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An introduction to the sensitivity and stability in risk models

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Abstract

In the studies done by the author about dynamic systems modelling, the basic concepts of systems and their application to ecological model theory have been formalised (Villacampa et al [3]). In the dynamic risk system modelling, it is not possible to obtain a system as a model, because it would suppose knowledge over reality. Moreover, it is usual to consider that a model is more real when more variables are studied and when there are a lot of relationships between them. It has been checked that the increase of parameter does not contribute to obtaining a better simulation. It is necessary to define measurements to improve the dynamic systems modelling. In this way the authors begin to study the system diversity, because it is important to measure how a variable has an effect on certain processes. An important concept that the system theory studies is the model sensitivity. Initially, some variables and usually numeric data of the same are used for modelling. The conclusions of constructed modelling are precisely the data on which the conclusions depend. The sensitivity generic study tries to know in what measure the model behaviour is altered by the modification of some data. If small variations cause important modifications in the model global behaviour then the model is very sensitive in relation to some variable. If risk systems are considered then it is important to submit the system to extreme situations and analyse the behaviour of the same. In these lines some risk indexes will be defined to determine the variables' sensitivity in front of extreme changes. This will permit analysis of the system behaviour in relation to several simulations of the same.

1.- Introduction

The authors start from inductive-deductive methodology, applied and published in other works (Villacampa et al [4], Usó et al [2]), for the sensitivity and stability study of risk models system. Starting from this methodology makes possible to determine one mathematical expression for a differential equation system set. These equations, state equations, are elaborated hypothetical shapes (deductive methodology) and depend on a variable set named flow variables. The mathematical expression of these variables is obtained starting from experimental data with an inductive methodology.

In this paper, we assume that the system study is defined starting from differential equation system named state equations: $\left\{\frac{dy_i}{dt}\right\}_i$. Each differential equation will be supposely defined by a variable set, named flow variables. In general, the flow variables are expressed mathematical shape, that is they are modelled mathematically, they are the reason for originating the flow equation in the modelling study of the said system realised by the authors, Usó et al [2], Villacampa et al [4]. The calculation of this equation is essential in the modelling study of every system. It is important to analyse the sensitivity of the equation facing the change realised in the experimental data, in order to be able to establish the produced effects in this equation to take us at system extreme situations. The best choice of this equation will have consequently a model that abides best by the established criterion of the modeller.

2.- Flow equation modelling in risk system

2.1.- Flow equation

In view of a risk system, a flow variable x_i is considered; we also suppose that x_i represents a certain phenomenon, which flow equation defined by:

$x_i = F(x_{i1}, x_{i2}, \ldots\ldots, x_{ip})$ (1) we know. Starting from experimental data it is possible to model such as equation and express it mathematically. This process is equal to writing the variable in a certain language which will depend on the method used in each case. The authors have used an inductive-deductive methodology on the modelling works Usó et al [2], Villacampa et al [4] . If we express the used language by $\Im(SM)$, then the flow equation (1) will be expressed mathematically by a word of the language $\psi_i \in \Im(SM)$.

2.2.- Equivalent flow equations

Let $\psi_{ij} \in \Im(SM)$ be in such a way that in risk model system the said works model a same process under established criterion by the modeller. Then we will say that the work sets $\{\psi_{ij}\}$ are synonymous of the process that it is being studied x_i.

Firstly, we will suppose to model a risk system defined by differential equation system in relation to experimental data and according to the methodology used by the authors. Furthermore, we start from a flow equation choice defined in the language and the equation that it has the greatest correlation (fitting) between the variables and experimental data will be considered. Therefore, we will know the flow equation and its associated correlation coefficient, $\psi \rightarrow r$, for data set and particular instances.

3.- Flow equation risk. Risk rates

Let's consider a flow equation ψ that has been obtained in the risk system modelling; then we put one variable in anomalous situation that carries some risk. A fixed value is assigned to it within experimental data modifying such a value a finite number of times determined by the modeller. If we consider j variations then the situation does not affect excessively over own equation that models the situation $j \to \infty$ in an interval.

3.1.- Flow equation risk rates

The flow equation $\psi = F(x_1, x_2, ..., x_n) = A_1 f_1(x_1) + ... + A_n f_n(x_n)$ is considered to model a certain process in a risk system. It will be also suppose that said equation is obtained starting from some experimental data if we apply the recognoscibility criterion to consider that it has the greatest multiple correlation coefficient r_0, from experimental data table T_0. The picked up data of the variables x_i will be facing generic time-set by $T_i = \{t_i, i = 1,..,n\}$ for n measures of the variables. Furthermore, a normalised representation of carried measuring number will be represented in the interval [0,1]. In the same way the equation initial correlation coefficient is represented at x=0, and its correlation coefficient for j measures with i=1,..., m in the abscise axe: $\forall j \in [0,1]$ we associate with $\frac{i}{j} \in [0,1]$.

3.1.1.- Flow equation ε-risk in relation to a variable

Next measures are defined to determine if the flow equation is valid for several risk situations produced in a variable.

