A topographical approach for multimodal structural optimal designs

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Abstract

This paper presents a two-stage multimodal optimization algorithm and its application to structural optimizations. The first stage of the two-stage algorithm serves to identify local optimum candidates which are defined as the best sampled design in each valley containing a local optimum from a number of sampled designs by using topographical identification techniques. The second stage of the optimization algorithm involves a number of local searches using the local optimum candidates as initial designs. A unique crystal growth approach is proposed to perform systematic identification of the local optimum candidates. This topographical multimodal optimization algorithm is applied to one illustrative structural optimization problem and comparison are made to results obtained by an existing topographical algorithm.

1 Introduction

Global optimization seeks the global optimum in problems with a nonconvex design space. Most practical engineering optimization problems cannot be proved to be unimodal and therefore it is important to use global optimization strategies to handle such engineering problems so as to attain a satisfactory solution. Global optimization methods can be classified into two major groups:

deterministic methods and probabilistic methods. Extensive description on the theories and characteristics of principal global optimization methods can be found in the literature [1,2]. Deterministic methods aim to obtain the global optimum with guaranteed accuracy. Since these methods are generally based on restrictive assumptions on the objective function, and are often only applicable to unconstrained problems, they are not practical to most engineering multimodal optimization problems.

Probabilistic methods basically provide no guarantee in the attainment of the global optimal design with limited computing effort. A global solution of a better quality can be generally achieved by investing more computing effort. Two important approaches in probabilistic methods are random search methods and clustering methods.

Random search methods include pure random search, singlestart and multistart. A pure random search generates a sequence of randomly distributed designs and at the end of the search the best design obtained is regarded as the global solution. The method is inefficient because the design distribution density is equal in promising and unpromising regions. Local refinement is also not performed in the pure random search.

In singlestart, a local search is executed from the best of a number of evenly-distributed sampled designs, and the local search results in the global solution. The reliability of this method to attain the global optimum is quite low when the covering of the design space is not sufficiently dense. In multistart, a local search is performed from each sampled design in the space and the best optimum located is regarded as the global solution. For a small number of sampled designs, the confidence level of obtaining the global optimum is not high. If a large number of sampled designs are used, precious computing resource may be wasted because many local searches lead to identical optima. These random search methods are simple in algorithm and thus often used by engineers who are not experts on global optimization. Modification on the random search schemes for improved efficiency is therefore highly desired in engineering design practice.

Clustering methods aim to perform only one local search in each optimum-containing valley and thus prevent redetermination of already known local optima. In clustering methods, four fundamental steps are sampling, concentrating designs, clustering, and local search. First, a number of sampled

designs are generated in the design space. Discarding a percentage of designs with higher objective function values and a few iterations of local search are applied to remaining designs to create the concentration towards regions of attraction. Cluster analysis techniques based on design distribution densities determine design clusters. From the best design of each design cluster, a local search is executed. A satisfactory performance of clustering methods relies on proper identification of optimum-containing valleys from sampled designs. Since traditional cluster analysis techniques are solely based on density information of designs distribution, two drawbacks remain. First, if a few steps of local searches fail to create proper concentrations on optimum-containing valleys, cluster analyses will result in incorrect determination on clusters which consequently lead to redundant identical local optima or failure to locate some optima. Secondly, a few steps of local search on each of remaining sampled designs creates additional computing cost which makes the algorithm less efficient.

