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A simple method for estimating the maximum softening damage index

C.L. Mullen, R.C. Micaletti, Jr., A.S. Çakmak
Department of Civil Engineering and Operations Research, Princeton University, Princeton NJ 08544, USA

Abstract

A simple method is proposed for estimating the maximum softening damage index proposed earlier by DiPasquale and Cakmak. The index requires that the time variation of the fundamental period of an equivalent linear system be computed over the course of a damaging earthquake event. The proposed method estimates this variation by computing empirical transfer functions from fast Fourier transforms of moving windows on measured input and output time series. The method is applied to records from a scale model of a 10-story reinforced-concrete building frame subjected to severe levels of damage. The computed damage indices are shown to agree well with those computed using a more elaborate time domain procedure. The proposed method is applicable to a large class of structures undergoing a damaging earthquake event.

1 Introduction

The maximum softening damage index (MSDI) has been proposed by DiPasquale and Cakmak[1] as a reliable index for use in seismic damage assessment of existing structures. The reliability of the MSDI has been demonstrated[1] through damage assessments made using actual strong-motion records from medium-rise reinforced-concrete building frame structures subjected to the 1971 San Fernando earthquake. Its theoretical basis has been established in terms of continuum damage mechanics principles[2].

The definition of the MSDI, δ_M, is quite simple, stating that the following relationship exists between the damage intensity measured by the index and the ratio between the initial undamaged fundamental period, T_0, and the maximum fundamental period, $T_M = max\{T(t)\}; t \in [0, t_0]$, of the softening system during a forced vibration event:

$$\delta_M = 1 - \frac{T_0}{T_M} \tag{1}$$

Measuring the time variation, $T(t)$, and then identifying the maximum, T_M, from a single pair of short duration input/output records turn out to be not that simple. Devising a simple, direct, automated, reliable procedure for this task is the motivation of the present work.

The estimation of natural frequencies from measured data is certainly not new, and a variety of solutions have been proposed in both time and frequency domains (e.g. see Pilkey and Cohen[3]). A time domain method was proposed by DiPasquale and Cakmak[1] and incorporated in a program called MUMOID[4]. The MUMOID algorithm identifies the complete set of modal parameters defining an assumed time-varying equivalent linear system of specified order in each of a set of nonoverlapping windows of measured time series. The algorithm solves a nonlinear optimization problem and requires an initial estimate of this complete set of parameters. MUMOID is interactive, may converge to multiple solutions given different initial values, and may fail to converge for certain sets of initial values. The latter two difficulties are characteristic of many nonlinear optimization routines. An alternative time domain algorithm has recently been proposed[5] for system identification of aerospace structures which solves a linear optimization problem and requires no initial estimate of system parameters. The present form of the routine is interactive, however, and requires an estimate of the order of the system. Judgment must then be exercised as to whether identified modes are in fact real or artifacts of the data analysis.

Here we propose a simple and direct frequency domain procedure we call the Moving-Window Transfer-Function (MWTF) method which extracts the frequency content of a system from a pair of measured input and response records using fast Fourier transform (FFT) subroutines. It is applicable to nonstationary, nonlinear system response exhibited by a wide range of structure classes during high-intensity (damaging) seismic events. The MWTF approach has been devised to meet the following objectives:

1. Directly compute frequency variations from data without recourse to a specific dynamic model of the system.
2. Require little or no input from the user.
3. Require little or no user feedback.
4. Use a reasonable amount of computational resources.
5. Produce a fast, reliable estimate of the MSDI.

2 Proposed MWTF Procedure

The frequency extraction procedure begins with the selection of time windows of the measured records which are suitable for FFT computations as well as for identifying time variation of the fundamental period. These turn out to be competing objectives. On the one hand, the FFT resolution is improved with a larger window size, because more data points are included. The number of data points must be even powers of 2 unless padding is introduced which obscures the result. On the other hand, the smaller the window size the better the chance of capturing frequency changes. Strong motion accelerograms of earthquake events typically provide records fixed at about 10-30 s with sampling rates of 100 or 200 samples per second. About 512 points are required to obtain adequate resolution of the FFT which translates into 2.06 or 5.12 seconds per window. Such a window size provides only 5-15 or 2-6 point estimates of the period over the record for the respective sampling rates and record lengths. The MWTF routine uses overlapping windows to provide additional sensitivity. In this study, windows are centered on time positions spaced 0.5 s apart yielding 20-60 point estimates for the respective record lengths. When overlapping windows are used, the point estimates are no longer independent.

