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Application of the 2DH advection-diffusion equation in the inshore zone

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Abstract

Two wave models, an average over the period and an intra period, are applied to derive the wave characteristics in the inshore zone. A wave induced current model is used for the prediction of the velocity field considering the effects of the secondary current. The surf zone current field is used for the application of the advection-diffusion model by means of which the transport of a well-mixed, surface and bottom pollutant in and outside the surf zone can be modelled.

1 Introduction

Numerical models based on the 2-D horizontal Fickian advection-diffusion equation are widely used in coastal engineering for the simulation of various phenomena such as oil spill time-space evolution, prediction of pollutant concentrations from sea outfalls, thermal diffusion of discharged warm water e.g. Borthwick & Joynes [2], Falconer et al. [8], Tanaka & Wada [17]. The same equation is also used for the suspended sediment transport modelling in a seabed evolution model, e.g. Li & Wright [12].

In a 2-D horizontal form the advection-diffusion equation is written

$$\frac{\partial c}{\partial x} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = \frac{\partial}{\partial x}\left(E_x\frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(E_y\frac{\partial c}{\partial y}\right) + S_1 + S_2 + .. \quad (1)$$

where u and v the mean over the depth horizontal current velocities in x and y directions respectivelly, c the mean concentration (of the pollutant or the suspended sediment), E_x and E_y the depth average dispersion-diffusion coefficients in x,y directions.The terms S_1, S_2.... simulate different processes in each model such as resuspension and deposition rate in a suspended sediment model, heat exchange in a thermal diffusion model, source, sink and decay rate faecal coliform in a water quality model.

Inside the inshore zone breaking wave induced currents are very important for the prediction of the nearshore circulation. In addition two important mixing mechanisms are operating: diffusion due to the wave breaking induced turbulence and convective dispersion due to the vertical structure of the horizontal wave and current velocities.

2 Wave models

In the shallow water area wave refraction and shoaling are simulated through the equations given by Chen & Wang [3] and Phillips [13]:

$$\frac{\partial}{\partial t}(k\cos\theta)+\frac{\partial\sigma}{\partial x}=0$$

$$\frac{\partial}{\partial t}(k\sin\theta)+\frac{\partial\sigma}{\partial y}=0 \tag{2}$$

$$\frac{\partial(k\cos\theta)}{\partial y}-\frac{\partial(k\sin\theta)}{\partial x}=0 \tag{3}$$

$$\frac{\partial E}{\partial t}+\frac{\partial}{\partial x}\left[E\left(u+c_g\cos\theta\right)\right]+\frac{\partial}{\partial y}\left[E\left(v+c_g\sin\theta\right)\right]+S_{xx}\frac{\partial u}{\partial x}+S_{xy}\frac{\partial v}{\partial x}$$
$$+S_{yy}\frac{\partial v}{\partial y}+S_{yx}\frac{\partial u}{\partial y}=-\varepsilon_d(f)+\varepsilon_g(f) \tag{4}$$

where k the wave number, θ the wave angle, σ the apparent frequency c_g the wave group velocity, ε_g and ε_d are the rates of energy generation and dissipation respectivelly, E the energy density distribution and S_{xx} e.t.c. the radiation stresses.

Inside harbours and behind breakwaters an intra period hyperbolic type mild-slope equation model is used (Copeland [5], Karambas et al. [11]).

3 Nearshore current model

The breaking wave induced current model is based on the following 2-D horizontal system of equations:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+g\frac{\partial\zeta}{\partial x}=\frac{\tau_{sx}-\tau_{bx}}{\rho h}-\frac{1}{\rho h}\left(\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}\right)+\nu_h\nabla^2 u$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+g\frac{\partial\zeta}{\partial x}=\frac{\tau_{sy}-\tau_{by}}{\rho h}-\frac{1}{\rho h}\left(\frac{\partial S_{xy}}{\partial x}+\frac{\partial S_{yy}}{\partial y}\right)+\nu_h\nabla^2 v$$

