Transactions on the Built Environment vol 21, © 1996 WIT Press, www.witpress.com, ISSN 1743-3509

Recent advances in the analysis of expandable structures

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Abstract

The methods of calculation for expandable structures made of bars, developed over the last few years, has been widely tested in models. Nevertheless, in order to apply these methods to real structures it has been necessary to correct some aspects of calculation. In this paper a new, improved method of calculation is shown. It takes into account the effect of the eccentricity of bars over the knots and the bending that the inner joint over the bars produces when the structure is open. Another important aspect for the stability of the real structures, as much as the problems of deployment and stiffeners, will be also studied.

1 Introduction.

For several years our team has been studying distinct aspects of the design and calculation of expandable bar structures. During this period diverse models of these structure have been analysed using physical models on some occasions and computer models on others. Both kinds of models have permitted us to design and later to debug numerous prototypes of deployable structures, using diverse kinds of bars and knots. Also, some efficient and rapid computer programs have been developed in order to be able to analyse such structures.

The theoretical problems that can be found in these structures were already sufficiently developed in 1992 and 93. Nevertheless, it became necessary to compare all these studies with the actual situation. The opportunity presented itself when in 1993 Seville City Council commissioned a team of

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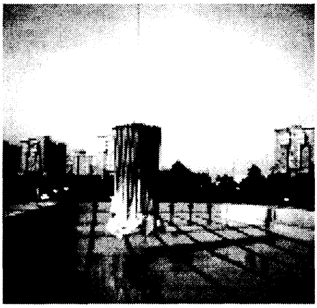
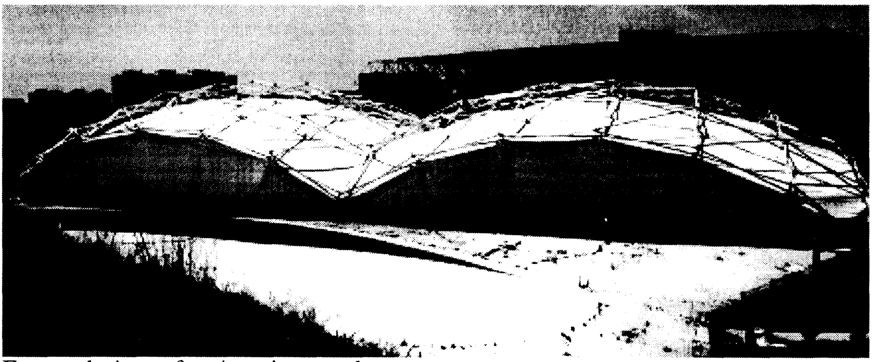
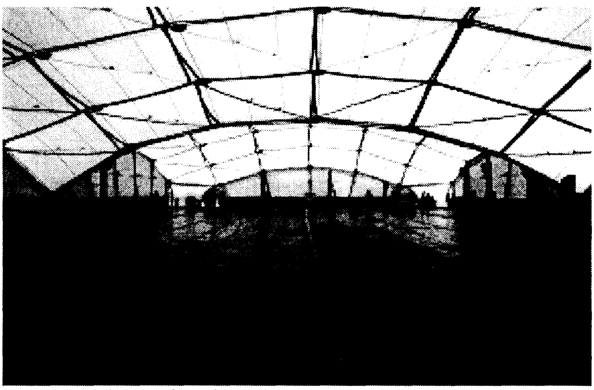
architects directed by Félix Escrig to cover an olympic swimming-pool. It involved the designing of a winter cover for the pool to permit its use in comfortable conditions. The covering had to be removable in summer, to leave the pool once again in the open air.

In the accompanying figures, some views of the deployment process, the finished cover and a detail of the knot, are shown. This last picture is specially relevant in order to understand the problems that the construction of the knots causes in the computer analysis of the truss.


Grid before deployment.


Grid after deployment.


External view of swimming-pool.


Internal view of swimming-pool.


Detail of lower knot.