Let it be the flow equation ψ and the time-set T and let it be r_0 as the obtained correlation coefficient. Every data, excepting the data of a variable x_i, will be kept constant according to several values in accordance with the modeller is choice. We are going to study what will be going on with the flow equation and with the correlation coefficient if the variable values remain constant in some rate of variability.

If we consider a time-set set $\{T_j\}_{j=1}^{m}$ in such a way that the experimental data in each one remain equal, excepting the values of the variable x_i that will remain constant to each one; then, let us r_{xi} (j) as the equation correlation coefficient ψ in that time-set T_j for each j.

<u>*Definition:*</u> *Flow equation ε^m-risk in relation to x_i.*

Let $\{ri\}_{i=1}^{m}$ the flow equation correlation coefficients for m-variations of the variable x_i and so for m-time-set. It is defined

$$A_{x_i}^{m} = r_o - \sum_{i=1}^{m} \frac{r_i + r_{i-1}}{2m} .= r_o - \frac{1}{2m} [r_1+r_o+r_2+r_1+.......+r_{m-1}+r_m] =$$

$$= r_o - \frac{1}{2m} [r_o+2r_1+2r_2+...+2r_{m-1}+r_m].$$

The flow equation ψ has a ε^j- risk in the variable x_i if and only if $| A_{x_i}^{m} | < \varepsilon$.

<u>*Definition:*</u> *Flow equation ε-risk in relation to x_i.*

The extreme case, where the infinite time-set $m \rightarrow \infty$ are carried out, is considered and the correlation coefficient is represented by $r_{xi}(x)$ in the abscise x, so that it is defined $A_{r_o} = r_o - \int_0^1 r_{xi}(x)dx$.

The flow equation Ψ has a ε-risk in the variable x_i if and only if $| A_{r_o} | < \varepsilon$.

If ε is small then a flow equation will be valid in front of produced risk situations in a determined variable. This will suppose that the equation is not sensitive.

4.- Sensitivity and stability of a model

In this paragraph, the sensitivity and stability of a model will be analysed facing realised variations in its variables, both to have new experimental data and to simulate the same. But it is important to measure generally the model sensitivity and stability and the variations of all its feasible case data in the reality.

4.1.- Flow equation sensitivity and stability

We are going to study the flow equation sensitivity and stability in front of data rate m-variations of all variables that it depends on. Measures will be obtained to allow us to simulate the experimental data variation and to analyse their consequences in the model.

Let ψ the flow equation whose determination coefficient is r in relation to the first experimental data acquired in a temporal discretization $t_0 = \{t_0^0, t_1^0, \ldots, t_n^0\}$. Assume that the new variations of the data to certain temporal discretizations is $t_1 = \{t_1^1, \ldots, t_n^1\}, t_2 = \{t_1^2, \ldots, t_n^2\}, \ldots, t_m = \{t_1^m, \ldots, t_n^m\}$ and r_i are the determination coefficients of said equation in relation to several time-set. The sensibility of the equation ψ in relation to temporal variations t_i is defined by $\Delta r_i = |r - r_i|$. The main sensitivity produced in the flow equation in relation to temporary m-discretizations is defined by $C_\psi^m = \sum \frac{\Delta r_i}{m}$. The equation stability to m-variations will be defined by $E_\psi^m = \frac{1}{C_\psi^m}$.

4.2.- State equation sensitivity and stability

Let the state equation be defined from P flow variables $\frac{dy}{dt} = F(x_1, x_2, \ldots, x_p)$. It supposes that each flow variable is expressed by flow equation ψ_i after a modelling process. Let $C_{\psi_{ii}}^m$ be the sensitivity of each flow equation ψ_i to data temporal m-variations.

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The state equation sensibility C_y is defined by $C_y = \dfrac{\sum C^{m}_{\psi_{ii}}}{P}$ and the stability in relation to m-variations produced in experimental data $e_y = \dfrac{1}{C_y}$.

4.3.- Model sensitivity and stability

A model of q state variables is defined by $\{y_i\}_{i=1,\dots,q}$. Let C_{y_i} be the sensitivity of each state equation in relation to m-variations produced in the experimental data. The model sensitivity in relation to such variations is defined by $C_m = \dfrac{\sum_{i=1}^{q} C_{y_i}}{q}$.

The model stability will be defined by $e = \dfrac{1}{C_m}$.

5.- Conclusions

Measures have been defined to allow us to study the best complex structural system modellings and the variations which are produced in front of variable several changes. This is analysed from two view points: on the one hand from the risk situation perspective to be determined by analyst; and on the other hand in relation to global realised variations in all its experimental data and in the reality observation boundaries.

To define the coefficients that analyse the sensitivity analysis of flow equations facing the variations in the variables data that they determine implies the best choice equation that models a process in relation to the situations that suits us more. In this way the stability of these equations will be able to be assumed. This involves the analysis of the state equation stability and its own model.

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