2 Topographical Identification Method

Törn and Viitanen proposed a topographical global optimization method using pre-sampled designs in 1994 [3]. Being a modification on the clustering method, the topographical approach uses both objective function values and location coordinates of pre-sampled designs to identify near-optimum designs in each optimum-containing valley. A threshold distance approach is used in their work to create a most even distribution of sampled designs in the design space. Eliminating executions of a few steps of local search on 'good' sampled designs in classical clustering methods, topographical algorithm can invest computing effort in distributing more pre-sampled designs. Based on a well covering of pre-sampled designs, more accurate identification on near-optimum designs in each valley is achieved with increased efficiency. Törn and Viitanen' topographical algorithm is briefly described as follow steps:

1)Sampler

1.1 Randomly generate a design.

1.2 Keep the design if the design is not located within a threshold distance to any design already accepted.

1.3 Continue 1.1 & 1.2 until all pre-sampled designs are attained.

2)Topography Construction

2.1 For each design, compare its objective function value to that of k nearest designs to it.

2.2 During the comparisons, any design having a worse objective function value is marked.

2.3 When the process defined in 2.1 & 2.2 is completed for all pre-sampled designs, unmarked designs are regarded as near-optimum designs.

3)Local Search

3.1 A local search is executed from each near-optimum design.

3.2 The best local optimum thus obtained is regarded as the global optimum.

Being simple in algorithm and efficient in a number of test problems [3], the topographical global optimization method using pre-sampled designs still has a drawback. The proper value of the critical parameter k is problem dependent and is very difficult to define. To be more elaborating, a suitable value of k may vary in different domains of the design space. This situation can be described by using the sampled designs in a one-dimensional function as shown in Figure 1.

The function has three local minima and the goal is to identify exclusively the best design in each valley from 14 designs on it. For varied values of k, near-optimum designs identified by Törn and Viitanen' algorithm are listed in Table 1.

Table 1. Near-optimum designs identified by varied k values

k value	1	2	3	4	5	6	7
Near-optimum designs	B,F,G,I, L,M	B,F,G, M	B,F,G, M	B,G,M	B,G,M	G,M	G

It is noted that near-optimum identification is all correct only in the cases, k=4 and k=5. Other k values result in either redundant local searches ($k \leq 3$) based on too many near-optimum designs or failures to attain the local (k=6) or even the global optimum ($k \geq 7$). The primary reason for improper identification

is the comparison of function values either between a design and another design not located in the same valley or between a design and a neighboring design of wrong direction. For example, as k is less than 4, F design compares its function value only with that of E, D, and C designs and with a better function value F design is regarded as a near-optimum. There is no comparison between F and G designs at this stage simply because G design is farther away from F design than E, D, and C designs. As k is greater than 5, B design is compared with G design which is located in another valley, and a proper near-optimum design thus fails to be identified. The observation showed that Törn and Viitanen' topographical algorithm using pre-sampled designs was effective if a proper value of k was selected. It is difficult to define a proper k value which is influenced by many factors such as the number and distribution pattern of sampled designs, and the contours of the design space. Furthermore, the situation of an algorithm demanding proper selections on critical parameters often requires hidden computing cost spent in multiple executions of the algorithm with varied parameter values.

3 Crystal Growth Approach Near-Optimum Identification

The crystal growth approach near-optimum identification techniques proposed in this paper are originated from the questions arose in the previous example as shown in Figure 1. How to implement an algorithm which will demand comparisons between designs which are supposed to compare and not demand comparisons between designs which are not supposed to? Then it comes out as the answer. The proposed algorithm for solving the problem shown in Figure 1 is as follows:

The one-dimensional version

1) Rank all designs according to objective function values.
2) Using the best design as the core, perform comparison to its nearest design. (G is the core, H is the nearest)
3) Compare the next nearest design from the core, to its nearest design among all designs having done comparison. (I is second nearest to G, and I is compared to H because H is closer to I than G is)
4) Continue the process until all designs are compared.

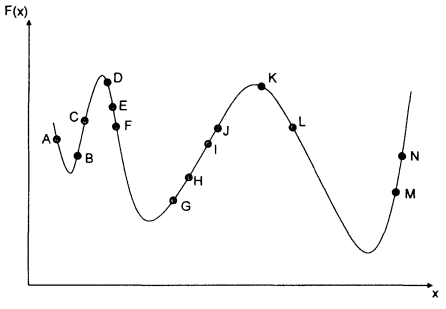

Figure 1. 1-D multimodal function.