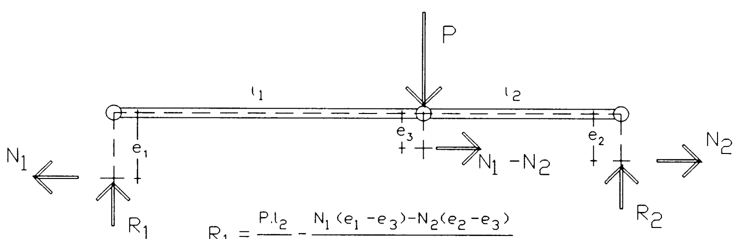
In other papers, different aspects of the design and construction of this dome will be studied. In this paper there will be explained the structural analysis of this cover and distinct aspects that have been studied for the calculation of the structure and that we believe are generally applicable to this type of grids.

2 Effect of eccentricity of knots.

The real dimension of the knots make it impossible for the bars to pass through the geometrical center of the joints. This problem has always been a worry for the investigators on expandable structures and appears in all types, as much in scissor modules as in bundle ones. The architect Emilio Pérez Piñero, pioneer in the design and construction of these structures tried to solve this problem using bend bars, as in his proposal for a expandable dome with a span of 34 m. The analysis of expandable grids with bended bars has been solved by our team as is showed in the references[ ], but our studies lead us to the opinion that the structural effectiveness of the system does not compensate the difficulties of construction.

Another possible solution consist in to assume the fact that the load in the knots has a certain eccentricity and in consequence to calculate the stiffness matrix of the bar considering these effects. This has been the system used for the analysis of the structure for the Seville swimming-pool.

Firstly, it is necessary to consider that the eccentric action of the axial forces on a bar modifies the bending moments distribution. Therefore they must be taken into consideration in the formulation of the stiffness matrix. This eccentricity can occur with respect to both axes but, by the proper definition of local axes, they normally occur in the direction of y-axis. The forces for any given bar are shown in figure



$$R_1 = \frac{P.l_2}{l} - \frac{N_1(e_1-e_3)-N_2(e_2-e_3)}{l}$$

$$R_2 = \frac{P.l_1}{l} + \frac{N_1(e_1-e_3)-N_2(e_2-e_3)}{l}$$

The strain energy for a bar is

$$W = \frac{1}{2}\cdot\int_a^b \left[\frac{M^2}{E\cdot I} + \frac{k\cdot V^2}{G\cdot A} + \frac{N^2}{E\cdot A}\right]\cdot ds$$

If, as is usual, the effect of the energy produced by shear forces is neglected, the displacements can be obtained by differentiation of the elastic energy with respect to the adequate forces in the manner

$$u_1 = \int_a^b \left[M\cdot\frac{\partial M}{\partial N_1} + N\cdot\frac{\partial N}{\partial N_1}\right]\cdot ds \qquad u_2 = \int_a^b \left[M\cdot\frac{\partial M}{\partial N_2} + N\cdot\frac{\partial N}{\partial N_2}\right]\cdot ds \qquad [1]$$

$$v = \int_a^b \left[M\cdot\frac{\partial M}{\partial P_1} + N\cdot\frac{\partial N}{\partial P_1}\right]\cdot ds \qquad w = \int_a^b \left[M\cdot\frac{\partial M}{\partial P_2} + N\cdot\frac{\partial N}{\partial P_2}\right]\cdot ds$$

Resolving these integrals we can obtain a flexibility matrix in the manner

$$\begin{bmatrix} u_1 \\ u_2 \\ v \\ w \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix} \cdot \begin{bmatrix} N_1 \\ N_2 \\ P_1 \\ P_2 \end{bmatrix}$$

In that

$$f_{11} = \frac{l_1}{E\cdot A} + \frac{e_{1y}^2\cdot l_1}{EI_y} - \frac{e_{1y}\cdot(e_{1y}-e_{3y})\cdot l_1^2}{l\ EI_y} + \frac{(e_{1y}-e_{3y})^2(l_1^3 + l_2^3)}{3\ l^2\ EI_y} + \frac{e_{1z}^2\cdot l_1}{EI_z} - \frac{e_{1z}\cdot(e_{1z}-e_{3z})\cdot l_1}{l\ EI_z} + \frac{(e_{1z}-e_{3z})^2(l_1^3 + l_2^3)}{3\ l^2\ EI_z}$$