For each of the selected windows, a smoothed pair of input and output spectra are obtained using the IMSL[6] routines, PFFT and SSWP. The first routine provides a periodogram of the record, and the second a smoothed, non-normalized, one-sided power spectrum. In this study, a Tukey-Hanning spectral window[6] is applied to obtain a reasonably smooth spectrum. Such smoothing is consistent with the response behavior expected for reinforced-concrete structures with relatively large damping and well separated modal frequencies. Smoothing may not be appropriate for lightly damped structures or those with closely-spaced modal frequencies. The transfer function relating the input and output random processes at the measurement locations is then computed using the following simple algebraic relation from linear random vibration theory[7].

$$|H_{XY}(\omega_k, t_j)|^2 = G_{YY}(\omega_k, t_j)/G_{XX}(\omega_k, t_j) \qquad (2)$$

where t_j indicates the time center for the j-th window, ω_k is the k-th discrete frequency which is a function of the FFT resolution, and $G_{XX}(\omega_k, t_j)$ and $G_{YY}(\omega_k, t_j)$ are the ordinates of the smoothed power spectra for the input process, X, and the output process, Y, respectively, for the j-th time window. Figure 1 shows a typical set of windowed input/output time series, power spectra, and transfer function computed using the above procedure for a case that is described later in this paper. The assumption made here is that, within the time window centered on the point, t_j, the actual nonlinear response can be represented as a time-varying, equivalent linear stationary process. This is consistent with the characterization made in MUMOID.

The application of the above steps requires several practical considerations in order to produce reasonable results using actual data which have been incorporated in a Fortran 77 subroutine called MWTF. Firstly, real events may have little energy at some frequencies or the computed FFT may yield essentially zero ordinates between some pairs of peaks. Either case results in unrealistically high estimates of the transfer function at that frequency. The MWTF routine, therefore, filters out the $(\omega_k, |H_{XY}(\omega_k, t_j)|)$ data pairs corresponding to input spectral values, $G_{XX}(\omega_k, t_j)$, below a specified value, G_0. Once the filtered data pairs have been obtained for each $j = 0, j_{max}$ window, the routine then searches for the peak $|H_{XY}(\omega_k, t_j)|$ values for each window. The ω_k for these peaks are assumed to correspond to the natural or modal frequencies of the structure in that window. Depending on the energy distribution of the input excitation, some of the actual modal frequencies may not be identifiable in the empirical transfer function for a given window.

