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On the ship trajectory tracking LQG controller design

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Abstract

The paper considers a problem of ship trajectory tracking control synthesis based on linear Nomoto model of ship dynamics complemented by linearized equations of ship motion kinematics. In the course of state estimation, however, in order to handle influence of errors due to model simplifications as well as environmental disturbances some extra variables, modeled as random walks (Fossen[1]), are introduced. The Kalman filtering technique was then used. This way the whole problem was formalized as LQG-like optimal regulator problem. A number of simulations confirms the efficiency of adopted solution technique even in extremal conditions . In the course of simulations the influence of waves is modeled via specifically prefiltered white noise (Lukomski at al.[2]).

1 Introduction

In the considered track keeping problem the ship motion is described by the nonlinear W-O model (de Wit&Oppe[4]) while the wave disturbances are modeled as a properly prefiltered white noise. The optimal tracking control synthesis is based however on its linearized form as well as introduced quadratic cost functional. Since some of the state vector components are unavailable for measurement as well as noise corrupted the Kalman estimator was introduced leading to the LQG-like solution technique. To assure however required estimator performance the linear process model was complemented by some extra variables introduced to handle model errors (due to linearization and environmental disturbances).

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2 Track errors definition

Assume that a trajectory to be tracked is composed from broken line segments defined by a sequence of vertexes $P_1(x_1,y_1)$, $P_2(x_2,y_2)$, ... , $P_i(x_i,y_i)$, ..., $P_n(x_n,y_n)$. Let us introduce also the following coordinate systems :

- earth-fixed coordinate system (X_g,Y_g) (these coordinates can be measured directly via GPS).
- relative (transformed) coordinate system (X_r,Y_r) whose center is located at the point $P_i(x_i,y_i)$ and with the axis OX_r directed along a segment P_iP_{i+1} (i=1,2,...,n)

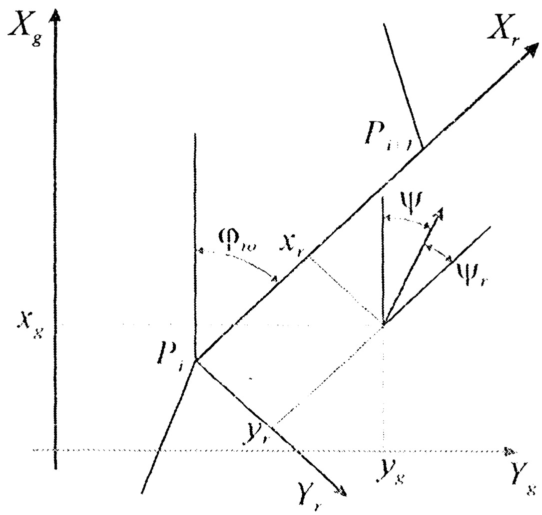

Figure 1: Earth-fixed and relative coordinate systems

The relative ship position (x_r,y_r) as well as its relative course ψ_r can be obtained through the following simple transformations :

$$\begin{aligned} x_{rt} &= x_g - x_i \\ y_{rt} &= y_g - y_i \end{aligned} \tag{1}$$

where x_{rt}, y_{rt} are the coordinates of the translated earth-fixed system and φ_{ro} is an angle of its rotation

$$\tan \varphi_{ro} = \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \tag{2}$$

So, the relative ship positions and the relative course are

$$\begin{aligned} x_r &= x_{rt} \cos \varphi_{ro} + y_{rt} \sin\varphi_{ro} \\ y_r &= - x_{rt} \sin \varphi_{ro} + y_{rt} \cos\varphi_{ro} \\ \psi_r &= \psi - \varphi_{ro} \end{aligned} \tag{3}$$

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Now it is reasonable to treat the coordinate y_r and the course ψ_r as the tracking errors corresponding to the given segment.

3 Ship Motion model

As a model that represents further a real ship dynamics we adopt here the following de Wit-Oppe's (W-O) ship dynamical model (de Wit&Oppe [4])

$$\begin{aligned}
\dot{x}_1 &= x_5 \cos x_3 - x_6 \sin x_3 \\
\dot{x}_2 &= x_5 \sin x_3 + x_6 \cos x_3 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -a\, x_4 - b\, x_4^3 + cu \\
\dot{x}_5 &= -f\, x_5 - W x_4^2 + S \\
x_6 &= -r_1\, x_4 - r_3 x_4^3
\end{aligned} \tag{4}$$

where the common control theoretical state space notation will be used :
$(x_1, x_2) = (x,y)$ - Cartesian coordinates
$x_3 = \psi$ - course
$x_4 = r$ - angular velocity
$x_5 = u$ - longitudinal velocity (surge)
$x_6 = v$ - transversal velocity (sway)
$u = \delta$ - rudder deflection as a control variable
S - propelling force

As the ship model parameters we take the dynamic maneuvering parameters of the m.s. Compass Island (de Wit&Oppe [4]). The units of time, length and angle are respectively one minute, one nautical mile and one radian. The parameters were determined as follows $a = 1.084$ /min, $b = 0.62$min, $c = 3.553$ rad/min, $r_1 = -0.0375$ nm/rad, $r_2 = 0$, $f = 0.86$ /min, $W = 0.067$ nm/rad^2, $S = 0.215$ nm/min^2.
It should be noted, however, that another ship dynamic models could be used here as well.