$$f_{12} = f_{21} = \frac{e_{1y}\cdot(e_{2y}-e_{3y})\cdot l_1^2}{2l\ EI_y} + \frac{e_{2y}\cdot(e_{1y}-e_{3y})\cdot l_2^2}{2l\ EI_y} - \frac{(e_{1y}-e_{3y})(e_{2y}-e_{3y})(l_1^3+l_2^3)}{3l^2\ EI_y} + \frac{e_{1z}\cdot(e_{2z}-e_{3z})\cdot l_1^2}{2l\ EI_z} + \frac{e_{2z}\cdot(e_{1z}-e_{3z}\cdot l_2^2}{2l\ EI_z} - \frac{(e_{1z}-e_{3z})(e_{2z}-e_{3z})(l_1^3+l_2^3)}{3l^2\ EI_z}$$

$$f_{13} = f_{31} = \frac{e_{1y}\cdot l_1^2\cdot l_2}{2l\ EI_y} - \frac{(e_{1y}-e_{3y})(l_1^3\ l_2 - l_1\ l_2^3)}{3l^2\ EI_y}$$

$$f_{14} = f_{41} = \frac{e_{1z}\cdot l_1^2\cdot l_2}{2l\ EI_z} - \frac{(e_{1z}-e_{3z})(l_1^3\ l_2 - l_1\ l_2^3)}{3l^2\ EI_z}$$

$$f_{22} = \frac{l_2}{E \cdot A} + \frac{e_{2y}^2 \cdot l_2}{EI_y} - \frac{e_{2y} \cdot (e_{2y} - e_{3y}) \cdot l_2^2}{l\ EI_y} + \frac{(e_{2y} - e_{3y})^2 (l_1^3 + l_2^3)}{3l^2\ EI_y} + \frac{e_{2z}^2 \cdot l_2}{EI_z} - \frac{e_{2z} \cdot (e_{2z} - e_{3z}) \cdot l_2^2}{l\ EI_z} + \frac{(e_{2z} - e_{3z})^2 (l_1^3 + l_2^3)}{3l^2\ EI_z}$$

$$f_{23} = f_{32} = \frac{e_{2y} \cdot l_1 \cdot l_2^2}{2l\ EI_y} + \frac{(e_{2y} - e_{3y})(l_1^3\ l_2 - l_1\ l_2^3)}{3l^2\ EI_y}$$

$$f_{24} = f_{42} = \frac{e_{2z} \cdot l_1 \cdot l_2^2}{2l\ EI_z} + \frac{(e_{2z} - e_{3z})(l_1^3\ l_2 - l_1\ l_2^3)}{3l^2\ EI_z}$$

$$f_{33} = \frac{l_1^2\ l_2^2}{3l\ EI_y} \quad ; \quad f_{34} = f_{43} = 0 \quad ; \quad f_{44} = \frac{l_1^2\ l_2^2}{3l\ EI_z}$$

And by inversion of this matrix there can be obtained the stiffness matrix of the bar

$$\underset{\sim}{P} = \underset{\sim}{F}^{-1} \cdot \underset{\sim}{Z} \quad \Rightarrow \quad \underset{\sim}{K} = \underset{\sim}{F}^{-1} \qquad \begin{pmatrix} N_1 \\ N_2 \\ P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ v \\ w \end{pmatrix}$$

The compatibility matrix is similar to that which is obtained without considering the eccentricities in the knots, because it does not change the orientation of the local axes.

$$\underset{\sim}{Z} = \underset{\sim}{A} \cdot \underset{\sim}{X}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ v \\ w \end{pmatrix} = \begin{pmatrix} -\cos\alpha_1 & -\cos\beta_1 & -\cos\gamma_1 & \cos\alpha_1 & \cos\beta_1 & \cos\gamma_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\cos\alpha_1 & -\cos\beta_1 & -\cos\gamma_1 & \cos\alpha_1 & \cos\beta_1 & \cos\gamma_1 \\ -\frac{l_2\cos\alpha_2}{l} & -\frac{l_2\cos\beta_2}{l} & -\frac{l_2\cos\gamma_2}{l} & \cos\alpha_2 & \cos\beta_2 & \cos\gamma_2 & -\frac{l_1\cos\alpha_2}{l} & -\frac{l_1\cos\beta_2}{l} & -\frac{l_1\cos\gamma_2}{l} \\ -\frac{l_2\cos\alpha_3}{l} & -\frac{l_2\cos\beta_3}{l} & -\frac{l_2\cos\gamma_3}{l} & \cos\alpha_3 & \cos\beta_3 & \cos\gamma_3 & -\frac{l_1\cos\alpha_3}{l} & -\frac{l_1\cos\beta_3}{l} & -\frac{l_1\cos\gamma_3}{l} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ x_2 \\ y_2 \\ z_2 \\ x_3 \\ y_3 \\ z_3 \end{pmatrix}$$

