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The mechanics of masonry stairs

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Abstract

A simple statical analysis is made of both straight and curving ("geometrical") flights of stairs. The basic structural action is twist of individual treads, leading to shear stresses in the masonry; such stresses are more harmful than direct compression.

Introduction

Figure 1 gives a plan view of a simple masonry stair built against two walls of the enclosing chamber. Each of the treads (of which 4 are shown in the sketch of Fig. 2) consists of a slab of masonry ℓ x b x d (Fig. 3); one end of each tread is built in to the wall, while the other end is free. Thus the bottom tread in Fig. 2 rests on the ground; the front edge of the next tread rests on the rear edge of the bottom tread, and one end is supported also by the wall; and so on for successive treads. In practice, the treads may be notched together as sketched in Fig. 4(a); a common form of construction involves the cutting away of the soffit of each tread, Fig. 4(b), to give a smooth undersurface for the stair and also to lighten the overall weight. Such notching is of course essential to avoid the infinite stresses which would be generated in theory by a line contact; they also provide a very necessary margin of safety against defects in the stair. The basic mechanics of the stair as a whole may be examined, however, by reference to the idealised representation of Figs. 2 and 3.

Live Load

The fundamental problem is to determine how the forces generated by dead and live load are distributed through the stair system to the walls and to the ground. A start may be made by considering the effect of a single load P placed at the centre of the last (top) tread in Fig. 2 (assuming for the moment that the stair stops at this point). It is clear that equilibrium of the top tread could be established by the forces shown in Fig. 5(a); the right hand support force of $\frac{1}{2}P$ is provided by the wall into which the tread is built, while the left hand force of ½*P* is provided by support given by the tread below. It may be imagined that in fact these idealised concentrated support forces will be replaced by some distributed system as sketched in Fig. 5(b). Whatever the distribution, the total upward support must have the value *P*, and other conditions of moment equilibrium must also be satisfied.

The actual distribution of the support forces is essentially unknowable. If the exact geometry of each tread were known, together with its elastic properties; if the stair had been perfectly constructed, or alternatively surveyed to find its actual state; if the elasticity and other properties of the support walls had been ascertained; if, in short, a complete knowledge of the stair were available, then, in theory, a laborious calculation would yield the values of the forces in the stair. Even so, such a calculation would be valid for only a limited period; small accidental settlements, for example, can lead to marked changes in internal force systems. All this is well known to structural analysts (although it is sometimes hidden from view behind computer packages), and leads to the adoption of simple equilibrium systems such as that shown in Fig. 5(a). Such systems, involving point forces, give a fundamental insight into overall structural behaviour; it must always be remembered that the point forces will be replaced in practice by some distributed system, as for example that of Fig. 5(b).

Equilibrium of the top tread can be achieved, then, by the forces of Fig. 5(a). The rest of the stair will then experience a loading force of ½*P* as shown in Fig. 6. To establish a plausible equilibrium system for the second tread, it is helpful to consider a very small movement of that tread. If, for example, the supported (shaded) end of the tread were slightly loose in the wall, then it may be imagined that tread 2 would rotate slightly about the support line on tread 3, so that edge *AB* would deflect slightly downwards under the action of the load ½*P*. Rotation would be limited by the fact that the lower corner *B*′ would come into contact with support from the wall, so that a vertical propping force would be generated at *B*′.

Such ideas of deformation need not be examined analytically, but they lead at once to the (essentially correct) notion that each tread could be maintained in equilibrium by forces acting at the four corners *A*, *B*, *C* and *D*. Such a system is statically determinate; the forces acting at corners *B*, *C* and *D* to support the load ½*P* at *A* can be found at once, and they are shown in Fig. 7. It will be seen that up and down forces of ½*P* are engendered at the encastred end of tread 2; correspondingly, the free end of tread 2 is supported by a force of ½*P* from tread 3. The tread is attempting to twist in the wall, and is restrained by the torque corresponding to the up and down forces of ½*P*. (The consequences of this torque will be examined later.) It should be emphasized that the treads are not actually loose in the wall, and that stresses are low so that the stair as a whole is virtually rigid; nevertheless, the tendency to twist is there, and the torques will actually arise.

It was seen that tread 3 provides a support to tread 2; tread 3 is acted upon in turn at its free edge by a force of ½*P*. The equilibrium analysis of tread 3 is identical with that of tread 2, and so for tread 4; the load of *P* placed on the top tread is carried, according to this simple analysis, by a load of ½*P* transmitted from tread to tread at their free edges, with each tread being subjected to a torque as it attempts to twist in the wall. It may be imagined that the forces will die away down the stair; each tread has been thought of as being simply supported in the wall, but the actual encastrement will allow the development of some bending action, so that the treads can act to some extent as cantilevers. Such cantilever action will in fact be weak, and the simple analysis shows that bending is not needed for the stability of the stair as a whole.

It may be noted that each tread of the stair in Fig. 7 is subjected to a pure torque of value

$$T = \tfrac{1}{2}Pb \qquad (1)$$

where the width of the tread is b (Fig. 3).

