# **Similitude law evaluation for composite structures using optical techniques**

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# **Abstract**

The aim of this paper is to contribute to similitude law development applied to composite structures. These laws permit one to extrapolate the small-scale model behavior to the real scale one. Existing approaches have been established following two different methods. They are summarized in this paper and applied to impact loadings on two laminated plate scales. In order to complete data collected by "conventional" instrumentation (force transducer, displacement sensor, accelerometer...), an optical device such as a high-speed CCD camera, associated with optical techniques for the monitoring of markers, was used. These techniques make it possible to compare displacement lines corresponding to each scale. It is shown that existing similitude laws, used for elastic materials, do not allow one to simulate the behavior of the real scale when this one is damaged.

*Keywords: optical measurement, similitude laws, impact, composite materials.*

# **1 Introduction**

For several years, composite materials have played a significant part in the realization of structures used for transportation (aeronautical, nautical, automotive...). During manufacturing or storing of such structures, they can be damaged locally by tool drops without there being any visible deterioration. That is why designers have to appreciate the criticality of such damages to check that this kind of accident is not harmful for the structure during its life. Nevertheless, tests are often very expensive and difficult to set up, especially when the structures' dimensions are large (fuselages of aircraft, ship hulls, etc.). An alternative way then is to employ small-scale models. The use of these reduced scale structures calls for the identification of similitude techniques allowing the behavior extrapolation from the model to the real scale.



Firstly, this paper presents two existing approaches used to develop similitude laws applied to composite structures. In a second part, an experimental study achieved on two laminated plate scales subjected to impact is presented. Finally, existing similitude laws are compared to experimental results and discussed.

#### **2 Similitude laws applied to composites structures**

Two basic approaches are used to develop scaling rules (see Abrate [1]). The first one is based on a dimensional analysis using Buckingham's Pi theorem [2]. The second one starts with the dynamic equations of the system ([3, 4]).

Morton [5] studied scale effects, using a dimensional analysis, in the case of beams subjected to impact loading. For homogeneous isotropic beams, considering a non damaged behavior, it is possible to identify thirteen influential parameters in order to completely model the test : the beam geometrical parameters (length  $l$ , thickness  $h$  and width  $b$ ), the beam specific properties (Young's modulus  $E$  and Poisson's ratio  $\nu$ ), the impactor features (Poisson's ratio  $\nu_i$ , Young modulus  $E_i$ , volumic mass  $\rho_i$ , the radius  $R_i$  and the impact velocity  $V_i$ ), the central deflection of the sample  $w$  and time  $t$ .

Using Buckingham's Pi theorem [2], ten nondimensional parameters can be formed (table 1).

Geometrical parameters	$\Pi_1 = \frac{w}{l}$	$\begin{array}{cc} \n\end{array} \n\begin{array}{c} \n\Pi_2 = \frac{l}{h} \n\end{array} \n\begin{array}{c} \n\Pi_3 = \frac{b}{h} \n\end{array} \n\begin{array}{c} \n\Pi_4 = \frac{R_i}{h} \n\end{array}$	
Materials parameters		$\Pi_5 = \frac{E_i}{E}$ $\Pi_6 = \nu$ $\Pi_7 = \nu_i$ $\Pi_8 = \frac{\rho_i}{\rho_i}$	
Tests conditions	$\Pi_9 = \frac{\rho_i V_i^2}{E} \left  \Pi_{10} = \frac{t V_i}{h} \right $		

Table 1: Nondimensional parameters.

If the prototype is a true replica of the model and if the same materials are used, then all the geometrical terms are equal between the prototype and the model. Equalizing all these terms, the impactor velocity has to remain the same. The  $\Pi_{10}$ term implies that the time have to be scaled. Finally, considering the scale factor between the model and the real structure  $\lambda$ , impact energy is scaled by  $\lambda^3$  and the impact force by  $\lambda^2$ .

Nettles et al. [6] used this approach to study quasi-static indentation and impact of unidirectional reinforced laminated plates. Experimental results were compared to parameters obtained by similitude laws (contact force, displacement, damage area and indentation). Differences are significant.

Dormegnie et al. [7] observe the same conclusions in the case of laminated omega-shaped structures subjected to crash.

The second approach we can find in the literature is based on the dynamic theorem and was particularly developed by Qian and Swanson [3, 4] for plates. Experiments were realized on composite plates considering scale factors  $\lambda$  of 1, 3 and 5. Deflection and contact force results exhibit good accuracy of these laws. Nevertheless, it's more difficult to predict damage size.

Many authors have established similitude laws applied to composite structures based on this approach. For instance, Ungbhakorn and Singhatanadgid or Razaeepazhand and Simitses have used it to study cylindrical shells buckling under axial loading [8, 9]. A similar investigation was presented by Chouchaoui et al. for various loadings [10].