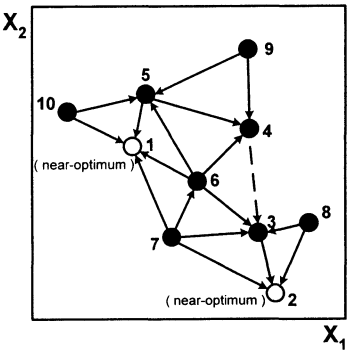

Figure 2. Crystal growth approach.

At the end of the process, only *B*, *G* and *M* are three non-inferior designs in comparisons, and they become near-optimum designs. In order to suit for problems of multiple dimensions, the algorithm is modified to as follows:

Crystal growth near-optimum identification algorithm

1) Rank all designs according to objective function values. The best design is the first core.
2) Bond the core, and its two nearest designs form a triangular set. Comparisons are made between members of the triangular set.
3) Bond the next nearest design from the core, to its nearest two designs among all designs having been bonded into triangular sets, to form a new triangular set.
4) Continue the crystal growth process until all designs are bonded; Any design is marked if its function value is inferior than that of any given design which it has compared to.
5) Repeat Steps 2-4 by using the next best unmarked design as the new core until all unmarked designs have been used as cores.

The algorithm can be illustrated in the example as shown in Figure 2. There are a total of 10 designs in the two dimensional case. The objective function value of each design is used as the index for the design. Design 1 (D_1) has the lowest objective function value and is thus used as the first core. D_5 and D_{10} are its two nearest designs and they are bonded into D_1 and form a

triangular set. Obviously D_5 and D_{10} will be marked due to inferior function values. D_6 is the next design closest to D_1 and is bonded to D_1 and D_5. In similar processes, D_7, D_4, D_9, D_3,D_8 ,D_2 are sequential bonded and leave D_1, D_4, and D_2 unmarked. Although D_1, and D_2 look promising as near-optimum designs, D_4 is an obvious miss-judgment. Treat the second best unmarked design D_2 as the new core and repeat the process to bond all other designs. When D_4 is the design to be bonded to D_3 and D_6 which are two nearest designs among D_2, D_3, D_6, D_7, and D_8, already bonded at this moment, D_4 is marked due to its inferior value than D_3. Whole crystal growth algorithm is completed when remaining four designs are bonded without any new outcome. D_1 and D_2 are deemed as near-optimum designs by this algorithm.

Many experiments have been performed on different number of designs for a new design to be bonded to. Results showed the number, two, provided very stable and satisfactory identification outcome with a highest computing efficiency in problems of varied dimensionality and number of local optima. Therefore, the crystal growth approach is considered as a parameter-free algorithm.

4 Two-stage Multimodal Optimization Algorithm

The crystal growth near-optimum identification techniques are used in a two-stage multimodal optimization algorithm using pre-sampled designs such as previously described Törn and Viitanen' topographical algorithm. Except that the Törn and Viitanen' topography construction algorithm is replaced by crystal growth algorithm, the sampler and the local search steps are identical.

5 Illustrative Problem

The illustrative nonconvex structural optimization problem involves a two beam grillage structure subjected to distributed static loads as shown in Figure 3. The design variables are the cross sectional areas of each beam A_i. The moments of inertia I_i and sectional modulus Z_i are related to the cross sectional areas A_i by empirical relations obtained by Clarkson [4] as follows:

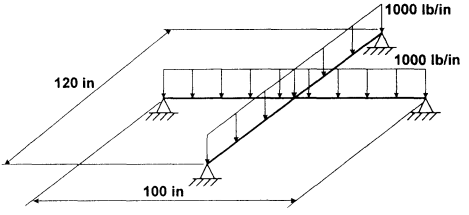

Figure 3. Two-beam grillage.

