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Application of the Muskingum-Cunge method for dam break flood routing

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Abstract

This paper deals with the application of the Muskingum-Cunge model to the case of wave propagation on a dry bed due to the sudden collapse of a dam. The model is based on variable parameters which are computed according to flow variability. The employed equation of motion takes into account wave steepness. The model was applied to simulate an experimental case carried out by the United States Army Engineer Waterways Experiment Station and the results were compared to those obtained by Chen (1980) who employed complete Saint Venant equations.

1 Introduction

The Muskingum method was developed as a conceptual model for flood routing computations. Many authors have contributed to solve the problem of the choice of parameters. Among these, Cunge's contribution [2] is of great importance, showing how the Muskingum model is analogous to a simplified form of Saint-Venant equations. Cunge gave us the possibility of clearly seeing the physical meaning of the parameters in the Muskingum model. Thanks to Cunge, the Muskingum method can be regarded as a method "firmly based on identifiable physical characteristics of the river channel and the properties of the flood event" [13].

This paper deals with the application of the Muskingum-Cunge model to wave propagation due to the sudden collapse of a dam. The Muskingum-Cunge method is generally used to compute flood propagation generated by hydrological events. It is less frequently used for floods generated by a dam break. In this case, in the equation of motion the inertia terms, compared with pressure, friction, and gravity terms, can have their importance and have not to be neglected. In a dam-break situation, if the above-mentioned terms are negligible, it would be interesting to study if the Muskingum-Cunge method can be employed for the computation of wave propagation. In such a case, the problems to be faced concern model parameter computation according to flow variability and flood wave steepness, which cannot be neglected. Moreover, the problem of computing wave front advance on a dry bed should also be solved.

2 Muskingum-Cunge method

The equation for the Muskingum method is:

$$Q_{j+1}^{n+1} = C_1 \cdot Q_j^n + C_2 \cdot Q_j^{n+1} + C_3 \cdot Q_{j+1}^n \tag{1}$$

in which Q is the discharge; j and n are respectively space and time discretization indices. The coefficients C_1, C_2 and C_3 are defined as:

$$C_1 = \frac{\Delta t + 2\vartheta k}{\Delta t + 2(1-\vartheta)k} \;;\; C_2 = \frac{\Delta t - 2\vartheta k}{\Delta t + 2(1-\vartheta)k} \;;\; C_3 = \frac{2(1-\vartheta)k - \Delta t}{2(1-\vartheta)k + \Delta t} \tag{2}$$

where k and θ are the parameters which, in the Muskingum method, have to be determined for a given reach by calibration; Δt is the time discretization interval.

According to Cunge [2], by expanding the function Q_j^n about the point (jΔx, nΔt) in Taylor series, one can show that the eqn (1) is a second-order approximation of the diffusion equation:

$$\frac{1}{c}\frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x} - D\frac{\partial^2 Q}{\partial x^2} = 0 \tag{3}$$

if k=Δx/c and

$$\vartheta = \frac{1}{2} - \frac{D}{c \cdot \Delta x} \tag{4}$$

In the above equations Δx is the space step, D is the diffusion coefficient and c is the flood wave celerity, which is defined by:

$$c = \left(\frac{dQ}{dA}\right)_x = \frac{1}{B}\left(\frac{dQ}{dh}\right)_x \tag{5}$$

where A is flow cross-sectional area, B is the surface width of flow and h is the depth of flow.

If there is a remarkable flow variability, parameters k e θ have to be considered as variable in time and space. Moreover, when a remarkable flood wave slope is present, the term ∂h/∂x cannot be neglected in comparison with bed slope S_0, so the equation of motion must be written as:

$$S_f = S_0 - \frac{\partial h}{\partial x} \tag{6}$$

in which S_f is the friction slope.

In order to obtain ∂h/∂x the same approximation which leads to Jones formula [17] can be applied. To do so, assuming the kinematic-wave approximation, it follows that:

$$\frac{\partial h}{\partial x} = -\frac{1}{c}\frac{\partial h}{\partial t} \tag{7}$$

so the eqn (6) can be written in the form:

$$S_f = S_0 + \frac{1}{c}\frac{\partial h}{\partial t} \tag{8}$$

The diffusion coefficient is given by the expression [5]:

$$D = \frac{Q}{2B\left(S_0 - \frac{\partial h}{\partial x}\right)} = \frac{Q}{2BS_f} \tag{9}$$