4 Wave disturbances

The oscillatory motion of waves acting on the ship is modeled as a specifically prefiltered white noise. The wave shaping (colourized) filter is defined by the following transfer function (Fossen, Lukomski at al. [1,2])

$$\psi_H(s) = \frac{K_w s}{s^2 + 2\eta\omega_n s + \omega_n^2} w_H(s) \tag{5}$$

or in the state-space representations

$$\begin{aligned} \dot{\xi}_H &= \psi_H \\ \dot{\psi}_H &= -2\eta\omega_n\psi_H - \omega_n^2\xi_H + K_w w_H \end{aligned} \tag{6}$$

where w_H is zero-mean Gaussian white noise process and the coefficients K_w, ω_n, η referring to different sea states were calculated according to (Lukomski at al. [2]). This model is often called as a model of high-frequency motions (Fossen [1]).

5 Controller synthesis model

Since in view of common control theoretical techniques to synthesize state-feedback track-controller the linear process model is required we linearize aforementioned W-O equations which results in the following Nomoto first order model complemented by linearized kinematical eqn. (2) i.e. $\dot{y}_r = U\psi_r + v$. Neglecting the sway velocity v we obtain

$$\begin{bmatrix} \dot{y}_r \\ \dot{\psi}_r \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & U & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -a \end{bmatrix} \begin{bmatrix} y_r \\ \psi_r \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix} \delta \tag{7}$$

U denotes the resultant velocity (as approximation of surge x_5) and the model parameters are the same as the ones in the linearized W-O model of section 3.
The considered tracking error contains therefore of two components: heading (course) error ψ_r and the cross track error y_r . Taking as a cost criterion the quadratic functional

$$J(\delta) = \int_0^\infty (\lambda_1 y_r^2 + \lambda_2 \psi_r^2 + \delta^2)dt \tag{8}$$

where λ_y, λ_ψ - are the weightings

one can see that this approach allows for formulation the above problem in terms of the standard LQR (linear quadratic regulator) formalism (Bryson&Ho [3]) . The optimal track controller can be thus defined by equation

$$\delta(t) = -\boldsymbol{K}\boldsymbol{x} = k_1 y_r + k_2 \psi_r + k_3 r \tag{9}$$

here $\boldsymbol{K}$ is the optimal feedback gain matrix and $\boldsymbol{x}=[y_r, \psi_r, r]'$ is the state vector.

6 State estimator design

As can be seen from previous section for the given process control the state vector of the model (7) is required. To assure however the appropriate estimator performance the model (7) is not sufficient. In order to handle the ship-model mismatch errors as well as environmental disturbances errors we introduce to the model (7) some extra variables (d_y, r_b) modeled as random walks.
So assuming available, (although noisy corrupted), GPS measurements (x_r,y_r) as well as the ship course ψ_r (or in fact their simple transformations) we estimate the whole state vector basing on the following linear model.

$$\begin{aligned} \dot{y}_r &= U\psi_r + v + d_y \\ \dot{\psi} &= r \\ \dot{r} &= -ar + c\delta + r_b \end{aligned} \tag{10}$$

where r_b is a slowly-varying parameter due to environmental disturbances representing the yaw angle bias while d_y is the drift term.
Treating r_b and d_y as random walk processes and neglecting the sway velocity v we obtain the following standard linear state equations

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{G}\mathbf{w} \tag{11}$$

where the augmented state vector is $\mathbf{x}=\lfloor y_r \;\; \psi_r \;\; r \;\; r_b \;\; d_y \rfloor'$ and the matrices **A, B** and **G** are as follows

$$\mathbf{A} = \begin{bmatrix} 0 & U & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -a & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ c \\ 0 \\ 0 \end{bmatrix}; \quad \mathbf{G} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{12}$$

u=δ and $\mathbf{w}=[w_1,w_2]'$; w_i (i=1,2) are zero-mean Gausssian white processes with covariance matrix Q. The model parameters are the same as the ones in the linearized equations of model (4).
The GPS and a gyro compass measurements (transformed also via (1)- (3)), can be written as :

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{13}$$

where

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

hence $z = [y_r + v_1 \;\; \psi_r + v_2]$.

$v = [v_1, v_2]'$ - denotes measurement noise with covariance matrix **R**.
The Kalman estimator stationary gain L was therefore found on the basis of model (12)-(13). In view of that the control gain K and the estimator gain L were calculated with the help of slightly different although still simple models.