Simitses and Rezaeepazhand adopted a similar approach to develop laws for laminated plates subjected to quasi-static bending [11] and for vibration response of cylindrical shells [12].

It is interesting to note that, contrary to Qian and Swanson's first study, these authors assign a scale factor to the structure materials properties.

To conclude, described methods allow us to establish relationships between two scales. Therefore, in the case of laminate plates subjected to impact, all plate dimensions as well as the impactor radius and the deflection are scaled by  $\lambda$ . Concerning materials specific characteristics and impact velocity, they remain the same between the model and the prototype. Time is scaled by  $\lambda$ .  $\lambda^3$  assigned to impact mass implies that energy is scaled too by  $\lambda^3$ . Finally, contact forces are scaled by  $\lambda^2$ .

# **3 Experimental study**

In order to experimentally appreciate these laws, impact tests on two laminated plate scales were achieved using optical devices for displacement field measurement.

### **3.1 Specimens**

Two different scales of plane plates were manufactured. Specimens were made using unidirectional carbon/epoxy pre-preg 914C-TS(6K)-5-34% and the ply thickness is about 125  $\mu$ m. First samples (samples A) characteristics were arbitrarily chosen. Therefore, the stacking sequence for samples A is :

$$
(0)_2(90)_3(0)_2(90)_3(0)_2 \\
$$

After the first stage of plies stacking, samples have been cured one hour at 175C and 7 bar pressure, plus 4 hours postcure at 190C. Approximately twelve hours later, they were cut out with the following dimensions : length  $= 100$  mm, width  $=$ 50 mm.

The second scale characteristics were determined using similitude laws previously described with a scale factor  $\lambda = 2$ . Specimen thickness, controlled by the



stacking sequence, was obtained by scaling each ply thickness ("Sub-Ply Level Scaling" method proposed by Jackson [13]).

#### **3.2 Experimental devices**

#### **3.2.1 Apparatus**

A drop tower was used for these experiments (figure 1). The dropping weight is raised by a winch and located to the wanted height thanks to optical cells. It is then conducted during its fall and stopped after the first impact by an anti-bouncing system in order to avoid a second nondesired impact. The drop height can vary between 20 cm and 3 m making it possible to reach speeds up to 7,5 m.s*<sup>−</sup>*<sup>1</sup>. The dropping weight can reach 30 kg corresponding to a 900 J impact energy.

Tests carried out with this drop tower can be instrumented thanks to different types of sensors. Therefore, a first laser sensor (50 mm displacement range) is used to measure the impactor displacement in order to evaluate the impact velocity. A second one with a lower range, allows to measure the displacement of one structure's point.



Figure 1: Drop tower.

To measure contact force between the dropping weight and the structure, a piezoelectric force transducer is set between the dropping weight and the impactor. It is also possible to set up an accelerometer to measure the dropping weight acceleration during impact.



#### **3.2.2 Optical measurement of displacement field**

In order to complete data collected by the "conventional" instrumentation previously described, video acquisitions were realized with an high-speed CCD camera used with 512 x 256 pixels resolution and 2500 frames/s frequency.

Markers were glued on the section and the lower face of the specimens (figure 2).



Figure 2: Markers set-up.

Video obtained is split up into a sequence of frames and these ones are then transformed into grayscale in order to obtain a better contrast between the markers and the specimen.

Initial and final marker positions are located thanks to a specific software (figure 3). Then, it calculates each marker center and follow it during all the test. Therefore, it become possible to obtain point displacement on two specimen lines. For this paper, only displacements of markers 2 will be presented.



Figure 3: Markers monitoring (sample A).

### **3.3 Test conditions**

On the one hand, since similitude laws previously described are restricted to an elastic behavior of structures (section 2), test conditions for samples A were chosen

to avoid significant damage ( $m = 1$  kg and  $V_i < 2$   $m.s^{-1}$ ). To avoid a too important stiffness of the specimens, simply supported conditions were chosen. The distance between supports was chosen depending on the samples dimensions. The impactor diameter was arbitrarily chosen.

Tests conditions for samples B were determined following similitude laws as shown in Table 2.

On the other hand, in order to point out that these laws cannot predict the behavior of a damaged structure, an higher velocity was chosen ( $V = 2.3$   $m.s^{-1}$ ).

	samples A	samples B
Dropping weight mass $(kg)$	1.075	8.6
Impact velocities $(m.s^{-1})$	1.75, 1.85 and 2.3	1.75, 1.85 and 2.3
Impactor diameter $(mm)$	10	20
Boundary conditions	simple supports	simple supports
Distance between supports $(mm)$	100	200

Table 2: Test conditions.