3 Computation of the variable parameters

To compute discharge Q_{j+1}^{n+1} by means of eqn (1), parameters k e θ, which depend on celerity c and discharge Q, need to be calculated. These quantities are evaluated by iteration, using a four-point average calculation of grid points (j,n), (j+1,n), (j,n+1) and (j+1,n+1) [11]. Moreover within this iterative cycle friction slope S_f is computed by means of:

$$S_f = \frac{1}{2}\left(S_{f_j}^{n+\frac{1}{2}} + S_{f_{j+1}}^{n+\frac{1}{2}} \right) \tag{10}$$

where $S_{f_j}^{n+1/2}$ and $S_{f_{j+1}}^{n+1/2}$ are evaluated by discretizing the eqn (8) in the form:

$$S_{f_j}^{n+\frac{1}{2}} = S_0 - \frac{1}{c}\frac{h_j^{n+1} - h_j^n}{\Delta t} \tag{11}$$

By expressing friction slope through Manning formula:

$$S_f = \frac{n^2 Q^2}{A^2 R^{4/3}} \tag{12}$$

in which n is Manning roughness coefficient and R is the channel hydraulic radius, eqn (8) is used to convert discharge values into flow depths.

As for space step choice, the minimum value of Δx is reported in the literature [17] as the value deriving from eqn (4), assuming θ=0, which is considered as the minimum value to be given to θ [7]. Assuming θ=0, eqn (4) gives:

$$\Delta x \geq \frac{Q}{B \cdot S_f \cdot c} \tag{13}$$

However, Ponce and Theurer [10] reported that, if the space step falls outside this limit, the obtained results are not physically unrealistic. According to the Authors, "there is no theoretical justification for a lower limit on either Δt or Δx". They also pointed out that, for the sake of accuracy, the condition $C_2 \geq \zeta$ is determinant, where ζ is a positive real number for which a value of 0.33 is recommended.

4 Application of the method for dam-break flood-wave propagation

In order to apply the above described method to dam-break flood-wave propagation, further problems of computation of wave-front advance on a dry bed and initial application of eqn (1) must be solved as well.

For the computation of the tip region of the wave, most authors, such as Sakkas and Strelkoff [15,16] and Katopodes and Schamber [6] follow the analysis made by Whitham [18], who pointed out that in the tip region of a wave the free surface slope becomes enough large to balance the frictional resistance and the flow velocity V is essentially uniform in the direction of flow. In this region the free surface profile is given by the equation:

$$\frac{dh}{dx} = S_0 - \frac{n^2 V^2}{R^{4/3}} \tag{14}$$

which is analogous to the equation of motion used in the model described in previous sections.

In order to compute wave front advance, wave front celerity and water velocity just behind the front are assumed equal [8, 14] so that the time τ necessary to the front to travel across the distance Δx is evaluated as $\tau=\Delta x/V$.

In order to solve the eqn (1) at a time step n=1 it is sufficient to observe that, if t_1 is the time when the wave front is in the abscissa $j\Delta x$, the front will reach the abscissa $(j+1)\Delta x$ at time $t_1+\tau$. At this time $Q_{j+1}(t_1+\tau)=0$. If we consider τ as a time lag period [4], the eqn (1), at time $t_1+\tau+\Delta t$, can be written as:

$$Q_{j+1}(t_1+\tau+\Delta t) = C_1 \cdot Q_j(t_1+\tau) + C_2 \cdot Q_j(t_1+\tau+\Delta t) \tag{15}$$

5 Numerical application

The above-described model was applied to an experiment carried out at the Waterways Experiment Station (WES) by the U.S. Corps of Engineers [3]. The experiments were carried out in a rectangular plastic-coated plywood flume 1.22 m wide and 122 m long, having a slope of 0.005. The flume terminated in a free overfall. Water was impounded by a 0.305 m high model dam situated midway of flume length. Manning roughness coefficient was n=0.009. The sudden, total collapse was simulated by suddenly removing the model dam.

In this application, the WES test condition 1.1 was considered, which is characterized by a breach width of 1.22 m and a breach depth of 0.305 m. The test was carried out with a dry bed downstrem of the dam. For this test, hydrographs were measured at distances of 6.10 m, 7.62 m, 9.14 m, 22.86 m, 24.38 m, 25.91 m, 44.20 m, 45.72 m, 47.24 m from the dam.

As far as breach discharge hydrograph is concerned, the measured one was employed.

In order to compare model performance with that of complete Saint Venant equations, the results of simulations carried out by Chen [1] and available for abscissas 7.62 m, 24.38 m, 45.72 m were considered. They are among the most reliable simulations of WES tests reported in the literature.

As for integration step choice, the two methods described in section 3 were applied as follows.

