



Applications of quasi-optimizing control method to structural response control system for seismic excitations

Y. Inoue, Y. Mukai & E. Tachibana

Department of Architectural Engineering, Graduate School of Engineering, Osaka University, 2-1 Yamada-Oka, Suita, Osaka 565-0871, Japan

Email: yinoue@arch.eng.osaka-u.ac.jp

Abstract

A "quasi-optimizing control" method is proposed as a practical programming algorithm for aseismic response control systems of building structures. By limiting control forces of each control device into a few kinds of trial control forces, the all values of the set of "digital-index functions" for all combinations of predicted structural response are computed on step-by-step, and the most adequate control forces are determined as the quasi-optimal control forces by finding out the set that makes the digital-index function to be minimum. A three-degrees-of-freedom system arranged an active brace on each story is adopted for investigating this control algorithm, through numerical simulations and experimental tests. As a result, it assured that remarkable control effects were obtained by introducing the quasi-optimizing control method for structural vibrations caused by seismic excitations.

1 Introduction

On the architectural engineering fields, remarkable developments of structural response control technology have been achieved. Over four hundred base-isolated buildings and over fifty vibration-controlled buildings had been constructed in the last one decade (e.g. Inoue[1]). In general, for relatively rigid buildings, base-isolating systems are adopted to prevent resonance of structural vibrations with earthquake excitations. On the other hands, for high-rise building structures having large aspect ratio, active or passive response control systems are used for reducing the wind-induced



structural vibrations. To reduce structural vibrations caused by earthquakes, vibration control systems have not been so popularized as much as the wind-resistant response control systems. Although there are a few practical buildings introduced active vibration control systems for earthquakes, those control devices are almost designed as to stop their operation under strong motions. In order to establish the aseismic control system against strong ground motions, several practical problems, such as capacities of control devices or optimality of control performances and so on, which should be solved have been still remained. In this paper, from the emphasis that the aseismic active control algorithm should be simple and easy to apply for practical use, a new active control algorithm named as a quasi-optimizing control method is proposed. Through some case studies with numerical simulations and experimental tests, the effectiveness of this active control method are investigated.

2 Quasi-Optimizing Control Method

The "instantaneous optimal control" method (e.g. Yang[2]) is very attractive for a control algorithm on the cases that earthquake motions are supposed as external excitations, since it can be considered step-by-step optimality of control system performances by introducing a time-dependent index function. However, in this method, it is very difficult to deal with the problems for capacities of control devices. In order to overcome such a practical problem, the quasi-optimizing control method has been proposed (e.g. Tachibana[3]).

When considering an n degrees-of-freedom structural system equipped with r control devices, let $\{X_s\}$ denote a state vector having $2 \times n$ components at the time of $t_s = \Delta t \times s$ (where Δt means a discrete control time interval and s is an integer number), and let $\{u_s\}$ denote a control force vector having r components. The equation of motion of this system can be described as a following expression by introducing the state vector.

$$\{\dot{X}_s\} = [A]\{X_s\} + [B]\{u_s\} + [D]\{w_s\}, \quad (1)$$

in which $[A]$, $[B]$ and $[D]$ are matrices defined from mass, damping and stiffness of the system, and arrangements of control devices. When supposing the disturbance vector $\{w_s\}$ as earthquake motions, it can be expressed by using the ground motion $\ddot{x}_{0,s}$.

Let control force of each control device assume to limit into a few kinds of trial control forces such as $\langle \bar{u}_j \rangle$ for the j -th device, where the notation $\langle \cdot \rangle$ represents a "set". For instance, in the case that those control



$$\begin{aligned} < \bar{u}_j > = \{ \bar{u}_j \mid \bar{u}_{j,1}, \bar{u}_{j,2}, \dots, \bar{u}_{j,N} \} \\ (u_{min,j} \leq \bar{u}_{j,a} \leq u_{max,j}, \quad a = 1, 2, \dots, N). \end{aligned} \tag{2}$$