$$\frac{\partial\zeta}{\partial t}+\frac{\partial Q}{\partial x}+\frac{\partial P}{\partial y}=0 \tag{5}$$

where u,v are the current depth average velocities, ζ the current surface elevation, h=d+ζ, Q=hv, P=hv, τ_s, τ_b are the surface and bottom shear stress respectively. The eddy viscosity ν_h is given in the next paragraph (eq. 14). Radiation stresses S_{xx}, S_{xy} and S_{yy} are calculated from the wave model results. In some cases the estimation of the vertical structure of the velocities u and v is required. From experiments and field measurements, the existence of a relatively strong offshore flow below trough level (undertow) is observed, to balance the mass flux due to wave motion above trough level. The simuation of this flow requires a full 3-D flow model or a quasi 3-D model (deVried & Stive [7]). We use here a simplified version of deVried & Stive method which is outlined below. It is assumed that the total current is split into a "primary" and a wave-induced "secondary" current:

$$u_t(z) = u_p + u_s(z)$$
$$v_t(z) = v_p + v_s(z) \tag{6}$$

by definition $\int_{-d}^{\zeta} u_s dz = 0$, $\int_{-d}^{\zeta} v_s dz = 0$

The primary current field is given from the solution of eqns. [5] with uniform distribution over the depth:

$$u_p = u$$
$$v_p = v \tag{7}$$

The wave induced secondary current is derived from the wave model assuming that the water column is divided into two layers (fig. 1):

-A surface (upper) layer above wave trough level containing the moving water surface due to wave motion. In this layer the mass flux M for non-breaking waves is given by:

$$M = \frac{E}{c} \tag{8}$$

with E the wave energy, while for breaking waves, deVried & Stive [7]:

$$M = \left(1 + \frac{7h}{L}\right)\frac{E}{c} \tag{9}$$

due to the surface roller contribution.

-A lower layer below wave trough level (ignoring the bottom boundary layer) which balance the mass flux above trough level M.

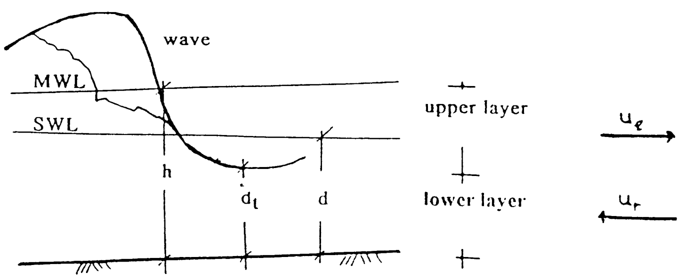

Figure 1: Definition sketch

As in the deVried & Stive method it is also assumed that the wave-induced secondary current has the direction of the wave-energy flux (on-offshore x-direction):

$$u_{s-upper} = u_e = \frac{M}{d_a} \tag{10}$$

$$u_{s-lower} = u_r = \frac{M}{d_t} \tag{11}$$

with $d_a = h - d_t$ and u_e, u_r the upper and lower layer secondary velocity respectivelly. In the longshore direction (y-direction) there is no wave induced mass flux M. The combination of eqns. [6] with eqns. [10] and [11] (a simple vectorial addition) will result to a 3-D velocity field.

As it is mentioned above the adopted approach is a simplified version of the deVried and Stive method since the effects of the shear stress at the wave trough level have been ingnored. Inclusion of the above effects results to a non-uniform distribution below wave trough level. However undertow measurements, show no significant variation over the depth. The bottom boundary layer is incorporated indirectly in the model (eqn.[1]) via the turbulent diffusion coefficient, eqn. [14].

4 Diffusion-dispersion coefficients

The coefficients E_x and E_y in eqn. [1] are given by:

$$E_x = \nu_r + D_x \tag{12}$$

$$E_y = \nu_r + D_y \tag{13}$$

where ν_r is the diffusion coefficient due to the wave breaking turbulence and D_x, D_y the wave-induced convective (or longitudinal shear) dispersion coefficients due to the time average and depth integration, Borthwick & Joynes [2], Smith [15], Fisher et al. [9].