#### **3.4 Results**

#### **3.4.1 Validation of the optical technique**

In order to validate the optical method, a marker has been glued on the impactor to compare usual measurements obtained by laser sensor and optical measurements. As shown figure 4, this method allows us to measure with good accuracy the system displacement. Indeed, the maximum variation observed on displacement for the same time t is  $0.8$  mm. Considering  $15$  mm maximum displacement, we can reach an accuracy of about 5%.

Therefore, this preliminary measurement shows that the technique of marker monitoring gives good results and can be used to determine specific points displacements of the sample.

#### **3.4.2 Analytical/experimental confrontation**

Similitude laws, described in section 2, indicate that the deflection and the time are scaled by  $\lambda = 2$ . Then, it's seems natural to compare these parameters to those experimentally obtained.

For each velocity level, the scale factor  $\lambda$  was assigned to deflection and time of samples A. Results obtained by this scaling, called "samples  $\lambda A$ ", have been compared to experimental results of samples B.



Figure 4: Methods comparison.



Figure 5: Analytical/experimental confrontation ( $V = 1.85$  m.s<sup>-1</sup>).

Figure 5 shows the impactor displacement according to the time measured from optical method. It is possible to determine the impact velocity calculating the segment MI slope. From this measurement, we can check the necessary condition for the velocity previously described in the similitude laws  $(V<sub>i</sub> = constant)$ . Point I represent the impact point and the contact duration is determined by the segment IJ ( $\Delta t = 0.025$  s). This contact duration remains the same between samples B and samples  $\lambda A$ . Thus, the similitude laws seem to predict with good accuracy this parameter. The maximum impactor displacement (point N) is obtained at  $t = 0.012$  s and the same value is found for samples  $\lambda A$  and samples B. Therefore, these impactor displacement measurements also allow us to validate similitude laws in this test configuration.

Figure 6 shows the markers 2 displacement for samples  $\lambda A$  and B at the maximum deflection ( $t = 0.012$  s). Support points are not represented in this figure (at  $z=0$ ).



Figure 6: Markers 2 displacement ( $t = 0.012$  s).

It is shown that the medium line positions of the two samples are very close. We only note a slight asymmetry between the two results, maybe due to an impact location shift. For these same conditions (non damaged plate), the medium line shapes are respected in accordance with the similitude laws.



Figure 7: Analytical/experimental confrontation ( $V = 2.3$  m.s<sup>-1</sup>).

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Figure 8: Markers 2 displacement ( $t_1 = 0.01$  s).



Figure 9: Markers 2 displacement ( $t_2 = 0.02$  s).

Results obtained for <sup>1</sup>.<sup>75</sup> m.s*<sup>−</sup>*<sup>1</sup> impact velocity are not presented in this paper because they are very close of those obtained for <sup>1</sup>.<sup>85</sup> m.s*<sup>−</sup>*<sup>1</sup> velocity.

In the case of an impact velocity of <sup>2</sup>.<sup>3</sup> m.s*<sup>−</sup>*<sup>1</sup>, results are quite different. For this velocity level, the structure behavior is not elastic, a significant damage on samples B can be observed. Figure 7 shows the impactor displacement according to time.

For time lower than  $t_1 = 0.01$  s, the structure behavior being elastic, we can observe similar results with previous tests. The similitude law is validated. However, for a larger time, both impactor displacement curves diverge. Indeed, whereas contact duration is about  $0.038$  s for samples B, it remains lower than  $0.025$  s for sample  $\lambda A$ . In the same way, for a given time, displacements are definitely different.

Until  $t_1$ , both structures exhibit an elastic behavior but, as soon as this duration is over, samples B are damaged to a significant degree by delamination. At this point, the curves shape become very different.

These results are exhibited by the representation of medium lines evolution for several different times.

For a time lower than  $t_1$ , again we find curves very close between samples  $\lambda A$  and B (figure 8). Indeed, displacement variation remains in the measurement scatter. Nevertheless, after damage (figure 9), curves become really distinct with a displacement variation equal to  $6.5$  mm.

To conclude, tests presented in this paper exhibit the limitations of these similitude laws. Although very useful to predict with good accuracy an elastic behavior, these laws do not allow to describe the scaling of damaged structures.

## **4 Conclusions**

In order to validate similitude laws applied to composite structures, impact tests were achieved on two laminated plate scales. Several impact velocities have been chosen to evaluate structure behavior according to impact conditions (mass and velocity). Optical methods, used for the markers following, allowed to determine structures medium line deformation. These tests exhibit that usual similitude laws can be used to predict the behavior of a scaled structure providing that the material behavior is elastic. It has be shown, thanks to these tests, that these laws, initially established to describe an elastic behavior, cannot take into account the structure damage.

The applicability of these laws, initially developed in order to model the elastic behavior of material for different scales, can not be enlarged to composite structure damage. It is thus necessary to define new similitude laws to take into account the damage phenomenon.

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