Computation A: space steps were computed by means of condition (13), which gives the following values from upstream to downstream: $\Delta x_1=\Delta x_2=12.19$ m, $\Delta x_3=\Delta x_4=10.67$ m. The last two values of Δx give minimum θ values which are a little lower than 0 ($\theta_{min}=-0.04$). Thus, the values of $\Delta x_{3,4}$ should be increased a little. Being the minimum value of θ so little, the above values were employed to simulate the hydrograph measured at abscissa 45.72 m. Moreover, because the θ values are a little negative, the condition $\Delta t=2\theta\Delta x/c$ suggested by Koussis [10] could not be employed. This condition corresponds to impose $C_2=0$. For the Δt computation at each time C_2 was thus imposed equal to 0.1, which gives enough little time steps.

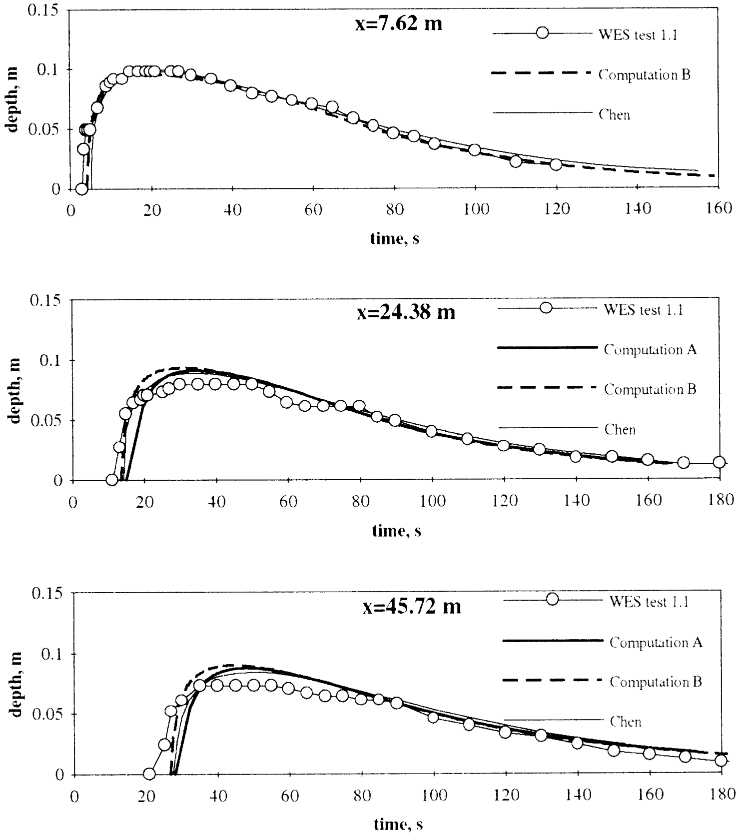

Figure 1: Stage Hydrographs for WES Test Condition 1.1

Computation B: the values assumed for space step are the following: Δx=1.905 m from the dam to the abscissa 7.62 m; Δx=2.0955 m from the abscissa 7.62 m to the abscissa 24.38 m; Δx= 2.134 m from the abscissa 24.38 m to the abscissa 45.72 m. In order to compute the time step, the condition $C_2 \geq 0.33$ was assumed, with a minimum value of 0.85 s, which always allowed us to respect this condition.

6 Computation results

Numerical application results are plotted in Figures 1, 2, and 3, reporting stage, velocity and discharge hydrographs computed at abscissas 7.62 m, 24.38 m, 45.72 m respectively.

Being the first space step in computation A greater than 7.62 m, at this abscissa only the result from computation B is available. The computed depths are in agreement with experimental results and Chen computation. The qualitative pattern of velocity is similar to Chen's, but values are a little higher. Discharge too shows lightly higher values in the peak area. For abscissas 24.38 m and 45.72 m a comparison with computation A results can be made.

