Parameter estimation of linear and nonlinear Muskingum models for river flood routing

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Abstract

The importance of flood routing in rivers has been vastly recognized in hydraulic engineering practice. Field data scarcity often prevents the use of the Saint Venant equations to route floods in natural streams. As a consequence, approximate techniques such as the Muskingum models are commonly employed. The linear form of the Muskingum model has been widely applied to river flood routing. However, a nonlinear relationship between storage and discharge exists in most rivers, making the use of the linear form of the Muskingum method inappropriate. In this paper, three nonlinear forms of the Muskingum model are used. Different optimization techniques are applied to river flood routing for the estimation of the nonlinear Muskingum models parameters for the calibration period. To evaluate the perfomance of different parameter estimation techniques in the calibration period, the main inflow hydrograph and the partial inflows from tributaries and local ungaged inflow for the calibration period are routed to produce a computed outflow hydrograph for a given parameter set for each model. Then, the computed and observed outflow hydrographs for the calibration period are compared.

1. Introduction

In hydraulic engineering, the flood routing problems are solved by using partial differential equations for unsteady flow in open channels. In hydrology, however, the approach of solving the flood routing problem is distinguished from that of hydraulics. The hydrologic method makes no direct use of those unsteady differential equations, but, tries to establish some relation between stage, storage, outflow and inflow, to approximate their solutions. In general, hydrologic method is simpler than hydraulic one and gives satisfactory result. In this paper, a well-known hydrologic method, called the Muskingum method is used (e.g. Kulandaiswamy[7], Diskin[1], Gill[2,3,4], Koussis[5], Ponce & Yevjevich[12], Ponce[11], Stephenson[15], Singh & McCann[14], Strupcrewski & Kundzewicz[16]).

The linear form of the Muskingum model is written as:

$$S_t = K\left[X\, I_t + (1-X)\, O_t\right] \tag{1}$$

where S_t is the storage within the routing reach at time t; I_t and O_t are the rates of inflow and outflow at time t, respectively; K is the storage time constant for the routing reach; and X is a weighting factor varying between 0 and 0.5. Strupczewski & Kundzewicz [16] have shown that the theoretical values of X range from $-\infty$ to 0.5. To perform river flood routing, Eq. (1) is solved in conjunction with the following continuity equation:

$$\frac{S_t - S_{t-1}}{\Delta t} = \frac{I_t + I_{t-1}}{2} - \frac{O_t + O_{t-1}}{2} \tag{2}$$

The numerical solution of Eqs. (1) and (2) results in the Muskingum routing equation as:

$$O_t = C_0\, I_t + C_1\, I_{t-1} + C_2\, O_{t-1} \tag{3}$$

in which C_0,C_1 and C_2 are coefficients that are functions of K,X and discretized time interval Δt; $C_0+C_1+C_2=1$.

The parameters K and X are estimated by plotting accumulated storage versus weighted flow from past flood hydrographs using all flows through the routing reach. This trial and error graphical procedure can easily be replaced by the Least Squares Method (LSM) (e.g. Gill[2,4]). The slope of the line for the correct value of X determine the value of K. However, the value of K thus determined is average K for the reach, but if tributaries enter the reach, propably not the inflow will rise and fall simultaneously. In such rivers the storage will be influenced by the variation in inflow and a more accurate evaluation of its effect can be made by separately routing each inflow from its point of entry into the reach to the lower end of the reach. To accomplish this, it is necessary to determine individual coefficients for each component of the flow (e.g. Papazafiriou[10]). This may be approximated by the following equations:

$$K_M = \frac{(Q_c)}{(Q_c\ M)}\, K \tag{4}$$

$$K_c = M\, K_M \tag{5}$$

where K is the average K for the reach in units of days or hours; K_M is the K per km; K_c is the individual K for separate routing of inflows, Q_c is the total inflow at one point during a given flood; and M is the distance in km from point of inflow to lower end of reach.

The ungaged local inflow sometimes may be great enough to present additional problems which cannot be ignored. If the ungaged area is small and the gaged tributaries can be considered as representative of the ungaged drainage, the ungaged flow can be included by increasing the gaged tributary flow in proportion to the drainage areas involved. If the ungaged local flow is large in relation to gaged drainage, a more exact procedure requires the use of rainfall-runoff procedures.