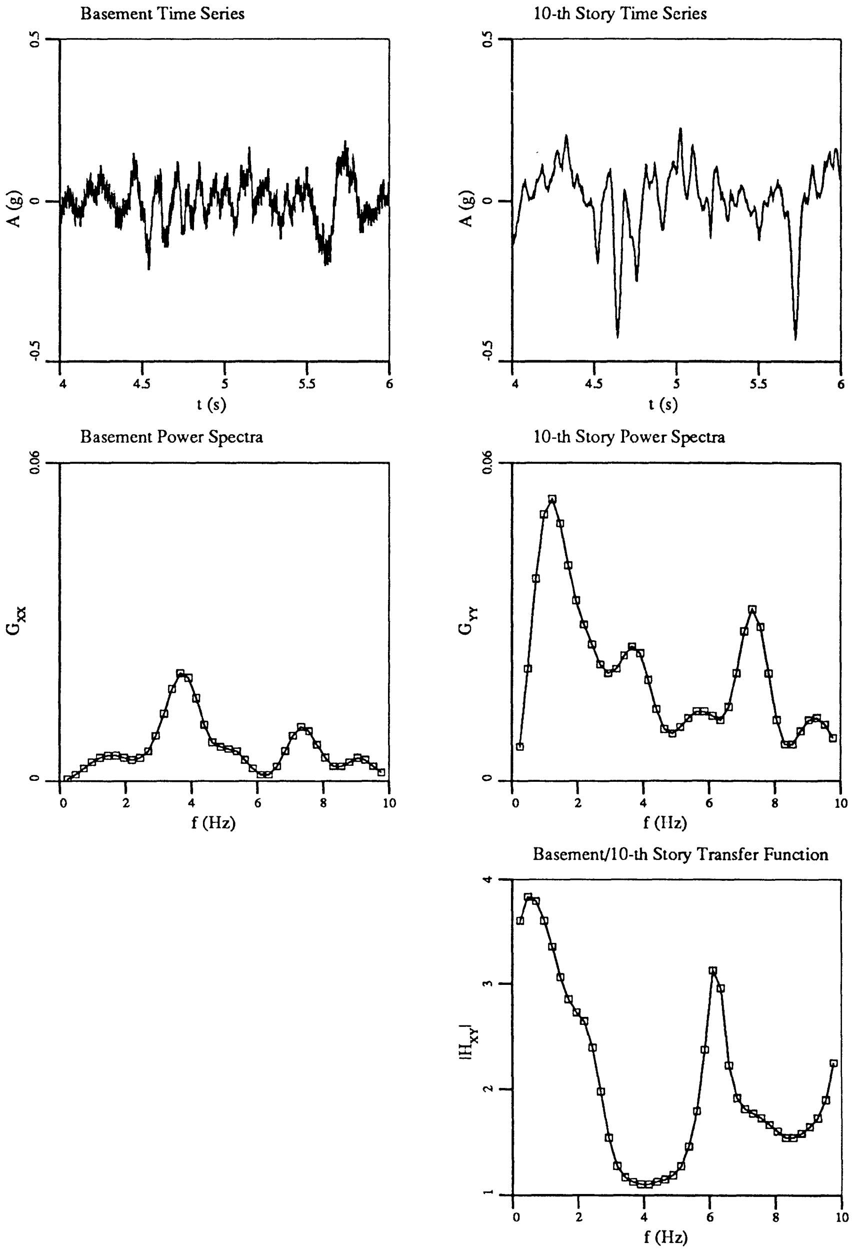

Figure 1. Typical MWTF processing of windowed acceleration records (UIUC H1-1 model test at $t_j = 5$ s).

