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Computer analysis of impact-induced vibration in single degree of freedom systems

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Abstract

With the advent of low priced personal computers and the availability of low cost mathematical analysis and spreadsheet software, the solution of advanced vibration problems may be solved by either exact or numerical solutions with relative ease. Specifically, this applies to single degree of freedom systems with or without damping and forcing functions. Vibration due to impact creates a forcing function of a fairly simple type, thus lending itself to a straightforward computer solution. This paper discusses the use of the program Mathcad to obtain exact mathematical solutions and the use of the spreadsheet program Microsoft Excel to obtain approximate numerical solutions of impact induced vibrations on single degree of freedom problems and the expansion of these methods to multiple degree of freedom problems.

1 Introduction

Frequently in the design of new buildings and the modification of existing buildings, the strength and serviceability of structural members must be checked for the effects of impact. This is particularly true in industrial type structures such as manufacturing facilities, fabrication shops and power plants. These impact loads can be generated by reciprocating machinery, punch presses, machine lockups and from hoists and cranes as well as numerous other less common sources.

Due to the complexity of impact problems, a rigorous hand analysis is usually out of the question for a practicing engineer. The time involved to perform the required calculations by hand far outweighs the benefits involved, thus approximate methods are normally used. Approximate methods, depending upon the type of load, may err on the side of conservatism or unconservatism. This provides either the possibility of over design due to overdetermination of impact load effects or a potential unsafe condition with underdesigned framing and/or connections.

With the advent of low priced personal computers and reasonably priced mathematical software and programming languages, the ability to perform sophisticated dynamic analysis is put into the hands of the practicing engineer. Sophisticated structural analysis software is also available, but at considerable expense, with some software costing several times what a complete computer system might cost. However, an engineer somewhat knowledgeable in structural dynamics and computer programming may pick up one of several software packages and prepare a program or spreadsheet to perform advanced dynamic analysis. Specifically, the analysis of single degree systems with vibration induced by impact loads.

Specific programs available to perform the type of analysis/spreadsheet programming are spreadsheet programs such as Microsoft Excel, Lotus, Quattro Pro, etc., and mathematical formula analysis programs such as Mathcad, Maple, etc. Additionally, programming languages such as Visual Basic, Visual C, Fortran, Qbasic, etc., are available for the more ambitious engineer.

2 Sources of Impact Loads

As previously stated, impact loads in industrial type buildings come from many sources. Reciprocating machinery which is run constantly induces a stepped forcing function. The force being the weight of the cam or other source of reciprocation and any weights attached thereto and moving accordingly. Punch presses apply a random impact force. The force is equal to the counterweights used as well as adjacent elements that move in unison with the punch. Many times this information is available from the equipment manufacturer. Equipment lockups also provide an impulsive load of most likely, an unknown magnitude. These loads could possibly occur due to a thrown turbine blade binding a turbine, or other similar types of potential situations. Impact loads from cranes and hoists can be due to sudden starts and stops of bridge cranes, cable slippage under load and so on. The forcing functions of these type of impacts must be considered on a case by case basis.

3 Single Degree of Freedom Systems

A single degree of freedom is one with a single displacement coordinate, i.e., movement along a single axis. Almost every structural element in a typical industrial building can be idealized as a single degree of freedom system. Exceptions being trusses, continuous beams and systems of flexible elements. With the proper assumptions and boundary values, even these elements may be simplified to be modeled as single degree of freedom systems.

Common types of simple single degree of freedom systems are one-way spanning plates such as concrete slabs, steel floor plates and metal bar grating. Single span and cantilevered beams with either hinged or fixed supports are very common single degree of freedom systems as well as are columns and strut members analyzed on a local basis.

Analysis of most single degree of freedom systems is similar depending upon the method of load application. The two most common impact loading conditions are axial and flexural. A flexural impact load is most likely to occur in one way plates and beams while an axial impact load is most likely to occur in columns and struts, although this is not necessarily always the case. Formulations for analysis of both of these types of impact loads are readily available.

