprinkler and drip are used. The area distribution in the region equipped for a particular irrigation method is 25,944, 4,020 and 997 ha respectively[12]. The mean application efficiency for surface, sprinkler and drip irrigation methods as estimated during measuring campaigns in 1998 and 1999 [13], [14], [15] are respectively 60, 70 and 80 %. The total available soil water for the major soil type in the region is about 160 mm (water)/m(soil depth) [16]. Determination of optimal cropping pattern for the region requires a good knowledge of the availability of the water in the reservoir and of meteorological conditions for the region.

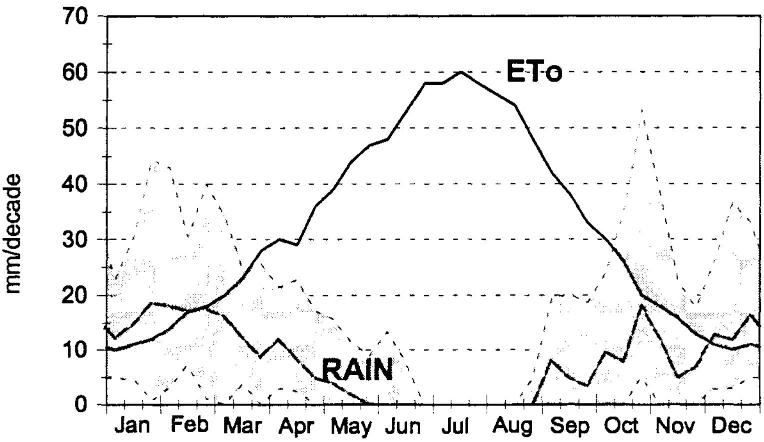

Figure 1: Mean 10-day, 20 and 80% dependable rainfall (shaded area) and mean 10-day reference evapotranspiration (full line) for the study area.

3.2 Crop data

The main parameters required for the optimisation procedure for the major crops grown in the region are given in Table 1. The planting month for each crop is specified in column 3. The maximum area for each crop using surface, sprinkler and drip irrigation methods corresponds with the developed area under the specific irrigation method. Social economic constraints require a minimum production for certain crops. In this study the minimum area corresponds with the 1994 survey. The yield response factors (Ky), for each crop, used in this study are obtained from [1]. The market price for each crop is given in column 4 [17]. The maximum yield under the given management (Ymax) per irrigation method for each crop is given in columns 5, 6 and 7 [1]. The indicative costs in US$/ha per irrigation method for each crop is given in columns 8, 9 and 10 [18].

Transactions on Ecology and the Environment vol 48, © 2001 WIT Press, www.witpress.com, ISSN 1743-3541

Table 1. Planting months, crop prices, maximum yields and production costs per irrigation method (sources: [1],[17],[18])

Name of Crop	* Sensitivity	Planting month	Price (US$/T)	Ymax (T/ha) Surface	Ymax (T/ha) Sprinkler	Ymax (T/ha) Drip	Cost (US$/ha) Surface	Cost (US$/ha) Sprinkler	Cost (US$/ha) Drip
Sorghum	L	May	35	5	-	-	120	-	-
Maize	MH	May	69	6	-	-	300	-	-
Alfalfa	LM	Jan	150	20	35	-	420	1160	-
Potato	H	Feb	327	25	30	35	450	1160	1450
Melon	MH	Apr	343	25	-	35	450	-	1740
Chilli	MH	Apr	614	15	20	25	450	1160	1595
Tomato	MH	Apr	287	40	50	65	450	1160	1595
Vegetable	H	May	250	60	70	-	450	1160	-
Grape	LM	Apr	539	15	25	30	730	2200	2920
Peach	MH	Jan	886	20	25	35	730	2200	2920
Apple	MH	Jan	673	30	40	50	730	2200	2920
Pear	MH	Jan	673	30	40	50	730	2200	2920
Citrus	LM	Jan	885	25	35	40	730	2200	2920
Olive	LM	Jan	510	45	-	-	730	-	-

* sensitivity to water shortage: L-low, LM-low medium, MH-medium high, H-high, (source: [1])

3.3 Model runs

MIOS model (described in section 2) is used to derive the seasonal water production function for each crop and to process input data files required by CSUDP. The CSUDP, generalised Dynamic Programming package [19], based on Dynamic programming optimisation technique, is used to perform the optimisation and to determine the optimal seasonal allocation of area and water to each crop for various available water amounts. The available seasonal amount of water in the reservoir was varied from 155.25 to 179.41 (maximum capacity) million m^3. The total area under fruits in the region is 8500 ha and cannot be cultivated by other crops; therefore only water allocations are done. In this study, the full supply is allocated to fruit crops because of their high economic value.

