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Numerical and experimental investigation of the exciting wave loads on a vertical truncated cylinder

S.A. Mavrakos, G.J. Grigoropoulos
Department of Naval Architecture and Marine Engineering, National Technical University of Athens, GR-15780 Zografos, Greece

Abstract

The paper deals with the numerical and experimental investigation of the exciting wave loads on a vertical truncated cylinder placed in a channel and exposed to the action of regular waves. The effect of wave reflections from the tank walls have exactly been taken into account in the numerical study using the method of images. An insight into the relevant importance of the tank confinement effects has been given through a parametric numerical study. The numerical predictions are compared with experimental data and the relevant importance of higher order effects for steeper waves is demonstrated.

Introduction

In model testing in towing tanks, it is important to be able to assess the effects of wave reflections from the tank walls. Tank wall effects may affect significantly the flow field around the physical model resulting in deviations from the open-sea large-scale prototypes. Spring & Monkmeyer[1] reported departures of the order of 76% in the pressure distribution around a single cylinder due to flume wall influence. Similar findings were reported by a number of investigators[2-4].

One of the most notable features of all the above mentioned studies is that the channel walls exert an important influence on the radiation and the diffraction properties, the latter to a lesser extent (Yeung & Sphaier[2]). Such influence is characterized by the presence of "spikes" at wave frequencies corresponding to the occurrence of symmetric transverse resonant modes in the channel. To assess the effect of the flume confinement on the various hydrodynamic characteristics, both integral formulations and the method of images have been proposed in the literature. In the first case, the potential solution is written in terms of an unknown surface distribution over the body surface (Thomas[3]), whereas in the second, use is made of the fact that the flow field associated with an infinite row of cylinders is equivalent to a single cylinder placed between two parallel vertical walls[1,2,4]. McIver[5], using matched asymptotic expansions, presented an approximate solution for a cylinder in a channel.

The various formulations proposed in the literature to solve the interaction problem between regular waves and an arbitrary arrangement of vertical cylinders, can be principally applied also to arrays of image cylinders. In this context, exact and approximate formulations are reported in the literature. The former are usually based on direct matrix inversion methods[6] or on multiple scattering formulations[7-9]. The latter rely on the assumption that the bodies are widely spaced and, thus, the circular waves emanating from each body can be approximated as a plane wave on the other ones[10].

In the present paper both numerical predictions and experimental data of the interaction between regular waves and a vetrical truncated cylinder placed in the wave tank of the Laboratory for Marine and Naval Hydrodynamics, NTUA, are given. In the numerical work, a parametric study was undertaken to assess the influence of the wall tank confinement effects on the diffraction loads in terms of the main parameters of the problem (diameter and draught of the cylinder, channel width). The method of images and the formulation developed by Mavrakos & Koumoutsakos[8] have been used. According to this, the single body hydrodynamic characteristics are combined through the physical idea of multiple scattering to derive exact series representations of the velocity potential around each cylinder of the array, including the evanencent wave modes as well. The derived expressions are obtained by superposing various orders of successively radiated/scattered waves emanating from all the cylinders in the array. The necessary isolated body hydrodynamic characteristics are obtained using the method of matched axisymmetric eigenfunction expansions. In the experimental work the exciting loads and the heave motion are measured and compared with the numerical predictions which include second-order contributions as well.

Formulation of the problem

We consider a vertical truncated cylinder of radius a which is located on the centerline of a channel of width l (Fig.1). The cylinder is exposed to the action of a plane wave train of frequency ω and amplitude H/2 propagating in water depth d along the channel. A cylindrical coordinate system (r_0, θ_0, z_0) is introduced with origin on the channel bottom and its vertical axis directed upwards. To describe the fluid flow around the cylinder, the method of images will be used. According to this method, the problem is equivalent to one with an infinite array of parallel cylinders having their vertical axes all in a plane perpendicular to the channel walls. In the following this array interaction problem will be elaborated. The cylinders in the array are separated between each other by a distance l. For the p-th cylinder, we define local coordinates (r_p, θ_p, z_p) as shown in Fig.1.