In cases where a nonlinear relationship between storage and discharge exists, using a linear form of Muskingum model may introduce considerable error. In these cases, the nonlinear models given in Eqs. (6), (7) and (8)

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respectively may be more appropriate (e.g. Gill[2,4], Tung[17], Papamichail & Georgiou[9], Yoon & Padmanabhan[11])

$$S_t = K\left[X\, I_t^{p} + (1-X)\, O_t^{p}\right] \tag{6}$$

$$S_t = K\left[X\, I_t^{p_1} + (1-X)\, O_t^{p_2}\right] \tag{7}$$

$$S_t = K\left[X\, I_t + (1-X)\, O_t\right]^{m} \tag{8}$$

These models have additional parameters p, p_1 and p_2 and m, respectively, which cah be determined by using alternative parameter estimation methods.

The nonlinear models in Eqs. (6), (7) and (8) require O_t, to be solved by trial and error at every time step of flow routing. The routing equations for nonlinear models in Eqs. (6), (7) and (8) are given in Eqs. (9), (10) and (11), respectively:

$$K\left[XI_t^{p} + (1-X)O_t^{p}\right] - K\left[XI_{t-1}^{p} + (1-X)O_{t-1}^{p}\right] = \frac{\Delta t}{2}\left[I_t + I_{t-1} - O_t - O_{t-1}\right] \tag{9}$$

$$K\left[XI_t^{p_1} + (1-X)O_t^{p_2}\right] - K\left[XI_{t-1}^{p_1} + (1-X)O_{t-1}^{p_2}\right] = \frac{\Delta t}{2}\left[I_t + I_{t-1} - O_t - O_{t-1}\right] \tag{10}$$

$$K\left[XI_t + (1-X)O_t\right]^{m} - K\left[XI_{t-1} + (1-X)O_{t-1}\right]^{m} = \frac{\Delta t}{2}\left[I_t + I_{t-1} - O_t - O_{t-1}\right] \tag{11}$$

In routing, I_{t-1}, O_{t-1} and I_t for any routing period are known. The parameters K, X, p, p_1, p_2, m and K_c are also known. Therefore, O_t can be solved for succesive routing periods in Eqs. (9) or (10) or (11) depending on the model of choice. An iterative method is used to solve for O_t, since O_t appears nonlinearity in the equation. The bisection method (e.g. Press et al.[13]) is used in this investigation.

2. Parameter estimation of the nonlinear Muskingum models

The nonlinear Muskingum models considered in this paper consist of the parameters K, X, p, p_1, p_2 and m, which are to be estimated from observed river flow data. In this paper, four parameter estimation techniques are employed to minimize the sum of the squares of deviations between observed channel storage and computed channel storage over the total data points.

The objective function to be minimize for nonlinear Muskingum models in Eqs. (6), (7) and (8) are given in Eqs. (12),(13) and (14), respectively:

$$\underset{K,X,p}{\text{Minimize}}\ F = \sum_{t=1}^{N}\left\{S_t - K\left[XI_t^{p} + (1-X)O_t^{p}\right]\right\}^2 \tag{12}$$

$$\underset{K,X,p_1,p_2}{\text{Minimize}}\ F = \sum_{t=1}^{N}\left\{S_t - K\left[XI_t^{p_1} + (1-X)O_t^{p_2}\right]\right\}^2 \tag{13}$$

$$\underset{K,X,m}{\text{Minimize}}\ F = \sum_{t=1}^{N} \left\{ S_t - K\left[XI_t + (1-X)O_t \right]^m \right\}^2 \tag{14}$$

in which S_t is the observed channel storage at time t and N is the number of data points.

The nonlinear Muskingum models of Eqs. (6) and (7) can be reduced to linear forms if the values of the parameters p, p_1 and p_2 are assumed. Also the nonlinear model of Eq. (8) can be expressed (via logarithms) as:

$$\ln(S_t) = \ln(K) + m \ln\left[X^* I_t + \left(1 - X^*\right) O_t \right] \tag{15}$$

in which a weighting factor X takes as assumed X^*. When the values of accumulated storage Eq. (2) are negative the linear form of Eq. (15) can not be used. Under such linear forms of the nonlinear Muskingum models, the parameters values for K, X, p, p_1, p_2 and m can be estimated by using the Linear Least Squares Regression (LLSR) technique (e.g. Gill[2,4], Tung[17], Papamichail & Georgiou[9], Yoon & Padmanabhan[18]).

The optimal estimation of unknown parameters in the nonlinear Muskingum models can also be derived by solving the objective functions, Eqs. (12), (13) and (14) with an optimization technique. Three optimization schemes the Hooke-Jeeves (H-J), the Rosenbrock (ROS) and the Marquardt (MAR) methods (e.g. Kuester & Mize [6]), are applied to estimate the value of K, X, p, p_1, p_2 and m. The above optimization methods directly fit the nonlinear models to the data using nonlinear least-squares regression. The estimation proceeds iteratively from initial quesses of the parameters using the Hooke-Jeeves, Rosenbrock and Marquardt algorithms.