3.4 Results and discussion

The optimisation model aimed at aiding decision making of optimal irrigated cropping pattern under water scarcity constraints has been applied to the Ariana region, Northern Tunisia. Figures 2, shows the results of the optimisation for available seasonal amount of water ranging from 155.25 to 179.41 million m^3. As an example, for 167.75 million m^3, the allocation of water and area are shown in Table 2. Resources allocation (water and area) within the analysed range was similar for both sprinkler and drip systems and all the available area per irrigation method was allocated. This is the result of high irrigation efficiency of the two irrigation methods and most of the crops under these systems being of high value.

The allocation of resources for 160.25 and 172.75 million m^3 are compared in Table 3. Only the area under surface is presented (the area under drip, sprinkler and fruit does not change). Sorghum and maize (very low price) are allocated the minimum area. This is an example of the model being apple to consider social economic factors that may require production of certain crops even though they may be of low value. For sorghum, maize, alfalfa, chilli and vegetables it is seen that with decrease in water there is an increase in percentage area per crop. For potato, melon, and tomato it is seen that with decrease in water there is a decrease in percentage area per crop. Within the analysed range it was noted that with decrease in water the area allocated for alfalfa crop (low price and low medium sensitivity) increased from about 350 with full supply to 510 ha with 58% supply respectively. This is an indication that area under alfalfa can be increased with limited water supply. Vegetables are allocated 1000 ha with over 90% of supply in both cases, indicating that the crop has the highest priority in case of water shortage.

Table 2. Allocation of water and area to crops per irrigation method for 167.75 million m^3 available water

	Surface		Sprinkler		Drip			
	Water (Mm3)	Area (ha)	Water (Mm3)	Area (ha)	Water (Mm3)	Area (ha)	Total area	% Area
Sorghum	2.907	300					300	2.0
Maize	3.000	290					290	1.9
Alfalfa	3.000	510	2.500	200			710	4.7
Potato	2.000	350	1.592	220	0.299	50	620	4.1
Melon	6.047	1000			0.302	50	1050	6.9
Chilli	2.907	250	5.843	450	0.572	50	750	4.9
Tomato	7.093	1000	4.261	450	0.602	77	1527	10.0
Vegetables	5.547	1000	2.637	450			1450	9.5
Grape	5.164	500	3.860	450	0.371	50	1000	6.6
Peach	16.999	980	6.708	450	2.321	180	1610	10.6
Apple	17.346	1000	6.708	450	2.321	180	1630	10.7
Pear	17.346	1000	6.708	450	2.321	180	1630	10.7
Citrus	12.509	1000	4.838	450	1.674	180	1630	10.7
Olive	9.455	1000					1000	6.6
Total	111.317	10180	45.654	4020	10.782	997	15197	100.0

Table 3. Allocation of water and area to crops for 160.25 and 172.75 million m^3 available water to crops under surface irrigation method.

	160.25 million m^3 available water				172.75 million m^3 available water			
	Water (Mm3)	Area (ha)	Relative Supply	% Area	Water (Mm3)	Area (ha)	Relative Supply	% Area
Sorghum	2.904	300	0.98	8.0	2.907	300	0.98	5.6
Maize	3.000	290	0.99	7.8	3.000	290	0.99	5.4
Alfalfa	3.000	510	0.58	13.6	3.000	510	0.58	9.5
Potato	2.000	350	0.81	9.4	6.000	1000	0.84	18.7
Melon	2.048	400	0.73	10.7	7.046	1000	0.97	18.7
Chilli	2.904	250	0.80	6.7	2.907	250	0.81	4.7
Tomato	4.096	640	0.68	17.1	7.093	1000	0.74	18.7
Vegetables	5.048	1000	0.90	26.7	5.546	1000	0.96	18.7
Total	25.000	3740		100.0	37.500	5350		100.0

Transactions on Ecology and the Environment vol 48, © 2001 WIT Press, www.witpress.com, ISSN 1743-3541

With a given amount of water, it is possible to increase profit by extending the cropped area resulting in deficit irrigation in some fields compared to full supply to all fields. An example of this is shown in Figure 2, where an increase of 8.69 million US$ (i.e 247.65 - 238.96) can be achieved by extending the area by 1280 ha (i.e. from 14597 to 15877) if 179.41 million m^3 of water is available.

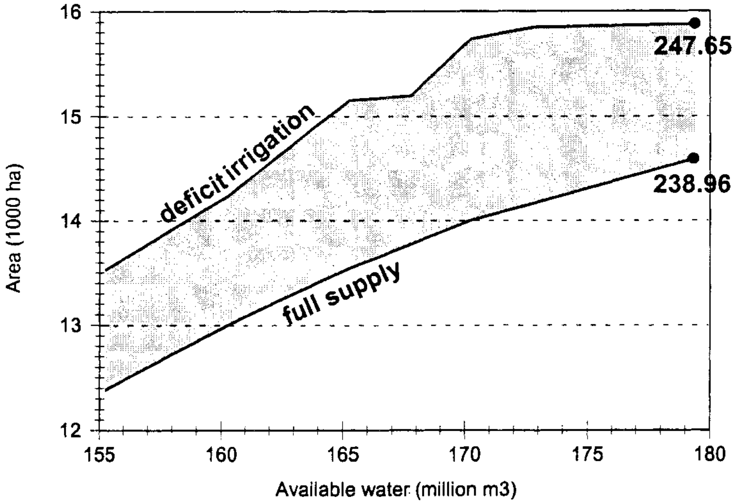

Figure2: Relation of optimal allocation of area and water for both full supply and deficit irrigation.