Turbulent diffusion coefficients

Turbulent diffusion coefficient is generally taken equal to the eddy viscocity coefficient which is given by de Vried and Stive [7]:

$$\nu_r = \frac{5}{24}\kappa h u_* + Mth\left(\frac{D_b}{\rho}\right)^{1/3} \tag{14}$$

where u_* is the wave and current bed friction velocity, D_b is the mean rate of breaking wave energy dissipation per unit area, κ the von Karman constant and Mt a constant to be determined. The first term of the right-hand side of eqn. [14] is similar to the term used in the steady or weakly varying flow models (such as tidal models). The additional mixing due to the non-breaking wave orbital motion is incorporated through the increase of the current friction velocity u_*. The second term represents the increase of the turbulence production due to the wave breaking. The rate of dissipation D_b is given by:

$$D_b = \frac{1}{4T} g \frac{H^3}{h} \tag{15}$$

where T is the wave period. D_b=0 for non-breaking waves.

Since only a small part of the turbulence generated in the roller area spreads downward, below trough level, we expect the value of Mt in this region will be smaller than the corresponding value of the upper layer where the major part of turbulence is trapped and dissipated in the roller. Svendsen [16], assuming that the turbulence length scale l to be 20-30% of the total depth h (l=0.2-0.3 h) found that only 2-5% of the total energy production is dissipated below trough level. This leads to a value of Mt≡ M_l in this region about 0.12. Following Svendsen's analysis a depth average value for Mt≡M_m=0.45 can be derived assuming that all the energy of the wave motion is transformed into turbulence, Battjes [1]. With the mean, M_m and lower region M_l values of Mt known and assuming that the lower region height to be 80% of the total depth h we can conclude that the value of Mt≡M_u=1.8 in the upper layer. In this region the turbulent length scale l is expected to be larger than 0.2-0.3h, due to the presence of the large vorteces, resulting to a higher value for Mt. Unfortunately the lack of experimental data above trough level does not permit to a more precise representation of the turbulence characteristics in this region.

The above approach led to three different values of M: M_l for the lower layer (below trough level, suspended sediment transport model), M_u for the upper layer (above wave trough level, surface pollutant transport) and M_m for the total depth (well-mixed over the depth pollutant transport).

Longitudinal shear dispersion coefficients for well-mixed pollutants

The non-uniform over the depth distribution of the horizontal velocities results to a shear-dispersion mechanism (Fisher [9]) which is generally more significant than the turbulent diffusivity. Wave oscillatory motion as well as the vertical structure of the horizontal velocities have been considered for the estimation of the dispersion coefficients D_x and D_y.

The total dispersion coefficient for a well-mixed or solute pollutant is given by:

$$D_x = D_c + D_o + D_s$$
$$D_y = D_c \tag{16}$$

when D_c arise from the non-uniform vertical distribution of the primary current velocity, u_p, v_p, D_o from the oscillatory wave motion, u_w, v_w, D_s from the secondary vertical structure, u_s, v_s .

For a turbulenct shear flow, assuming a logarithmic velocity profile, the dispersion coefficient D_c is given by the Elder's expression, Chikwendu [4]:

$$D_c = \frac{0.4041}{\kappa^3} hu. \tag{17}$$

with u_* is the primary current friction velocity.

Since eqn. [17] is widely used in the open channel flow simulation such as tidal and river flows, it is adopted here for the primary current dispersion coefficient estimation, under the assumption that the wave-induced current has a logarithic velocity profile.