The application of these equations to the structure under review has let us confirm that the effect of the eccentricity of the knots is not specially relevant but it is not negligible. In the following table there can be observed the comparative results of several bars of the most representative bars of the corner modulus of the dome, under its own weight and under snow. The structure was

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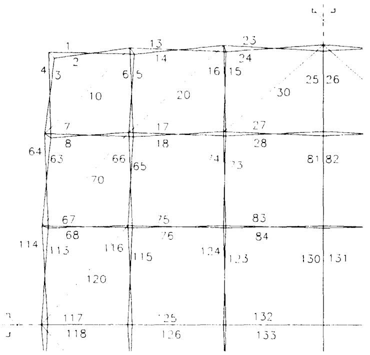
calculated considering the knots centred with the bars and considering the eccentric knots.



It can be observed that the effect to the eccentricity of the knots supposed a certain variation in the efforts over the bar. The change is more important for the axial forces and less so for the bending moments. The sense of the variation is not constant and thus some are increased, others diminished. The deflection of the whole structure are increased in all cases. This is a logical result, because the main effect of the eccentricity of the knots is to diminish the axial stiffness of the bar.

	Bars centred on knots				Bars eccentric on knots			
	N1 (t)	N2 (t)	M1 (mt)	M2 (mt)	N1 (t)	N2 (t)	M1 (mt)	M2 (mt)
4	2.05	1.89	0.06	0.02	1.25	1.22	0.07	0.00
5	-4.17	-3.13	0.03	-0.53	-4.19	-2.87	0.03	-0.48
8	-3.86	-3.42	0.02	0.64	-3.93	-3.20	0.02	0.59
15	-2.19	-2.39	0.03	-0.49	-2.33	-2.36	0.04	-0.48
17	-3.30	-0.63	0.01	0.77	-3.07	-0.32	0.01	0.77
18	-1.17	-3.87	0.01	0.42	-1.28	-4.06	0.01	0.41
25	-2.58	-4.94	0.01	-0.81	-2.29	-4.61	0.01	-0.76
28	-2.36	-5.08	0.01	0.63	-1.90	-4.74	0.01	0.60

3 Effect of the initial bending of bars.

Because of the real dimension of the knots and the need of carry out the structure in compact packages, it is necessary for the bars of each scissor to be separated by a certain distance after deployment. But as these bars are joined at a central point by an articulation, they have to bend slightly inwards. From the point of view of the analysis of the problem, this is equivalent to imposing a condition of imposed bending to bars of the scissor.

In the useful structures the lengths of the bars are enough for the deflection to be slight and consequently the bending moments are not excessive. The elastic strain energy is kept within reasonable limits, by which we understand that these structure are viable for architectural uses.

The analysis of these grids can be realised with the matrix formulation already explained, but is necessary to apply a new condition to the stiffness matrix in global coordinates in order to take into account the effect of the imposed bending by the existence of the inner joint.

$$\vec{d} = \vec{d}_1 - \vec{d}_2 \qquad \begin{aligned} \delta_{x_1} - \delta_{x_2} &= d_x = d \cdot \cos\alpha \\ \delta_{y_1} - \delta_{y_2} &= d_y = d \cdot \cos\beta \\ \delta_{z_1} - \delta_{z_2} &= d_z = d \cdot \cos\gamma \end{aligned}$$

This condition can be applied without difficulties modifying the stiffness matrix in a manner that satisfies the aforementioned restrictions. In this case only two variables x_i and x_j may be estudied, because the restriction is the same for other axis.