When using mathematical analysis software these formulations may be directly input using the original variables and actual mathematical symbols involved. This makes checking and validation of the program simple and the possibility of errors occurring unlikely. In spreadsheet programming the application of the formulations is somewhat more difficult due to conversions of variable names to those allowed in the program, the lack of calculus functions, etc. However with the cell replication capabilities of spreadsheets, the ability to create numerical method models becomes very simple. Also, these equations will calculate much faster than in mathematical analysis software. Lookup tables simplify the use of material data and member section properties so that only a material name, such as STEEL, and a section name, such as 80mm DIAMETER PIPE, need to be specified.

4 Program Setup

Programs should be set up in a standard format whether they be spreadsheet, mathematical analysis or stand alone software. This not only aids in debugging, but allows other users to easily understand the program and to make modifications if they become necessary. Many formats are possible. One which works well is as follows:

1. Program Title/Heading Information

2. Constants/Criteria — Most common values may be set as the default and modified by the end user.

3. Formulations/Variables - Calculations performed by the program to obtain a conclusion or end result.

4. Conclusion/End Result

5. Secondary Equations - Calculations to determine universal constants and other values by commonly known formulas and spreadsheet lookup tables.

6. Comments - Program description and instructions.

Items 5 and 6 may be left out of the main program flow so that they don't slow down an experienced user employing the program, while still being available for the inexperienced user.

5 Sample Programs

Rather than perform some rigorous analysis or derive complicated equations, it was thought wisest to provide examples of several sample programs. These programs are in both mathematical software and spreadsheet formats. The subject of these programs is Duhamel's integral for impulsive loads or which may be written:

$$\frac{1}{m*\omega_D}\int_0^t F(\tau)*e^{\xi*\omega*(t-\tau)}*\sin[\omega_D*(t-\tau)]d\tau \qquad (1)$$

Figure 1 illustrates a <u>Mathcad</u> worksheet to evaluate the integral by mathematical analysis. Based upon user input parameters, defining the beam geometry, physical constants and a forcing function, deflection and velocity values at specific points are obtained as well as a graph of deflections for a range of time values.

Figure 2 illustrates an <u>Excel</u> spreadsheet to evaluate the integral by numerical methods. User values are similar to those for <u>Mathcad</u>, although somewhat simplified.

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Evaluation of Impact Load on a Damped System

$F(\tau) := -667 \cdot N$ Forcing Function $k := 875590 \cdot \frac{N}{m}$ Beam Stiffness

$m := 6.8 \cdot kg$ Beam Mass $\omega := \sqrt{\frac{k}{m}}$ Undamped Frequency $\omega = 359 \cdot sec^{-1}$

$\xi := 0.30$ Damping Ratio

$\omega_D := \omega \cdot \sqrt{1 - \xi^2}$ Damped Frequency $\omega_D = 342 \cdot sec^{-1}$

Duhamel's Integral for a Damped Single Degree of Freedom System:

$$y(t) := \frac{1}{m \cdot \omega_D} \cdot \left[\int_0^t F(\tau) \cdot e^{-\xi \cdot \omega \cdot (t - \tau)} \cdot \sin\left[\omega_D \cdot (t - \tau)\right] d\tau \right]$$

$$v(t) := e^{-\xi \cdot \omega \cdot t} \cdot \frac{F(t)}{m \cdot \omega_D} \cdot \sin\left(\omega_D \cdot t\right)$$

Evaluation for a Specific Value:

$y(.02 \cdot sec) = -0.67 \cdot mm$ $v(.0011 \cdot sec) = -93.599 \cdot \frac{mm}{sec}$

Evaluation for a Range of Values:

$t := .000, .002 .. .2$

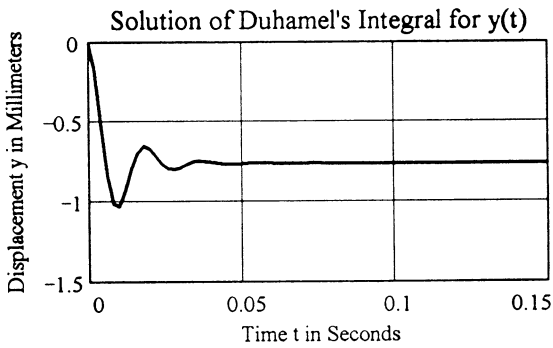

Figure 1: Mathematical analysis model for solution of Duhamel's integral for a constant impulsive load