The 'digital-index function' J_s is defined as,

$$J_s = \{ \dot{x}_{s+1} \}^T [Q_d] \{ \dot{x}_{s+1} \} + \{ \dot{x}_{s+1} \}^T [Q_v] \{ \dot{x}_{s+1} \}, \tag{3}$$

where $\{ \dot{x}_s \}$ and $\{ \ddot{x}_s \}$ represent the displacement and velocity vector at the time instant t_s , respectively, $[Q_d]$ and $[Q_v]$ mean those weight matrices. By limiting all trial control forces within the capacity of each device, since no spillover of control forces exists in the quasi-optimizing control method, the item of the index for the control forces is excluded in this digital-index function. When $\{ \dot{x}_s \}$ and $\{ \ddot{x}_s \}$ will be calculated from the following transition equation by selecting each set of $\{ \bar{u}_s \}$,

$$\begin{Bmatrix} \dot{x}_{s+1} \\ \ddot{x}_{s+1} \end{Bmatrix} = [A] \begin{Bmatrix} \dot{x}_s \\ \ddot{x}_s \end{Bmatrix} + [B] \begin{Bmatrix} w_{s+1} \\ w_s \end{Bmatrix} + [D] \begin{Bmatrix} \bar{u}_{s+1} \\ \bar{u}_s \end{Bmatrix}, \tag{4}$$

the values of J_s for all combinations of $< \bar{u}_j >$ can be computed. In which $[A]$, $[B]$ and $[D]$ are matrices defined from mass, damping and stiffness of the system, and arrangements of control devices.

The most adequate control forces are determined as real control forces by finding out the set that makes the digital-index function J_s to be minimum. Those digital-optimizing procedures can be written as following flows:

- 1) Set the basic coefficients of structure (Mass matrix $[M]$, Damping matrix $[C]$ and Stiffness matrix $[K]$).
- 2) Set the trial control forces $< \bar{u}_j >$, in which the all component of $< \bar{u}_j >$ should be selected as to be no larger than $u_{max,j}$ and no less than $u_{min,j}$ in Exp. (2), and weight matrices $[Q_d]$ and $[Q_v]$ in Exp.(3)
- 3) Get $\{ \dot{x}_s \}$ and $\{ \ddot{x}_s \}$, and ground acceleration $\ddot{x}_{0,s}$ from sensors at the time $t_s = \Delta t \times s$.
- 4) Predict controlled response $\{ \dot{x}_{s+1} \}$ and $\{ \ddot{x}_{s+1} \}$ at the time t_{s+1} for all combination of $< \bar{u}_j >$ from Exp. (4).
- 5) Calculate the all values of J_s for all $\{ \dot{x}_{s+1} \}$ and $\{ \ddot{x}_{s+1} \}$ by Exp.(4).
- 6) Determine the set of control forces $\{ \bar{u}^*_s \}$, which makes J_s minimum, as the quasi-optimal control force vector.

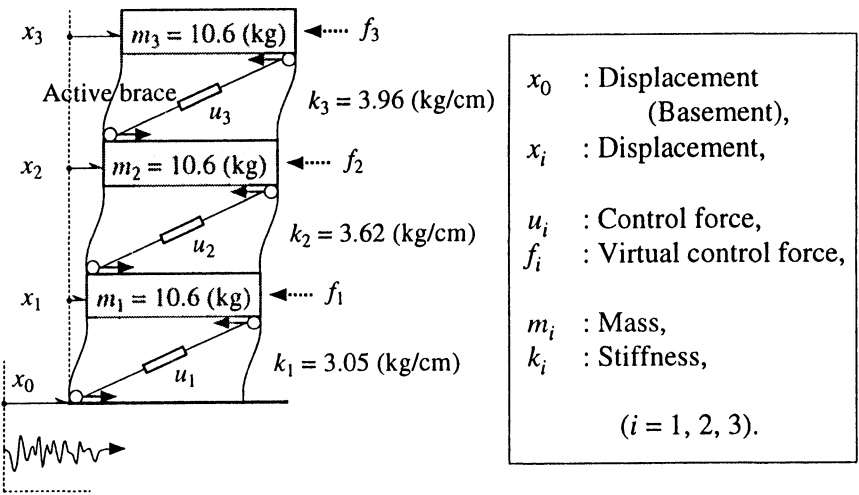


Figure 1. Multi-located aseismic active brace control system.