$$\begin{pmatrix} & & & \cdot & & \cdot & & \\ & & & \cdot & & \cdot & & \\ \cdot & \cdot & \cdot & s_{ii} & \cdot\ \cdot & s_{ij} & \cdot & \cdot \\ & & & \cdot & & \cdot & & \\ & & & \cdot & & \cdot & & \\ & & & \cdot & & \cdot & & \\ \cdot & \cdot & \cdot & s_{ji} & \cdot\ \cdot & s_{jj} & \cdot & \cdot \\ & & & \cdot & & \cdot & & \\ & & & \cdot & & \cdot & & \end{pmatrix} \cdot \begin{pmatrix} \cdot \\ \cdot \\ x_i \\ \cdot \\ \cdot \\ \cdot \\ x_j \\ \cdot \\ \cdot \end{pmatrix} = \begin{pmatrix} \cdot \\ \cdot \\ P_i - F_x \\ \cdot \\ \cdot \\ \cdot \\ P_j + F_x \\ \cdot \\ \cdot \end{pmatrix}$$

Summing both rows to eliminate the unknown reaction Fx and sustituing the second row by the known equation $x_i - x_j = d \cos \alpha$. To keep the symmetry of the stiffness matrix it is necessary for column j to have all its terms null except the ij one that will be equal to -1 and the jj that will be to equal to 1. To achieve this we take any n row and clear off y_j in function of y_i. The result will be

$$\begin{pmatrix} & & & \cdot & & & 0 & & \\ & & & \cdot & & & 0 & & \\ \cdot & \cdot & \cdot & s_{ii}+2s_{ij}+s_{jj}+1 & \cdot & \cdot & 1 & \cdot & \cdot \\ & & & \cdot & & & 0 & & \\ & & & \cdot & & & 0 & & \\ & & & \cdot & & & 0 & & \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ & & & \cdot & & & 0 & & \\ & & & \cdot & & & 0 & & \end{pmatrix} \cdot \begin{pmatrix} \cdot \\ \cdot \\ x_i \\ \cdot \\ \cdot \\ \cdot \\ x_j \\ \cdot \\ \cdot \end{pmatrix} = \begin{pmatrix} \cdot \\ \cdot \\ P_i + P_j + (s_{ij}+s_{jj}-1)\, d\cos\alpha \\ \cdot \\ \cdot \\ \cdot \\ d \cdot \cos\alpha \\ \cdot \\ \cdot \end{pmatrix}$$

The other terms of the row i will be $s'_{in} = s_{in} + s_{jn}$

The other terms of the colum i will be $s'_{ni} = s_{ni} + s_{nj}$

Lastly, the terms of the load vector will transform in $P'_n = P_n - s_{nj} \cdot d \cdot \cos\alpha$

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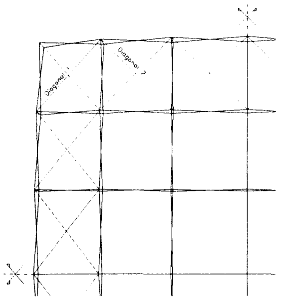
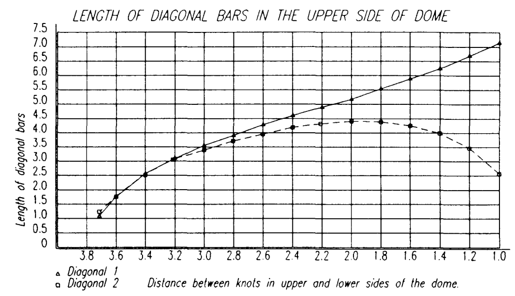
In practice the influence of the initial bending of the bars is very little for the normal dimensions. Furthermore it is opposite to the effect of the eccentricity of the knots, which is to an extent why the effects could compensate one another.

4 The stiffening problem

The studies carried out on this kind of domes show that effective stiffening is indispensable for grids made of squared modulus. Obviously a single square modulus lacks transversal rigidity and needs, in order to be stable, a traction cable or a compression bar. The most simple solution would be the use of cables, because they can be folded with the package of bars and upon being deployed, limit the opening and fix the grid in its final position. Nevertheless there are two serious problems. The first is that the wind could produce an inversion in the forces and the tractioned cables could become compressed, making them useless. The second problem can be observed in the figure. Measuring the length of both diagonal bars during the deployment, we see that the doted one designated as 2 grows constantly, whereas the continuous one designated as 1 grows until it reaches a maximum and later decreases. This means that diagonal 2 can be made with cables whose length would be the final one. In the other hand if diagonal 1 has its maximum length, they remain untensed in the final position and if they have its final length, they must be stretched unacceptably during the deployment.