Live Load at Edge of Stair

If the live load *P* is placed at the free edge rather than the centre of a tread, the force transmitted from tread to tread has value *P*, rather than the ½*P* shown in Fig. 7. Thus the resulting torque on each tread would have double the value of equation (1).

Dead Load

The forces generated by the self-weight of the stair may be found immediately from the above analysis. The stair is sketched in Fig. 8, where the weight of each tread is *W* (shown for convenience as a point load). The forces on tread 2 are merely the forces of treads 1 and 2 in Fig. 7 superimposed, and so on down the stair to the general tread *n*. Analysis of this general tread, Fig. 9, shows that the structural action at the centre of the tread can be represented by a torque *T* and a bending moment *M*, where

$$\left.\begin{aligned} T &= \left(\frac{2n-1}{2}\right) W\frac{b}{2} \\ M &= \frac{W\ell}{8}\ . \end{aligned}\right\} \qquad (2)$$

Thus the torque on a tread increases down the stair, while each tread is subjected to the same (small) bending moment *M* (as if it were a simply supported beam).

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The Quarter Landing

Figure 1 shows a quarter landing. Figure 10 sketches this quarter landing together with the treads just above and below; a live load P is being transmitted down the upper flight. The outermost corner of the quarter landing is subjected to a point load ½P; effectively, the quarter landing has to support only its own weight, and otherwise acts as a "newel" to transmit loads from the upper flight direct to the lower.

The Geometrical Stair

Figure 11 shows the plan view of a stair built within a circular chamber; an idealised tread is shown in Fig. 12. The mechanics of such stairs follows directly from the analysis given above. For example, Fig. 13 shows the top tread subjected to a live load P (cf. Fig. 7); the load of ½P is transmitted from tread to tread down the free edges, as before. Because of the taper of the tread, however, the torque exerted by the wall is reduced by a factor β, where β is defined in the plan of Fig. 11.

The value of ½P depends, of course, on the exact location of the live load. When the case of dead load is considered (cf. Fig. 8), the weights W of each tread will act through the centre of gravity of the tread rather than the geometrical centre, so that the value of ½W transmitted by each successive tread will be in reality slightly less. However, the difference is small, and the expressions of equations (2) will give good indications of the magnitudes of the structural forces (with the value of the torque T reduced by the factor β as appropriate).

It may be noted from Fig. 11 that for $\beta = 0$ the stair becomes a turret stair with a central newel; from Fig. 13 the loads are transmitted down the newel without torque on the treads.

Some Stress Calculations

A bending moment M acting on a structural member of cross-sectional dimensions b x d (Fig. 3) will produce a maximum bending stress

$$\sigma = \frac{6M}{bd^2} \, . \qquad (3)$$

Timoshenko (Theory of Elasticity, McGraw-Hill, 1934) discusses the torsion of a rectangular section by a torque T, and gives the expression for maximum shearing stress

$$\tau = \frac{1}{k_2}\frac{T}{bd^2} \, . \qquad (4)$$

The numerical constant k_2 is a function of the ratio b/d, and is tabulated by Timoshenko.

For the numerical calculations, the dimensions $\ell \times b \times d$ of each tread will be taken as 1000 x 300 x 150. The live load P will be taken as 800 N (corresponding to a man of mass 80 kg); this same figure of 800 N corresponds roughly to the weight of a single tread, and will be taken as the value of W.

Thus from equations (2) the value of bending moment is determined as 100 Nm, and equation (3) then gives the bending stress as 0.09 N/mm^2, which is a negligible value.

From the first of equations (2), the value of the torque T is essentially $\frac{1}{2}Wb$ per tread (for a flight of n treads, the torque on the lowest tread is $(n - \frac{1}{2})$ times this value). For the basic torque $\frac{1}{2}Wb$, equation (4) gives a stress

$$\tau = \frac{1}{2k_2}\frac{W}{d^2} \tag{5}$$

and for $b/d = 2$, Timoshenko gives $k_2 = 0.246$. Thus for $W = 800$ N and $d = 150$ mm, equation (5) gives the shearing stress per tread to be 0.0723 N/mm^2. Thus the shearing stress at the bottom of a flight of 20 steps will be 19.5 times this value, i.e. 1.4 N/mm^2.

A large geometrical stair of 100 steps, and having $\beta = 2/3$ say, would have a corresponding maximum shearing stress of 4.8 N/mm^2. A shearing stress on one plane of a material will engender tensile tresses on another plane, and tensile stresses in masonry of the order of 5 N/mm^2 are high.

Conclusion

It is likely that, in practice, a stair will transmit forces both vertically between treads, as in Fig. 7, and also horizontally if the steps are notched together. However, it is difficult to imagine that individual treads will be free of torque; hence shear stresses, and corresponding tensile stresses, will be present in the stone. Modest flights will, however, not be subject to excessively high stress levels, either from dead or from live loads. Individual treads in straight flights and in geometrical stairs behave in much the same way. There are no particular problems arising from quarter (or half) landings used in straight flights.