3 Numerical Simulations and Experimental Tests

In order to assure the effectiveness of the quasi-optimizing control method, numerical simulations and experimental tests are executed. A three degrees-of-freedom system which is installed three active brace devices in every stories is adopted (as shown in Fig.1). Structural properties of this testing system are shown in Fig.1, and those constants are approximately adjusted to the survey values of the experimental model (e.g. Inoue[4]). El Centro (1940) NS is used as input ground motions, and this wave record is scaled down to the maximum acceleration amplitude of 30 cm/s² according to the scale of the structural model. In the following case studies, the set of trial control forces is defined as a same-step type as follows:

$$\begin{aligned} \bar{u}_{j,a} &= u_{max,j} \times (a - L) / L, \quad a = 0, 1, \dots, N, \\ < \bar{u}_{j,a} > = < \bar{u}_{j,0}, \dots, \bar{u}_{j,2L} > = < -u_{max,j}, \dots, 0, \dots, u_{max,j} >, \end{aligned} \quad (5)$$

in which L is an integer number.

To find out the suitable number of trial control forces and capacities of control forces, control performances by introducing the quasi-optimizing control methods are investigated by numerical simulations. The reductions of response and the required control forces are shown in Figs.2. In those



As the estimation indexes of the reduced response and the required control forces, a DRMS and a FRMS are defined as follows:

$$\begin{aligned} \text{DRMS} (i) &= x_{rms} (i) / x'_{rms} (i) , \\ \text{FRMS} (i) &= u_{rms} (i) / (k_i \times 1) , \quad i = 1, 2, 3, \end{aligned} \quad (6)$$

in which $x_{rms} (i)$ and $x'_{rms} (i)$ mean the RMS (root-mean-square) values of the i -th story's controlled and non-controlled displacements, respectively. FRMS (i) is defined as the ratio of the RMS value of the i -th story's control force $u_{rms} (i)$ divided by the i -th story's resistant force for the unit deformation of 1 cm. As seen in Figs.2, it is assured that effective reductions of response are gained by selecting only a few kind of trial control forces, when the enough capacity of control force is introduced into each device.

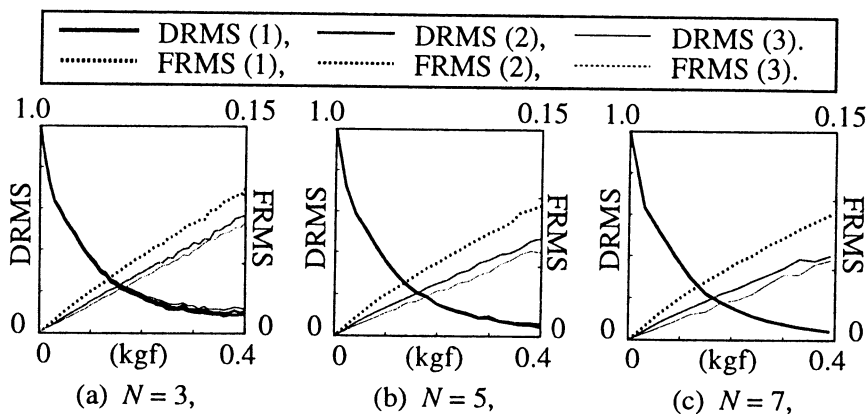


Figure 2. Control performances for numbers of trial control forces.