Smith [14] , assuming a linear shear profile for the oscillatory horizontal velocity proposed the dispersion coefficient D_o:

$$D_o = \frac{1}{2}\left(\frac{a}{\omega}\right)^2 \nu_t \tag{18}$$

where $\omega = \dfrac{2\pi}{T}$, $\quad a = \dfrac{u_d}{h} = \dfrac{u_{surface} - u_{bottom}}{h}$

For a breaking wave, in the upper layer of the crest phase the horizontal velocity changes from a near celerity c value at the surface to a much smaller value of order 0.2-0.3c (Stive [15]). The estimation of the a/ω term in eqn. [18] requires the use of a non-linear breaking wave model, since irrotational theory is not valid in the surf zone. In order to estimate the order of magnitude of D_o we apply Stive's wave conditions for a breaking wave:

$h=0.15$ m, $T=1.79$sec , $\omega=2\pi/(0.2T)$ (crest phase of the wave when $u_d \neq 0$)

Substituting these values in eqn, [18] we have:

$$D_o \cong 0.052\nu_t \tag{19}$$

The mean over the period value of D_o is expected to be less than the prediction of eqn. [19] since $D_o=0$ in a time interval about 0.8T.

Despite the empirical approach we followed, it is concluded the contribution of D_o in the dispersion coefficient D_x is not significant and can be ingnored.

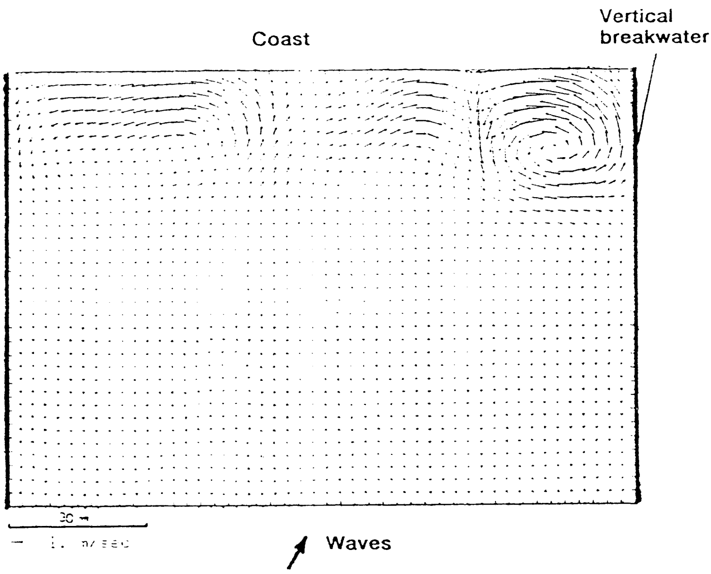

Figure 2: Wave-induced current field.

Finally due to the secondary current $u_s(z)$ vertical structure, fig. [1], the dispersion coefficient D_s is estimated from the method of zones proposed by Chikwendu [4]. In this method the flow is divided into two zones of thickness d_t, d_a and flow velocities u_e, u_r. Assuming that each zone is well mixed Chikwendu derived the following expression for the dispersivity:

$$D_s = \frac{(d_t d_a)^2 (u_e + u_r)^2}{h^2 2 \nu_r} \tag{20}$$

5 Application

Equation [1] is integrated numerically using a time-splitting procedure with a fourth order interpolation formula (Borthwick & Joynes [2]). The model is applied to predict the concentrations of a solute pollutant from an inshore sea outfall in a small coastal area bounded by two jetties in Thermaikos bay. The concentration at the outlet (y=70m, near the shoreline) was set to be C_0=1 kg/m^3, and the input wave parameters: T=8 sec, H=2.0 m, θ=45°.The wave induced current field is shown in fig [2] and the predicted concentration in fig. [3].

Conclusions

The 2DH advection-diffusion equation is applied to the modelling of pollutant and suspended sediment transport in the inshore zone. Two kind of hydrodynamic models have used: a wave propagation model and a current model considering the wave induced secondary flow. The diffusion dispersion coefficients are estimated in terms of the breaking wave characteristics.