7 Results of simulations

The performed *Simulink* simulations were based on the W-O *nonlinear* model of ship dynamics (sec.3) with the above synthesized controller K and the stationary Kalman filter gain matrix L both obtained via *Matlab Control Toolbox* facilities. The wave disturbances were modeled according to sec.4. while the constants λ_y, λ_ψ of performance functional eqn. (8) were taken as equal 1.
In the Figure 2 trajectories to be tracked consists of two segments. The first one is a part of OX axis (from the point (0,0) to (0,10)) while the second constitute a half-line originated at point (0,10) and with a slope of 60° against OX axis. The original ship position, its course and angular velocity are (0,-1.5), -60° and 0.5 rad/min respectively. The adopted distance scale is: 10units=1/6nm. Figure 3 depicts the original ψ and estimated $\hat{\psi}$ course, original y and estimated $\hat{y}$ cross tracking error and original r and estimated $\hat{r}$ ship turning rate. Figure 4 shows an original r and estimated $\hat{r}$ ship turning rate in a smaller scale. In order to make the original and estimated plots legible the measured signal plots were depicted without sensor noise .

Both the typical and hard wave disturbances (properly prefiltered white noise) were taken into account. Worth of noting is that the performance of the system with the exact state-based control and state-estimate-based control trajectories are practically not to differentiate even in the case of hard sea states. This fact as well as simulations depicted in the Figures 3 and 4 proves the high efficiency of the proposed method of estimation. During simulations the system performance was satisfactory till the maximum angle between subsequent segments of preset trajectory equal to 90°.

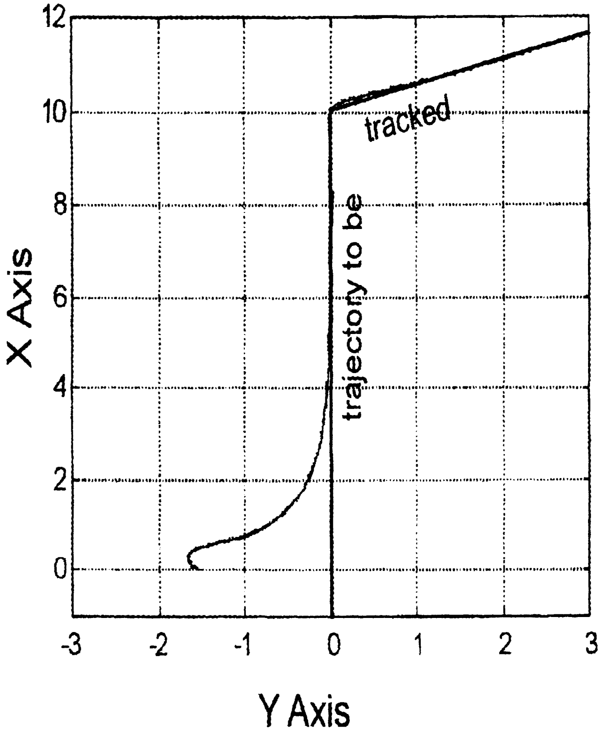

Figure 2 : Trajectory with the segment with a slope of 60°.

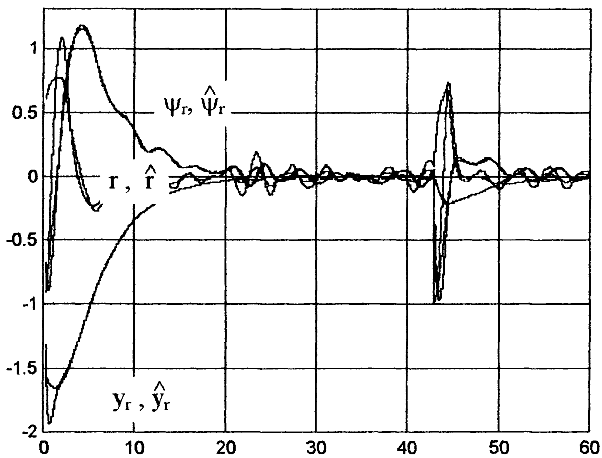

Figure 3 : Measured and estimated - cross track error, heading error and the ship turning rate.

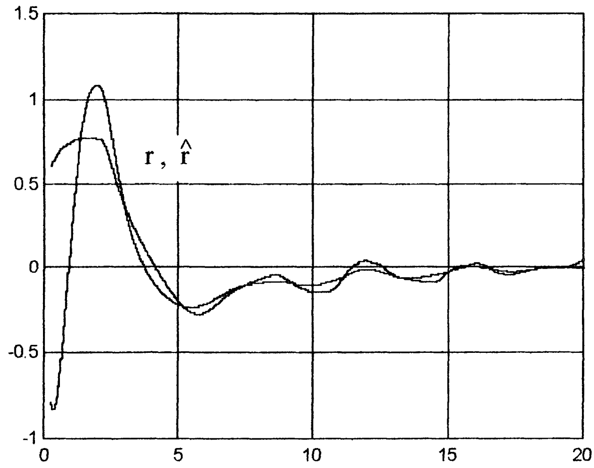

Figure 4 : Measured and estimated ship turning rate.

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