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Evaluation of Impact Load on a Damped System

F(tau)= -667 N   k= 876 kN/m
m= 6.8 kg   w= 359 sec
xi= 0.3   wD= 342 sec

I1=exp(xi*w*tau)/((xi*w)^2+wD^2)*(xi*w*cos(wD*tau)+wD*sin(wD*tau))

I2=exp(xi*w*tau)/((xi*w)^2+wD^2)*(xi*w*sin(wD*tau)-wD*cos(wD*tau))

y(t)=exp(-xi*w*t)*(Ad(t)*sin(wD*t)-Bd(t)*cos(wD*t))

tau	I11	I12	I21	I12	Ad(t)	Bd(t)	y(t)
0.000	0.000	0.000	0.00	0.00	0	0	0.000
0.001	0.001	0.002	0.00	0.00	-1	0	-0.045
0.002	0.002	0.003	0.00	0.00	-1	-1	-0.164
0.003	0.003	0.004	0.00	0.00	-2	-1	-0.328
0.004	0.004	0.004	0.00	0.00	-2	-2	-0.510
0.005	0.004	0.004	0.00	0.00	-2	-3	-0.686
0.006	0.004	0.004	0.00	0.00	-2	-4	-0.836
0.117	541	378	506	733	-252364	-488752	-0.761
0.118	378	123	733	910	-81994	-607083	-0.761
0.119	123	-211	910	1001	140977	-667515	-0.761
0.120	-211	-596	1001	971	397478	-647540	-0.761

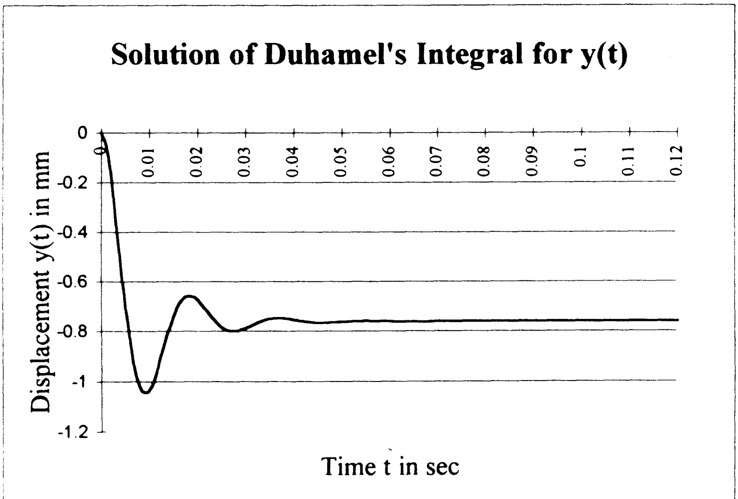

Figure 2: Spreadsheet analysis of Duhamel's integral for a constant impulsive load.

In comparing the two models it is obvious that the Mathcad model is superior in readability, ease of equation checking and reliability of units. The units in the Excel model are part of the formatting only and no not carry through in the numerical calculations. Formulations used in the spreadsheet are typed out as text for checking purposes and then copied to cells and converted to numeric formulas for use in the numerical analysis. The accuracy of the results of each are clear to see by comparing the displacement graphs, which are nearly identical. The spreadsheet program also provides results in a fraction of the time of the mathematical analysis program.

It should be noted that programming the Mathcad model was somewhat tedious and took approximately twice as long as the programming of similar items in the Excel spreadsheet. This leads to the notion that for reliable results for a single problem, spreadsheet analysis is most efficient, while for a generic application to be used over and over, a mathematical analysis model will provide the most efficiency.

While the program models presented here demonstrate the ability to analyze one degree of freedom systems, it is possible to make modifications and perform additional programming to create models for analysis of more degrees of freedom. Development of these models is beyond the scope of this paper, however, it is only a matter of providing a system of equations using the same analysis as presented previously. This can be accomplished with matrix operations in the mathematical analysis software and with multilayered worksheets using the spreadsheet approach.

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