It is very interested in the investigations about "where the acting point of each trial control force \bar{u}_j is arranged apparently". In general, the trial control force \bar{u}_j is regarded as the each control force u_j which acts as the inter-story force in the case which is introduced the active brace system (as shown in Fig.1), namely,

$$\{ \bar{u} \} = \{ u \}. \quad (7)$$

When the trial control force \bar{u}_j is supposed as the body force f_j acting on the each mass,



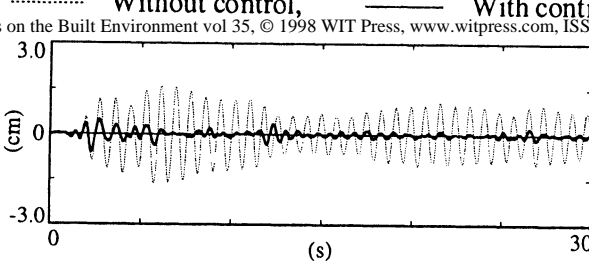
in which $[U]$ means the position matrix of the control forces.

In the next case studies, those two kinds of control force systems of the *Case-I* (named as a "dependent control force system") defined as the Exp. (7) and the *Case-II* (named as an "independent control force system") defined as the Exp. (8) are investigated. In the *Case-II*, it should be paid attentions that the all discrete component of the control forces of the active braces $\{u\}$ (which are determined from the all components of the trial control forces $\{\bar{u}\}$ by the transformation $[U^{-1}]$) should not be over capacities of those devices in the quasi-optimizing control method. Since the transformation $[U^{-1}]$ is defined as,

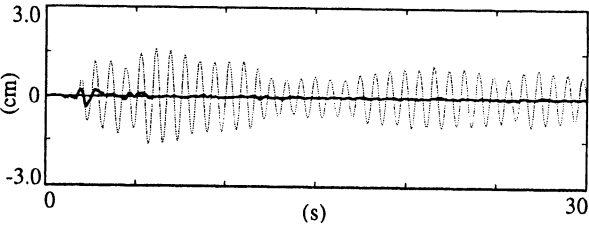
$$[U^{-1}] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad (9)$$

the capacity of the control force of the lowest story's active brace in the *Case-II* is the three times as much as that in the *Case-I*. So, it is reasonable that the capacities of the trial control forces $\{\bar{u}\}$ should be selected as to be 3 : 1 between the *Case-I* and the *Case-II*. By selecting the sets of trial control forces $\langle \bar{u}_j \rangle$ for the *Case-I* and the *Case-II* as 5 kinds of set which are $\langle -0.36, -0.18, 0, 0.18, 0.36 \text{ (kgf)} \rangle$ and $\langle -0.12, -0.06, 0, 0.06, 0.12 \text{ (kgf)} \rangle$, respectively, comparing studies are executed through numerical simulations and experimental tests. The weight matrices of response in the Exp. (3) are set as $\text{diag.}[Q_d] = 100$ and $\text{diag.}[Q_v] = 1$.

Displacements of the top floor are shown in Figs.3 and Figs.4. Figs.3 show the numerical results. Figs.4 show the experimental results. In those figures, the results of the dependent control force system (*Case-I*) are shown in (a), and the results of the independent control force system (*Case-II*) are shown in (b). By comparing those control force systems, the more effective reductions of response can be observed in the *Case-II*, on the both of numerical and experimental results. The control effects which are observed by experimental tests are less than those by numerical simulations. It is regarded that one of reasons for those results depend on the time delay of the practical control system. But, it seems that those control loss are quite little and not so significant, and that good control performances can be operated. Moreover, in the numerical results, effective reductions for the maximum displacements on the first shock motion of earthquake excitations are observed in the *Case-II*.

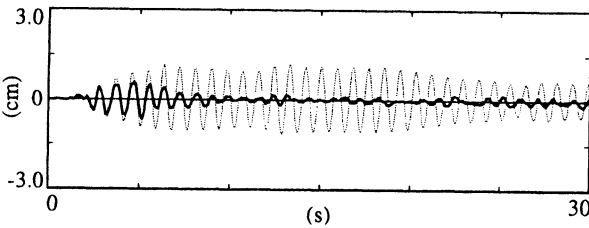


(a) *Case-I* (Dependent control force system),

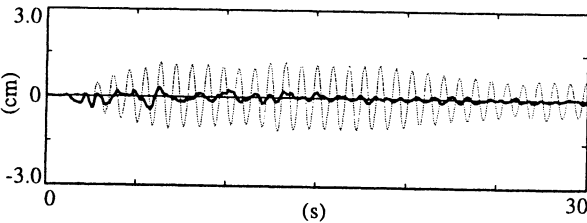


(a) *Case-II* (Independent control force system),

Figure 3. Displacements of the top floor (Numerical results).