Assuming that the flow is both irrotational and inviscid and that the waves are of small slope, classical linearized water wave theory can be employed. The fluid flow can be described by the potential function

$$\Phi(r_0, \theta_0, z_0; t) = \mathrm{Re}\left[\varphi(r_0, \theta_0, z_0)\, e^{-i\omega t}\right] \tag{1}$$

where the complex potential function $\varphi(r_0, \theta_0, z_0)$ can be expressed, on the basis of linear modeling, as a superposition of incident, φ_I, and scattered, φ_B, wave fields, i.e:

$$\varphi = \varphi_I + \varphi_B \tag{2}$$

The potential φ_B must satisfy the Laplace equation in the undisturbed fluid domain, the zero normal velocity on the channel's bottom, z = 0, and following conditions on the mean water surface, z = d, and the mean body's wetted surface, $S^{(p)}$, respectively :

$$\omega^2 \varphi_B - g\frac{\partial \varphi_B}{\partial z} = 0 \qquad , \; z = d \tag{3}$$

$$\frac{\partial \varphi_B}{\partial n^{(p)}} = -\frac{\partial \varphi_I}{\partial n^{(p)}} \qquad , \; on \; S^{(p)}, \; p = 1, 2, \ldots, \tag{4}$$

where $/ \; n^{(p)}$ denotes the derivative in the direction of the outward unit normal vector $\mathbf{n}^{(p)}$ to the mean wetted surface $S^{(p)}$ of the body p. Finally, a radiation condition must be imposed which states that propagating disturbances must be outgoing.

The velocity potential of the undisturbed incident wave system, φ_I, propagating along the channel (positive x-axis) can be expressed in the cylindrical co-ordinate frame of the p-th body as follows[8]:

$$\varphi_I(r_p, \theta_p, z_p) = -i\omega\frac{H}{2} \sum_{m=-} \Psi_{I,m}(r_p, z_p) e^{im\theta_p} \tag{5}$$

with

$$\frac{1}{d}\Psi_{I,m}(r_p, z_p) = i^m \frac{Z_0(z)}{d\, Z_0'(d)} J_m(kr_p) \tag{6}$$

Here, J_m is the m-th order Bessel function of the first kind. $Z_0(z)$ is defined by:

$$Z_0(z) = \left[\frac{1}{2}\left[1 + \frac{\sinh(2kd)}{2kd}\right]\right]^{-\frac{1}{2}} \cosh(kz) \tag{7}$$

and $Z'_0(z)$ is its derivative at z = d. Frequency ω and wave number k are related by the dispersion equation:

$$\omega^2 = gk\tanh(kd) \tag{8}$$

The total velocity potential around each body (p = 1, 2, ...) of the configuration can be expressed in accordance to eqn. (5) as follows:

$$\varphi(r_p, \theta_p, z_p) = -i\omega\frac{H}{2} \sum_{m=-} \Psi_m(r_p, z)\, e^{im\theta_p} \tag{9}$$

The complex function $\Psi_m(r_p,z)$, which is the principal unknown of the problem, can be determined in the way proposed by Mavrakos & Koumoutsakos[8], by combining the single body hydrodynamic characteristics through the physical idea of multiple scattering to account for interference effects.

For bodies having the form of truncated vertical cylinders, the single body hydrodynamic characteristics can be established through the method of matched axisymmetric eigenfunction expansions. According to this method, the flow field around the body p is subdivided in coaxial ring-shaped fluid regions, categorized by the numerals I and II (Fig. 2). In each fluid region different series representations for the velocity potentials can be established which are then matched by continuity requirements of the hydrodynamic pressure and radial velocity along the vertical boundaries of adjacent fluid regions, using Galerkin's method. The procedure, which has been intoduced by Garrett[11] and extended by several investigators (Black, Mei & Bray[12], Yeung[13], Mei[14]), has been extensively reported by Kokkinowrachos, Mavrakos & Asorakos[15], and thus it will not be further elaborated in the present contribution.