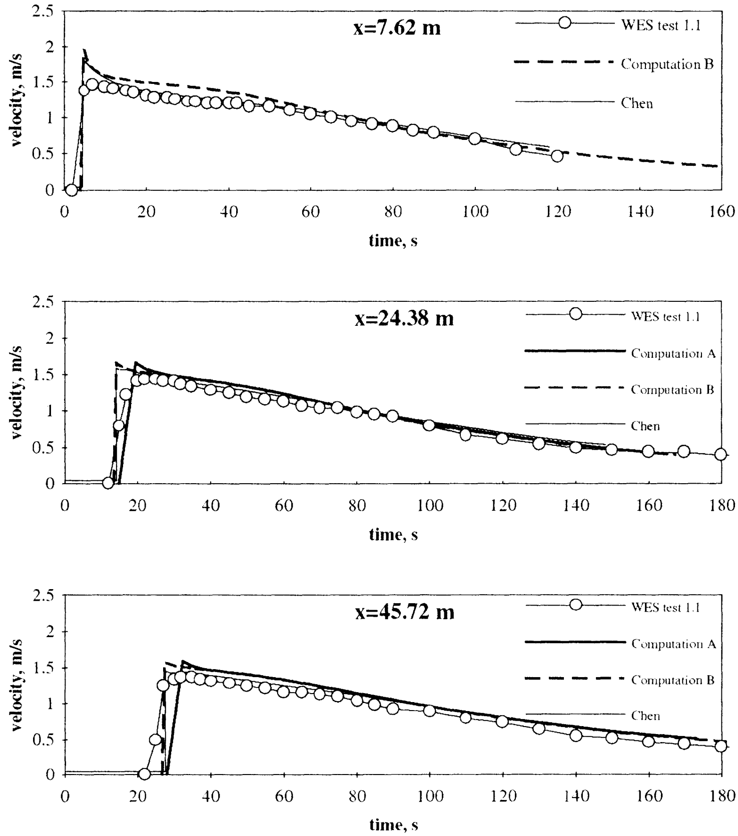

Figure 2: Velocity Hydrographs for WES Test Condition 1.1

Computation B hydrographs show that the rising limb is very close to Chen computation. Computation A hydrographs show a less steep rising limb in comparison with computation B: this is due to the higher first time step imposed by the condition C_2=0.1. For depth and discharge, computation A gives peak values which are closer to Chen's than computation B values, but computation B is practically free from any volume balance error. As far as wave front advance time is concerned, both computations A and B gave good results: in particular, computation B tends to remain very close to experimental values.

7 Conclusions

The analysed model proved to be successful in treating the propagation flood wave without the rising limb. This wave is typical of the instantaneous and total collapse of a dam. Front advance velocity was assumed equal to water velocity just behind the front, calulated at abscissa j, and this proved to be in agreement with the experimental values and those computed by Chen who employed Rankine-Hugoniot equations. In the equation of motion, wave profile slope was taken into account and this allowed us to obtain depth and velocity hydrographs

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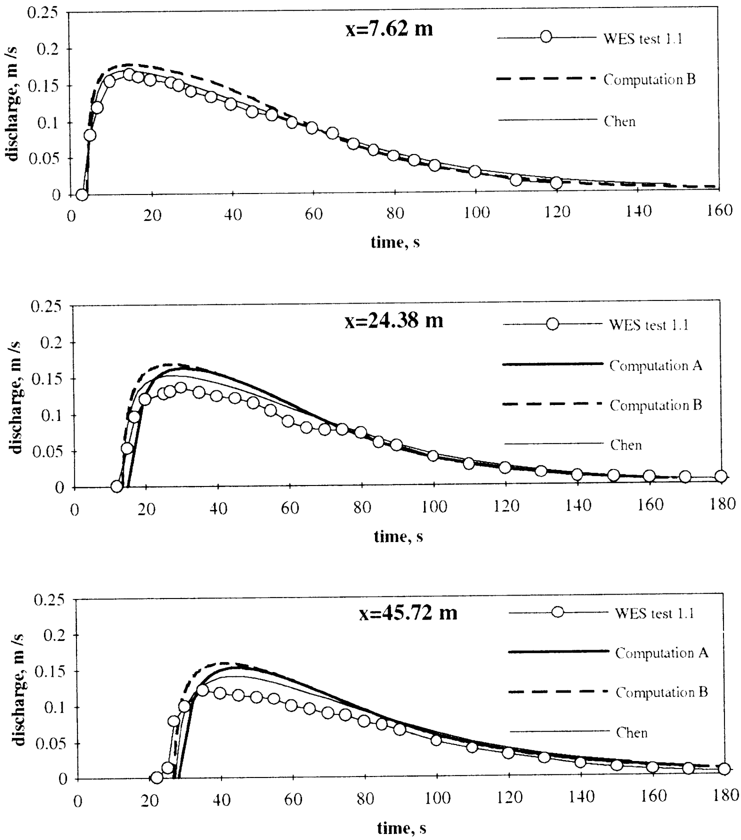

Figure 3: Discharge Hydrographs for WES Test Condition 1.1

very close to experimental ones and to those obtained by means of complete De Saint-Venant equations. Computation carried out by means of condition (13) (computation A) gave an increased wave attenuation in comparison with computation B (lower space steps and $C_2 \geq 0.33$). However the second computation is practically free from any volume balance error.

The above-presented model will be applied to the other experimental cases carried out by the United States Army Engineer Waterways Experiment Station in order to evaluate the model applicability in different flow conditions, also in the light of the analyses reported in the literature, e.g. by Ponce et al [9], Katopodes and Schamber [6] and Ponce [12].

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