The statistics used as efficiency criteria for comparing the performance of different estimation methods and of linear and nonlinear models are: (1) Residual variance of observed and routed outflows (RV); (2) Deviation of peak of observed and routed outflow (DPO); (3) Deviation of peak time of observed and routed outflows (DPOT); and (4) Sum of absolute deviations between observed and routed outflows (SD) (e.g. Tung[17], Yoon & Padmanabhan[18]).

3. Applications

The linear, Eq. (1) and the nonlinear Muskingum models, Eqs. (6),(7) and (8) are applied to river flood routing using an example from Linsley et al.[8]. The Sewickley-Wheeling reach of the Ohio river, all gaging stations and tributaries are shown in Figure 1.

Records for the flood of March 15^{th} to 31^{st} 1936 were used. Time unit was 12 hours. Local inflow was included by increasing the gaged tributary flow at E. Liverpool in propotion to the difference between inflow and outflow to the routing reach (Linsley et al.[8]).

The main inflow at Sewickley, the partial inflow A from Hazen, Wurtemburg and Wampum, the partial inflow B from E. Liverpool and local inflow and the observed outflow at Wheeling are shown in Figure 2. The parameter values in Eqs. (6), (7) and (8) estimated by different optimization techniques are given in Table 1.

To evaluate the perfomance of different parameter estimation techniques in the calibration process, the inflow hydrograph at Sewickley and the partial inflows A and B with individual K [Eqs. (4) and (5)] are routed to produce a computed outflow hydrograph for a given parameter set and various models. Then, the observed and the routed outflow hydrographs at Wheeling obtained by the linear Muskingum model [Eq. (3)] by the bisection method of the nonlinear Muskingum models [Eqs. (9), (10) and (11)] using different parameter values are plotted in Figure 3 for Eq. (9), in Figure 4 for Eq. (10) and in Figure 5 for Eq. (11).

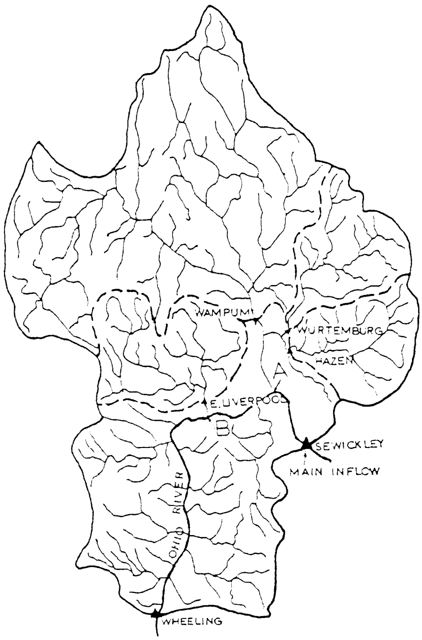

Figure 1: Map of the Sewickley-Wheeling reach of the Ohio River (Linsley et al.[8]).

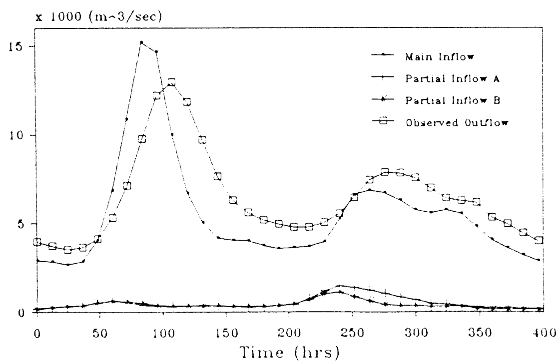

Figure 2: Inflow Hydrographs and observed outflow hydrograph.

Table 1. Values of parameters K, X, p, p_1, p_2 and m estimated by different methods for various models

Model	Method	K	X	p	p_1	p_2	m
Eq. (1)	LSM	21.7*	0.337*	-	-	-	-
Eq. (6)	LLSR	2.49	0.066	1.22	-	-	-
Eq. (6)	H-J	4E-04	0.05	2.1	-	-	-
Eq. (6)	ROS	6E-04	0.085	2.06	-	-	-
Eq. (6)	MAR	6E-05	0.089	2.32	-	-	-
Eq. (7)	LLSR	4.615	1.3E-4	-	1.8	1.15	-
Eq. (7)	H-J	0.0004	0.02	-	2.3	2.1	-
Eq. (7)	ROS	0.0007	0.0092	-	2.295	2.04	-
Eq. (7)	MAR	5E-05	0.0057	-	2.61	2.32	-
Eq. (8)	H-J	0.0002	0.1	-	-	-	2.2
Eq. (8)	ROS	0.027	0.08	-	-	-	1.65
Eq. (8)	MAR	6E-05	0.1	-	-	-	2.32