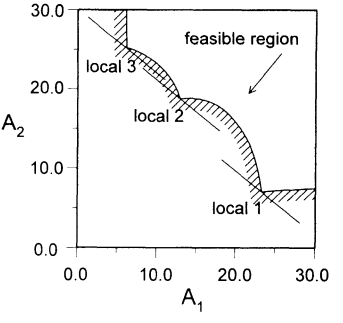

Figure 4. Disjoint design space.

$$I_i = 1.007(A_i / 1.48)^{2.65} \tag{1}$$

$$Z_i = (A_i / 1.48)^{1.82} \tag{2}$$

The beams were sized for minimum weight with stress constraints at the center point and at the location of maximum bending moment along the span to below allowable stresses 20,000 lb/in^2. Three optima exist in the nonconvex design space resulted from nonlinear constraint boundaries as shown in Figure 4. Using uniform threshold sampling, 25 designs were sampled in each of 5 experiments conducted with different random number seeds. The crystal growth identification algorithm and Törn and Viitanen' topographical algorithm with varied k values (k=2,3,4,5,6) were used to identify near-optimum designs and then local searches were followed. In order to compare crystal growth topographical algorithm to a multistart algorithm without near-optimum identification techniques, 3 to 6 evenly distributed initial designs were generated in four experiments and a local search was performed from each initial design. All above-mentioned numerical experiments were conducted five times so as to reduce sampling errors. Average results of all tests are listed in Table 2. Due to a large attraction region for the global optimal design in this problem, all experiments attained the global optimum. The second best local optimal design was attained in 80% of experiments using

topographical algorithms while the multistart method located this optimum from 60% to 80% of experiments. The third local optimum was attained by varied chances for different approaches. The ratio between the function evaluations and the number of distinct optima attained
represents the efficiency for an algorithm to capture optima. A good multimodal algorithm should not only spend less function evaluations for locating each optimum, but also be able to capture more optima. The above-mentioned efficiency ratio to be divided by the number of distinct optima attained by the specific algorithm will give a good overall performance index for each multimodal algorithm. It is noted that a lower performance index represents a better multimodal algorithm by this definition. The crystal growth approach with adaptive stopping criteria has an excellent performance index 11.7 in this problem. Although Törn and Viitanen' topographical algorithm with k=4 has an overall best performance index, 11.1, all k values presented an average of 13.7 performance index. For an algorithm requesting a definition of a critical parameter k, the use of an average performance index for this algorithm should be more than reasonable. The multistart which is already aided by even initial designs distributions obtained by using a maximum threshold distance have worst performance indices with an average 20.7.

Table 2. Comparison of different multimodal optimization algorithms

	Crystal growth method	Törn and Viitanen' algorithm					Multistart			
		k=2	k=3	k=4	k=5	k=6	3 pts	4 pts	5 pts	6 pts
optimum 1 (global)	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
optimum 2	80%	80%	80%	80%	80%	80%	60%	60%	60%	80%
optimum 3	80%	80%	80%	80%	60%	40%	60%	60%	80%	100%
distinct optima attained	2.6	2.8	2.6	2.6	2.4	2.2	2.2	2.2	2.4	2.8
function evaluations needed	79.2	134.4	98.4	75.0	70.8	61.6	87.6	77.6	149.8	177.0
performance index	11.7	17.1	14.6	11.1	12.3	12.7	18.1	16.0	26.0	22.6

6 Concluding Remarks

A crystal growth approach near-optimum identification algorithm was used in topographical multimodal optimization using pre-sampled designs. Preliminary results showed that crystal growth approach near-optimum identification algorithm provided a more stable and effective near-optimum identification output than Törn and Viitanen' topographical algorithm and classical multistart methods. The applicability for these topographical multimodal optimization techniques in problems of a large dimensionality and a large number of local optima calls for further investigation.

It is noted that the crystal growth approach near-optimum identification algorithm can also be used in any stage of a multimodal optimization method to identify near-optimum designs by using all designs previously evaluated (computing resource conservation) so that heuristic responses can be made to speed up the optimization.

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