The adopted solution was to add external stiffeners after deployment of the whole structure. It was a reasonable and effective solution, but needed a considerable effort of adjustment during the construction. But another interesting strategy could be suggested by the observation of the curves of the figure. It would consist of using stiffeners with a blockable central articulation. Obviously they would have to be situated in direction 2 in such a manner that in deployment they not reach an incompatible position and can be fixed manually into their final position, blocking the joint, without having to use complex mechanisms.

It is a notable fact that the structures with square modules do not always admit this solution, therefore the election of the correct direction for stiffeners,

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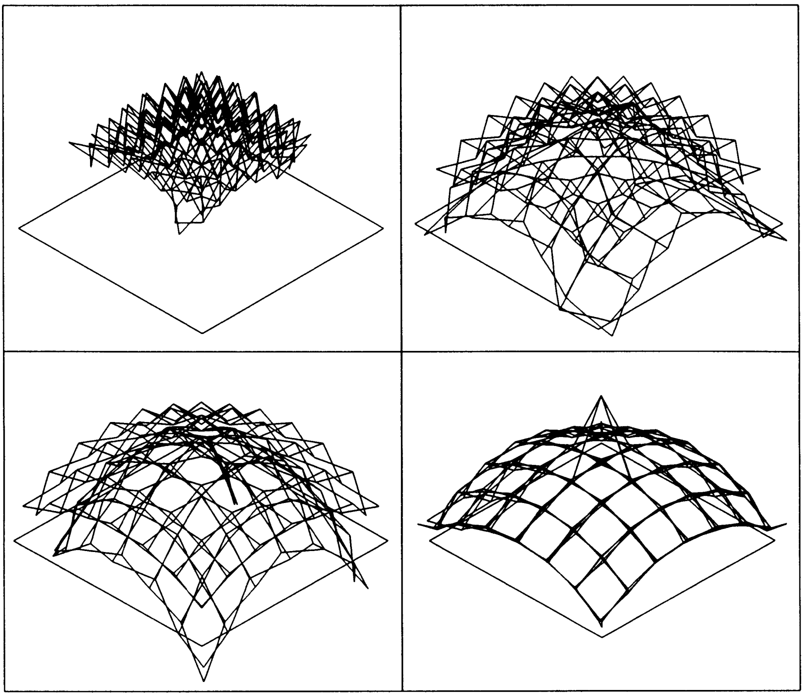
can be specially important.

5 The deployment problem

An expandable structure is, by its proper nature, a mechanism. If not it could not to be deployed. For build a structure able to resist the external loads, it is necessary to fix a set of points of its contour. After this it can begin to work as a conventional structure.

In the process of deployment the bars move theoretically as a solid body, and must therefore be submitted only to dynamic forces. These forces have a very reduced value due to the slowness of the process of deployment. Therefore it is reasonable to be expected that the forces in the deployment phase be minimal.



Nevertheless this is not a certainty. Experience shows that during this phase the friction over the knots is such that the behaviour of the grid is similar to that of a hanging structure in a set of joints and subjected to its own weight. It is necessary to calculate the structure in several positions and be sure that in the case of friction on the knots blocking the mesh temporarily in an intermediate position, the bars be able to resist without breaking.

In practice this obliges one to design carefully the hoisting systems and to calculate the structure in diverse positions. In the figures there can be observed the corresponding results to the Seville pool covering.

CONCLUSIONS.

We have seen through out this paper several practical problems that occur in the analysis of expandable structures and the corrections that must be made in calculation methods in order to obtain results in accordance with reality. The calculation method that is proposed is thought to be a considerable improvement over previous proposals. The differences are not great and in some cases they increase and in other decrease the tensions over the bar, but we believe that this method permits a more correct approximation to reality.
Likewise, the consideration of the effect of initial bending of bars is necessary, because, although of scarce magnitude, it always increases the tensions on the bars. Lastly, the correct design of the stiffeners and of the process of deployment are essential to get that the structure works adequately.

ACKNOWLEDGEMENTS

This paper has been made possible by a grant from the XUNTA DE GALICIA through a research project.

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