Similar analysis for normal and wet years can be done with the aid of the model and the results can be presented in Figures and Tables similar to the above. Such information can be very useful to regional agencies or irrigation authorities dealing with regional development of irrigation. Examples of such agency are the regional CRDA in Tunisia.

4 Conclusion

An optimisation model aimed at aiding decision making of optimal irrigated cropping pattern under water scarcity constraints was developed. The model determines the optimal distribution of areas and crops, the water allocation, and the profit. The model is relatively easy to apply, and has a great potential as a decision tool for cropping patters of a system under water scarcity constraints.

References

[1] Doorenbos, J., & Kassam, A.H. *Yield Response to water,* FAO Irrig. and Drainage Paper 33, FAO, Rome, Italy, pp. 193, 1986.

[2] Sagardoy, J.A. *Organization, operation and maintenance of irrigation schemes*. FAO, Irrig. and Drainage Paper No. 40. Rome, Italy. 165pp, 1982.

[3] Jensen, J.W. Water consumption by agricultural plants (Chapter 1). *Water Deficit and Plant Growth,* ed T. Kozlowski, Vol. 2. Academic Press: New York, pp. 1-22, 1968.
[4] Zhang, H., & Oweis, T. Water-yield relations and optimal irrigation scheduling of wheat in the Mediterranean region. *Agric. Water Manage.*, **38**: 195-211, 1999.
[5] Grahraman, B. & Sepaskhah, A.R. Use of a water deficit sensitivity index for partial irrigation scheduling of wheat and barley. *Irrig. Sci*, **18**:11-16, 1997.
[6] Trskiris, G.P. A method for applying crop sensitivity factors in irrigation scheduling. *Agric. Water Manage.*, **5**: 335-343, 1982.
[7] Kipkorir, E.C. & Raes, D. Transformation of yield response factors to sensitivity index. Unpublished, pp.4, 2000.
[8] Allen, R.G. Memorandum on application of FAO-33 yield functions. Department of Biological and Irrigation Engineering, Utah State University, Logan, Utah, USA. pp. 3, 1994.
[9] Kuo, S.F., Merkley, G.P. & Liu, C.W. Decision support for irrigation project planning using a genetic algorithm. *Agri. Water Manage.*,**45**:243-266, 2000.
[10] Allen, R.G., Pereira, L.S., Raes, D., & Smith, M. *Crop evapotranspiration (guidelines for computing crop water requirements).* FAO, Irriga. and Drainage Paper No. 56, Rome, Italy. 300pp, 1998.
[11] Raes, D. Mallants, D. & Song, Z. RAINBOW- Software package for analysing hydrological data. In W.R. Blain (ed.) *Hydraulic Engineering Software VI. Computational Mechanics Publications*, Southampton, Boston: pp. 525-534, 1996.
[12] Annuaire des statistiques agricoles 1994-1995. Ministère de l'Agriculture, DG/PDIA. October 1996.
[13] Sahli, A., Van Looij, J., Raes, D., Mechlia, N.B. Protocole de mesure de l'efficience de l'irrigation en aspersion. *Bulletin Technique N°4.* INAT, CGRE, Tunis, Tunisie, pp. 104, 2001.
[14] Sahli, A., Van Looij, J., Raes, D., Mechlia, N. B. Protocole de mesure de l'efficience de l'irrigation en goutte à goutte. *Bulletin Technique N°5.* INAT, CGRE, Tunis, Tunisie, pp. 92, 2001.
[15] Sahli, A., Van Looij, J., Raes, D., Mechlia N. B Protocole de mesure de l'efficience de l'irrigation de surface. *Bulletin Technique N°6.* INAT, CGRE, Tunis, Tunisie, pp 140, 2001.
[16] Boden, A. Fysische karakterisatie van de geirrigeerde bodems in de streek rond Tunis, Msc Thesis, KU Leuven University, Belgium, pp. 114, 2000.
[17] Journées d'Etudes "Stratégies régionales en matière d'économie d'eau" - Ministère de l'agriculture, Commissariat Régionale au Développement Agricole de l'Ariana, Ministère de l'Agriculture. November 1995.
[18] Burt, C.M., Clemmens, A.J., Bliesner, R., Merrian, J.L. & Hardy, L. *Selection of irrigation methods for agriculture*, ASCE, pp.129, 2000.
[19] Labadie, J. Generalized Dynamic Programming Package: CSUDP, Documentation and User Guide Version 3.2a. Colorado State University, Ft. Collins, pp. 77, 1999.