There is the possibility for very long flights of stairs that the total dead weight will lead to high torques on the lowest treads. If, in addition, every other step supports a person at the centre of the stair, then dead-load stresses will be increased by perhaps 50 per cent; if the people on the stair all move to the free edge, the dead-load stresses may well be doubled. These stresses are essentially tensile, and can lead to fracture of the stone treads at much lower stress levels than the corresponding crushing strengths of the material.

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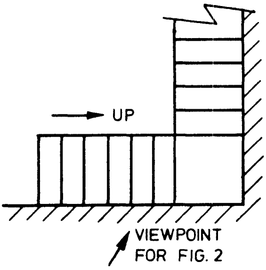

FIG. 1

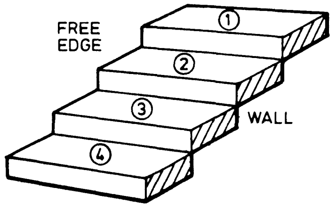

FIG. 2

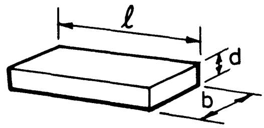

FIG. 3

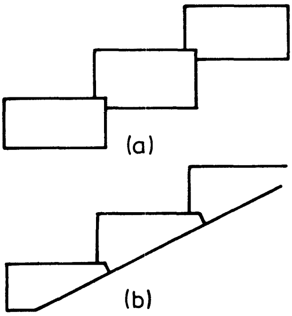

FIG. 4

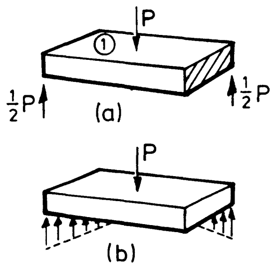

FIG. 5

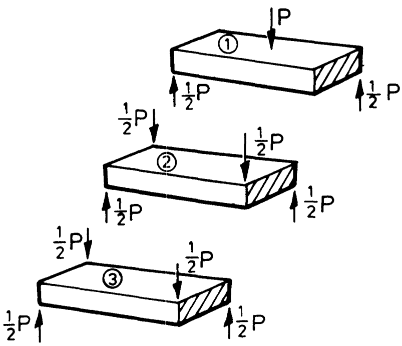

FIG. 7

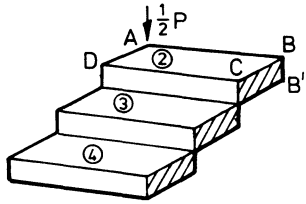

FIG. 6

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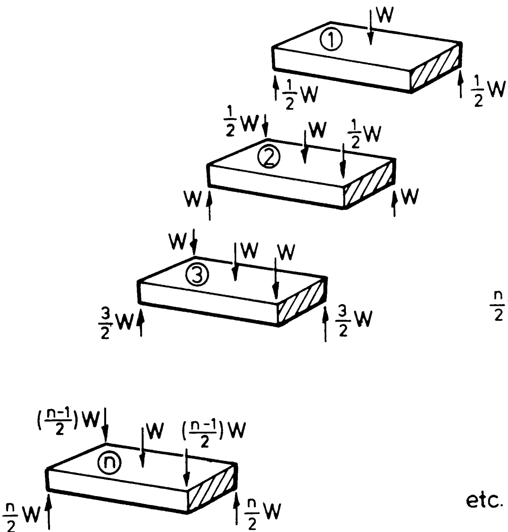

FIG. 8

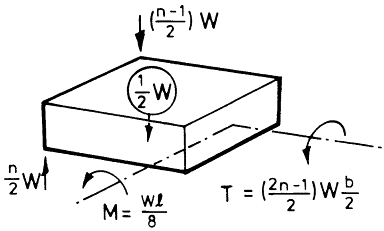

FIG. 9

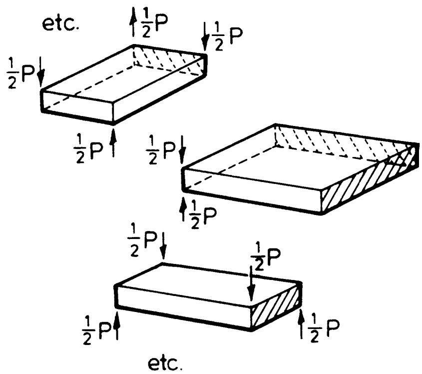

FIG. 10

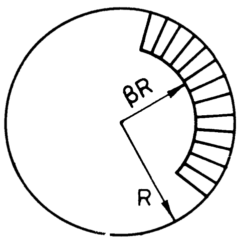

FIG. 11

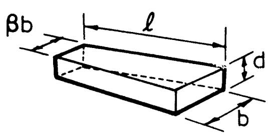

FIG. 12

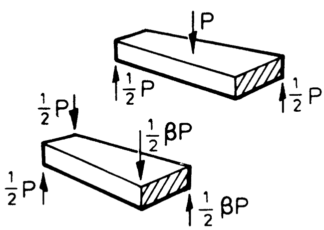

FIG. 13