(a) *Case-I* (Dependent control force system),



(a) *Case-II* (Independent control force system),

Figure 4. Displacements of the top floor (Experimental results).

4 Sensitivity Analyses for the Control System

System sensitivity for modeling errors on the control systems which is introduced the quasi-optimizing control method are investigated in the



lowing studies. The three degrees-of-freedom system which is shown in Fig.5 are used for those investigations. In this study, it is assumed that "distributions of structural mass and the first natural period are exactly observable", so that, no modeling error of mass constants exists on the internal model of the control system. Stiffness distributions are assumed as follow:

$$k_i = k_1 + \alpha \cdot (i - 1), \quad i = 1, 2, 3, \quad (10)$$

in which α means coefficient of stiffness distributions, according to the external model or the internal model of the control system, α is replaced on α_E or α_I in Exp.(10). Stiffness k_i of each story is determined by using the known quantity of the first natural period T_1 of the structural model (which is normally identified via spectrum analysis for free vibration tests). Stiffness distributions according to α in the case of $T_1 = 0.8$ s are shown in Fig.5.

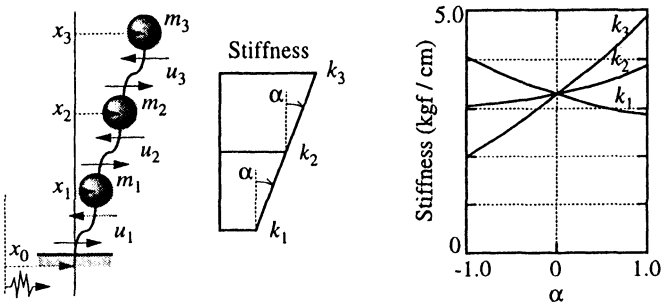


Figure 5. Structural model and stiffness distributions.

Comparative studies by using various combinations of the external models (the exact models which are provided by α_E) and the internal models (the identified models which are provided by α_I) are executed. By using three kinds of the external model ($\alpha_E = -1.0, 0, 1.0$), influences caused from modeling errors of stiffness distributions are investigated.

Figs.6 show the DRMS for each combination of the external and the internal models. The three kinds of the internal model ($\alpha_I = -1.0, 0, 1.0$) are used on each external model. Those figures are arranged to the matrix style form, on the meshed parts mean the agreement between the external and the internal models. So, it means that, on the left side of this part, stiffness of lower story are identified larger than the exact values, and on the right side of this part, stiffness of lower story are identified smaller than the exact values. By comparing those figures, it is assured that, in the case which stiffness of lower story on the internal model are identified smaller,



Figs.7 show the DRMS for three kinds of the control system which are introduced different capacities of control forces U_{max} ($U_{max} = 0.1, 0.2, 0.3$ kgf). For the three kinds of the external model ($\alpha_l = -1.0, 0, 1.0$), various internal models which have the values of α_l from -1.0 to 1.0 are examined. In those figures, marked points by filled circle (●) mean the agreement between the external and the internal models. By considering those figures, it is found that, on the left side from agreement point, the control system is insensitive for the modeling errors of stiffness, however, on the right side from agreement point, the modeling errors of stiffness affect to control performances sensitively.

5 Concluding Remarks

A practical aseismic active vibration control algorithms which is named a quasi-optimizing control method is presented. Through numerical simulations and experimental tests, effectiveness of this control method are assured. Moreover, system sensitivity for modeling errors, which is in the cases by introducing the this control method, are evaluated. Various cases of modeling errors are investigated from numerical simulations. As a result, it appeared that, when the models of lower stories' stiffness are larger than the real model, control effects are insensitive, so that, effective response reductions may be preserved.