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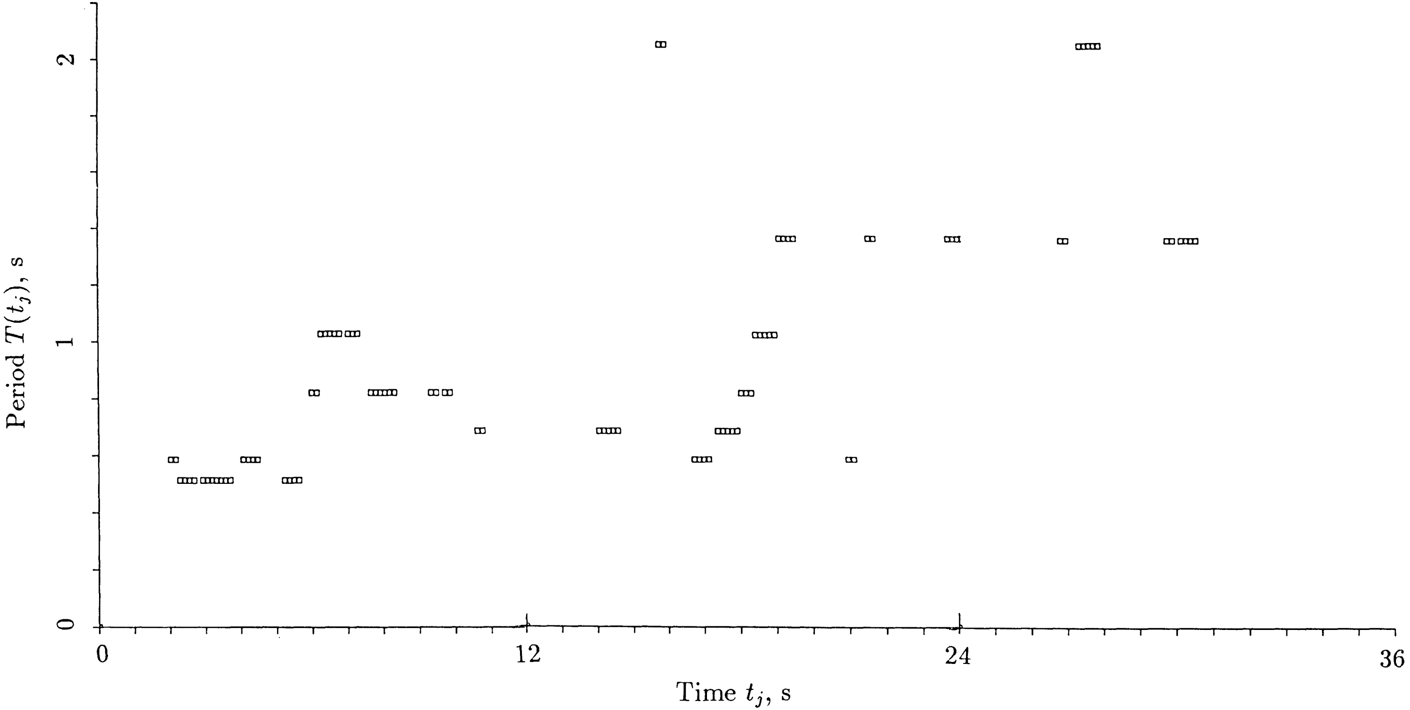
Subsequent processing is performed using period, $T = 2\pi/\omega$, as a basis rather than frequency, ω, and the magnitude of the transfer function, $|H(T_k, t_j)|$, is used as an indicator of the significance of modal periods operative in a given window. Specifically, for each j-th window, each T_k, $k = 1, k_{max}$ is examined to determine the maximum T_k in that window. If the structure is undamaged at the beginning of the event, the $(T_k)_{max}$ in the first few windows is used as an estimate of the initial undamaged fundamental period of the structure, T_0. For events beginning with a previously damaged structure, such an estimate of T_0 must be specified by the user. The $(T_k, |H(T_k, t_j)|)$ data pairs are filtered further by eliminating pairs with T_k values less than T_0. The surviving data pairs are then scanned sequentially from $j = 1, j_{max}$ for the pair having the maximum $|H(T_k, t_j)|$ in each window. Each T_k corresponding to that $|H(T_k, t_j)|_{max}$ in the j-th window is compared to T_0. If $T_k \geq T_0$, the data pair is retained, and if $T_k < T_0$, it is discarded.

The data pairs may now be analyzed as a single set of sequential, presumably first modal period values, $\{T_j\}, j = 0, j_{max}$, without regard to the transfer function values. Because of the above filtering criteria, T_j are not identified for all j. In order to estimate the T_M needed to compute δ_M, the maximum T_j must be determined considering all surviving T_j. To do this, the set, $\{T_j\}$, is scanned for "trends", proceeding sequentially from $j = 0, j_{max}$, omitting those j with no data. A "trend" is arbitrarily defined as being four (4) consecutive T_j having the same value. For a T_j to be considered as a possible T_M, it must be part of a trend. This constraint seeks to eliminate values which are probably not first modal periods but simply the maximum T_k in a window surviving the filtering process. After all T_j meeting the "trend" criterion have been identified, the maximum value is selected as T_M.

3 Application of MWTF Procedure

The MWTF routine has been used to analyze records from model tests conducted at the University of Illinois at Urbana-Champaign by Sozen and associates[8]. These tests were analyzed by DiPasquale using the MUMOID routine. The tests modeled a scale version of a 10-story, 3-bay reinforced-concrete structure under simulated seismic excitations. The base excitations were patterned after the 1940 El Centro earthquake NS acceleration time series and were scaled to increasingly higher PGA. The assembled test frame specimen, designated H1, was subjected to three successive simulations of increasing intensity with no repair or strengthening between runs. The label "H1-1" indicates the first simulation run performed using test specimen H1. Similar labels are applied to the second and third runs. The acceleration records from the base and the tenth story of the test structure have been processed for the purpose of computing the MSDI.