*Gill [2]

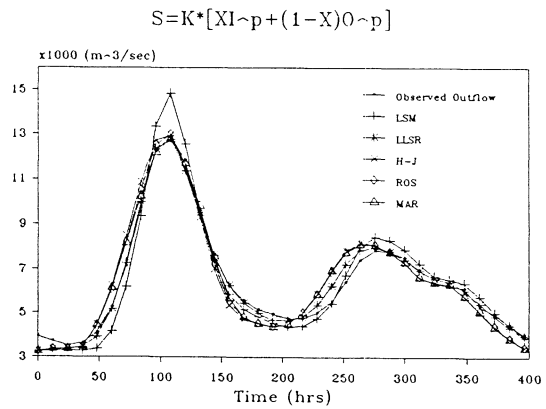

Figure 3: Observed and routed outflow hydrographs at Wheeling.

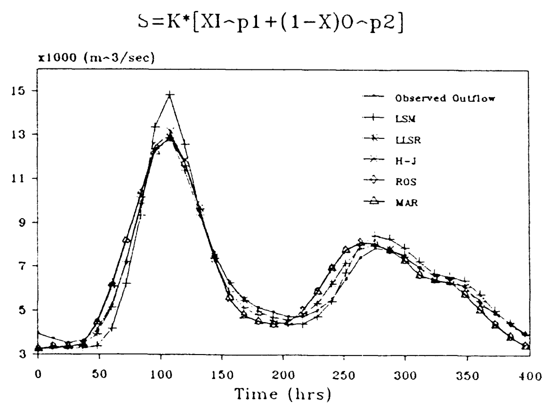

Figure 4: Observed and routed outflow hydrographs at Wheeling.

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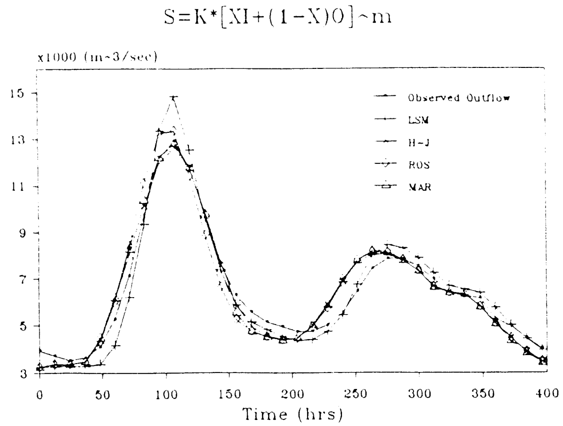

Figure 5: Observed and routed outflow hydrographs at Wheeling.

The efficiency criteria of the linear and nonlinear Muskingum models for various parameter estimation methods are summarized in Table 2.

Table 2. Efficiency criteria.

Models	Method	R.V.	DPO	DPOT	SD
Eq. (1)	LSM	357988	1890	0	15771
Eq. (6)	LLSR	90875	154	0	7481
Eq. (6)	H-J	459346	17	0	18537
Eq. (6)	ROS	355197	55	0	16139
Eq. (6)	MAR	342455	159	0	15396
Eq. (7)	LLSR	85587	65	0	7273
Eq. (7)	H-J	330886	203	0	15348
Eq. (7)	ROS	331427	91	0	15497
Eq. (7)	MAR	366000	117	0	15392
Eq. (8)	H-J	280109	288	0	14195
Eq. (8)	ROS	487250	422	0	19251
Eq. (8)	MAR	206615	81	0	12631

4. Summary and conclusions

The linear form of the Muskingum model commonly applied to river and channel flood routing may be inappropriate when an appreciate nonlinearity between weighted flow and channel storage exists. This study presents a routing technique with main and partial inflows for three forms of the nonlinear Muskingum models, using the bisection method.

When the nonlinear flood routing models are considered the task of parameter estimation, in the calibration process, becomes more involved. Four parameter estimations procedures are devised using the Linear Least Squares Regression (LLSR), the Hooke-Jeeves (H-J), the Rosenbrock (ROS) and the Marquardt (MAR) techniques. Comparisons were made of the nonlinear Muskingum models parameter estimation techniques and Gill's procedure (LSM) [2,4], including the use of the linear model. For the selected data, the nonlinear Muskingum models in DPO replicate the given outflow hydrograph more closely than the linear model. This demonstrates the limitation of the linear model and that nonlinear Muskingum models should be used. An interactive software was developed to select between linear and nonlinear forms of Muskingum models, in which parameters of nonlinear models can be estimated using four different estimation techniques and to route main and partial inflows based on the set of the estimated parameters.

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