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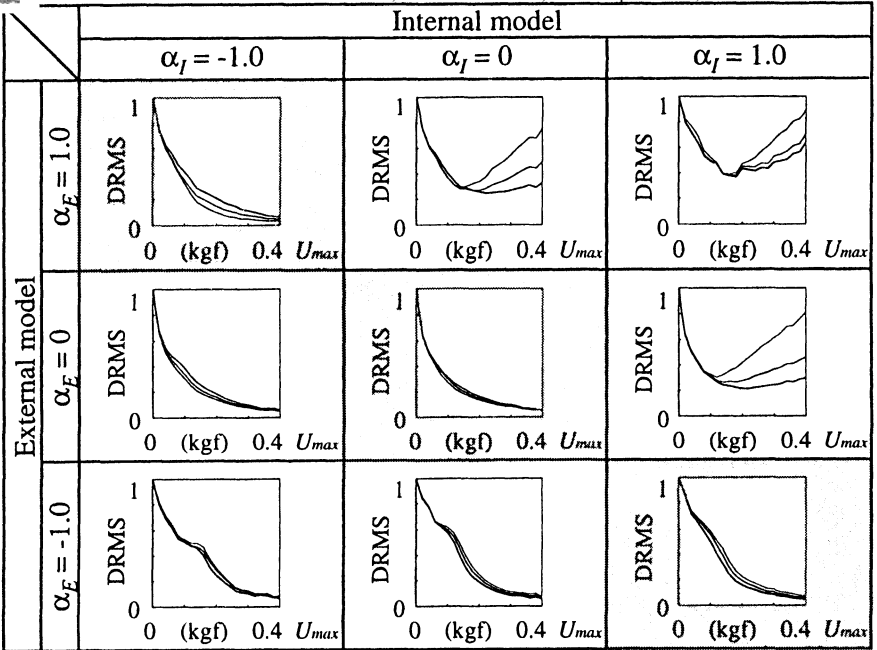


Figure 6. Sensitivity for modeling errors (fluctuations in U_{max}).

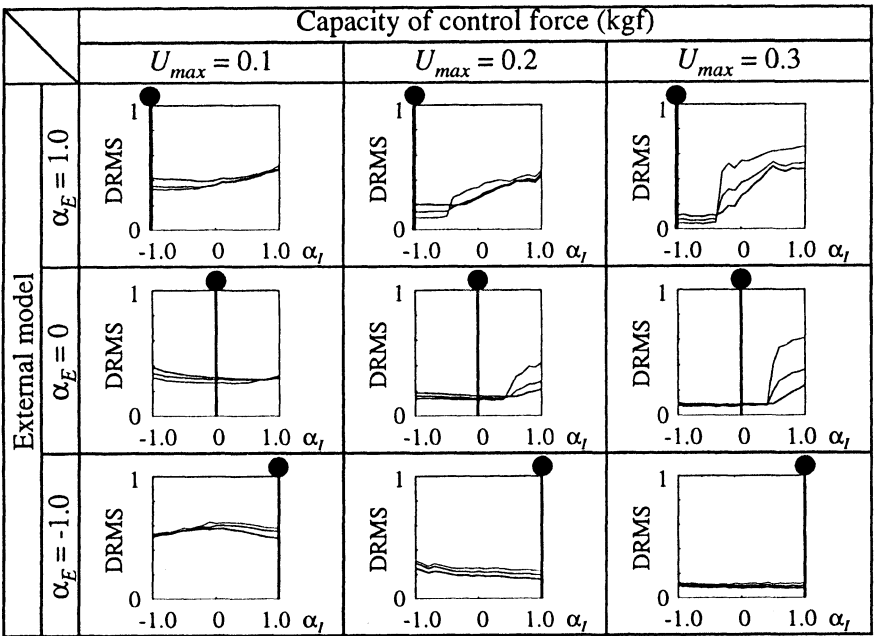


Figure 7. Sensitivity for modeling errors (fluctuations in α_I).