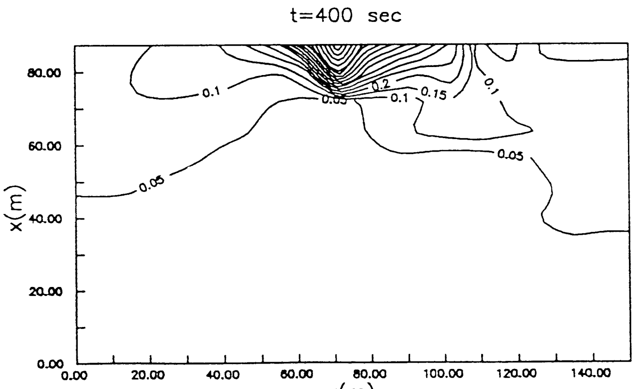

Figure 3: Predicted concentrations of a well-mixed pollutant.

References

1. Battjes, J. A. Modeling of turbulence in the surf zone, Proc. Symp. Modeling Tecniques, ASCE, pp 1050-1061, 1975.

2. Bothwick, A.G.I. & Joynes, S.A. Laboratory study of oil slick subjected to nearshore calculation, *Environmental Engineering*, 1992, Vol. 118, 6, pp 905-922.

3. Chen, Y.H. & Wang, H. Numerical model for nonstationary shallow water wave spectral transformation, *J. Geophysical Research*, 1983, vol. 88, C14, 9851-9863.

4. Chikwendu, S. C. Calculation of longitudinal shear dispersity using an N-zone model as $N \to \infty$, *J. Fluid Mechanics*, 1986, vol 167, 19-30.

5. Copeland, G. J. M. A practical alternative to the 'mild-slope' wave equation, *Coastal Engineering*, 1985, 9, 125-149

6. Copeland, G. J. M. Practical radiation stress calculations connected with equations of wave propagation, *Coastal Engineering*, 1985, 9, 195-219.

7. De Vried, H.J. & Stive, M.J.F. Quasi-3d modelling of nearshore currents, *Coastal Engineering*, 1987, 11, 565-601.

8. Falconer, R.A. Liu, S. Q. & Chen, Y. Application of higher order accurate schemes for advective transport in a 2-D water quality model, Hydraulic and Environmental Modelling: Coastal Waters (ed R.A. Falconer, S.N. Chandler, S.Q. Liu), Vol. 1, pp. 241-252, *Proceedings of the 2nd Int. Conf. of Costal, Esuarine and River Waters,* 1992.

9. Fischer, H. B. et al., Mixing in Inland and Coastal Waters, Academic Press, 1979.

10. Inman, D. L., R.J. Tait & C.E. Nordstrom, Mixing in the surf zone, *J. Geophysical Research*, 1971, vol. 76, 15, 3943-3514.

11. Karambas Th., Y. Krestenitis & C. Koutitas , A numerical solution of Boussinesq equation in the inshore zone, *Hydrosoft,* 1990, 3(1), pp. 34-37.

12. Li, G. & Wright, P. Numerical modelling of sedimentation around Johor Port, Hydraulic and Environmental Modelling: Coastal Waters (ed R.A. Falconer, S.N. Chandler, S.Q. Liu), Vol. 1, pp. 277-290, *Proceedings of the 2nd Int. Conf. of Costal, Esuarine and River Waters*, 1992.

13. Philips. O. M. *The Dynamics of the Upper Ocean*, 2nd ed., pp. 23-24, Cambridge University Press, New York, 1977.

14. Smith, R. Dispersion of tracers in the deep ocean, *J. Fluid Mechanics*, 1982, 123, 131-142.

15. Stive, M.J.F. Velocity and pressure field of spilling breakwaters, *Proceedings17th Int. Conf. on Coastal Eng.*, Hamburg, 1980.

16. Svendsen, I. A. Analysis of surf zone turbulence, *J. Geophysical Research*, 1987, vol. 92, C5, 5115-5124.

17. Tanaka, H. & Wada, A., Prediction of the diffusion of discharged warm water influenced by nearshore currents, *Coastal Engineering in Japan*, 1982, vol. 25, 227-238.