The results of the MWTF identification are summarized in Figure 2 and Table 1. Figure 2 shows the evolution of the fundamental period, $T(t_j)$, over the course of the three successive earthquake simulation runs. The computed MSDI values for each run are given in Table 1. The MWTF estimates are seen to be essentially identical to those obtained by DiPasquale using MUMOID. In computing the MSDI, both MWTF and MUMOID routines selected the estimate of T_0 from the first few windows of the initial run, and, since the subsequent runs started from a damaged state, this T_0 was used in computing the MSDI for all three runs.



a. Run 1 (PGA= 0.35 g)   b. Run 2 (PGA= 0.85 g)   c. Run 3 (PGA= 1.6 g)

Figure 2. Evolution of Fundamental Period, $T(t_j)$, computed by MWTF routine for UIUC H1 model test series.

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Table 1. Maximum Softening Damage Indices for UIUC Model RC Frame

RUN	MSDI, δ_M MWTF	MUMOID
H1-1	0.500	0.432
H1-2	0.625	0.617
H1-3	0.750	0.751

4 Discussion of Results

Both MUMOID and MWTF estimate the MSDI using response records from two places on the structure, one characteristic of the input excitation and the other characteristic of the principal damage response mode. For a structure not suspected to have been in a damaged state prior to the recorded event, the MWTF routine proceeds in an automated fashion without input from the user, producing both the MSDI and the evolution of $T(t_j)$ as shown in Figure 2.

In contrast, the MUMOID procedure requires initial estimates of n x 3 parameters where n represents the number of degrees-of-freedom of the equivalent linear system being assumed in the model. For each DOF, there exist 3 modal parameters- frequency, damping ratio, and modal participation factor. Given these initial parameters, MUMOID calculates the "effective" parameters of the equivalent time-varying linear system in each specified nonoverlapping window. Different initial estimates may result in different solutions. In addition, some computed "effective" parameters may appear to be physically unrealizable (e.g. negative damping ratio). Furthermore, convergence is not guaranteed even when supposedly realistic initial values are provided. The more DOF operative in the system, the more difficult it becomes to obtain convergence.

The MWTF procedure suffers primarily from the limitations of the recorded data. For short events or records having too low a sampling rate, the FFT algorithm may yield too little resolution. Adaptation of the MWTF procedure to cases substantially different from the application discussed here may require some tuning by the user. For example, the cutoff G_0 value described earlier may require adjustment. If the cutoff value turns out to be too high, not enough data pairs will survive to establish meaningful trends. If the value is too low, data pairs with unrealistically high transfer function values may survive, obscuring trends. Additional testing of the MWTF routine may yield some guidelines for controlling such behavior.

5 Conclusions

A simple and direct method for computing the MSDI has been proposed. The so-called MWTF procedure involves computation of empirical transfer

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windows of input/output time series pairs. The proposed technique has two main advantages over a previously proposed time domain procedure, MUMOID. Firstly, the MWTF routine incorporates standard frequency domain analysis and the efficiencies of the FFT procedure to provide direct insight to the vibrational frequency characteristics of the structure without restrictions imposed by assumptions regarding the broader vibrational characteristics of the system including: the order of the system, the number and selection of the active modes during various segments of an event, and the modal parameters such as damping ratios and participation factors. While such broader characteristics are useful for purposes of prediction, their inclusion in system identification procedures may introduce errors or poor performance when real data are used involving measured response of complex systems whose actual characteristics are unknown or poorly defined. Secondly, the MWTF routine is essentially automated requiring little or no user input or feedback. As with any automated routine, however, assessment of the reasonableness of the results is the responsibility of the user. In the application discussed here, the results appear quite reasonable.

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