Having determined the isolated body hydrodynamic characteristics in the way outlined previously, the multiple scattering formulation is applied to account for hydrodynamic interference effects among the cylinders in the array. According to this formulation, the velocity potential around each body of the arrangement is obtained by successively superposing various orders of propagating and evanescent wave modes radiated/scattered from all the cylinders of the array. The method enables successive satisfication of the imposed boundary conditions on each body. Thus, a considerable reduction of the computer storage requirements can be achieved by its implementation. For more details reference is made to previous work of one of the authors (Mavrakos and Koumoutsakos[8], Mavrakos[9]). By way of example, the series representations for the complex functions $\Psi^i{}_m$ (i = I, II), see eqn. (9), in each fluid domain around the p-th vertical truncated cylinder in the array, are given in the following:

(a) outer fluid domain (i = I), i.e. for $r_p \geq b_p$, $0 \leq z \leq d$,

$$\frac{1}{d}\Psi^I_m(r_p, z) = \sum_{n=0}\left[Q^{(p)}_{mn}\frac{I_m(\alpha_n r_p)}{I_m(\alpha_n b_p)} + F^{(p),I}_{mn}\frac{K_m(\alpha_n r_p)}{K_m(\alpha_n b_p)}\right] Z_n(z) \tag{10}$$

The first term in (10) represents the contribution of the incident to the total wave potential around the p-th body. It constitutes from the undisturbed incident wave plus various orders of scattering emanating from all the remaining bodies of the array. These scattered fields are properly expressed in the p-th body's co-ordinate system using the Bessel function addition theorem (Abramowitz and Stegun[16]). Especially, in the case of isolated body-wave interaction, it holds:

$$Q^{(p)}_{mn} = \frac{i^m}{d\, Z_0'(d)}\, I_m(\alpha_n b_p).\delta_{o,n} \tag{11}$$

The unknown complex coefficients $F^{(p),I}{}_{mn}$ are obtained using the aforementioned method of mached eigenfunction expansions. Moreover, I_m and K_m are the mth order modified Bessel function of first and second kind respectively, whereas $Z_n(z)$ are orthonormal functions in [0, d] defined by equation (10) for n = 0 and by

$$Z_n(z) = \left[\frac{1}{2}\left[1 + \frac{\sin(\alpha_n d)}{2\alpha_n d}\right]\right]^{-1/2} \cos(\alpha_n z) \tag{12}$$

for n > 1. The eigenvalues α_n are roots of the transcendental equation

$$\omega^2 + g\alpha_n \tan(\alpha_n d) = 0 \tag{13}$$

and the alternative notation $\alpha_0 = -ik$ is used in the sequel for the imaginary root.

(b) second fluid domain (i = II), i.e. $0 \leq r_p \leq a_p$, $0 \leq z \leq h\text{-}d$

$$\frac{1}{d}\Psi^{II}_m = \sum_{n=0}\varepsilon_n F^{(p),II}_{mn}\frac{I_m\left(\frac{n\pi r}{h_p}\right)}{I_m\left(\frac{n\pi b_p}{h_p}\right)}\cos\left(\frac{n\pi z}{h_p}\right) \tag{14}$$

where ε_n is the Neumann's symbol defined as: ε_0=1 for n=0, otherwise ε_n=2.

Experimental Investigation

In order to experimentally verify the analytical results, two sets of experiments have been conducted in the Towing Tank of the Laboratory for Ship and Marine Hydrodynamics, NTUA. For the experimental investigation a PVC circular cylinder with watertight bottom was used. In the first set of experiments the cylinder was fixed, while in the second the same cylinder was free to heave.

In both experiments, the cylinder was submerged by h = 1200 mm in the 2900 mm deep flume. The diameter of the cylinder was $2a$ = 200 mm, while the breadth of the Towing Tank is l = 4560 mm. The channel width to cylinder diameter ratio is sufficiently large, $l/2a$ = 24.8, and thus the effect of the tank walls on its hydrodynamic behaviour can be neglected. This is fully supported by the parametric numerical study in the following section as well. The experimental arrangement is shown in Fig. 3 for the case of the fixed cylinder. Using this balance, the vertical forces at the fore end and the aft end of the dynamometer L_1 and L_2 and the horizontal drag D are measured. Based on L_1 and L_2 and the longitudinal distance d between them, the pitching moment and the lifting force can be computed. A similar arrangement has been used in the second case too, but the dynamemeter has been replaced by another one monitoring the heaving motion and the horizontal drag. In both cases the wave amplitude, which was of the order of 30-50 mm, was measured 3000 mm in front of the cylinder.

In order to investigate the effect of harmonic contamination, which is always present in the sine waves generated in a wave flume, the measured time histories were analysed by means of a least-square technique developed in NTUA[17]. In addition, in order to investigate the relative importance of the first and the second-order forces, the sampled time histories were also analysed using Fast Fourier Transformation (FFT).

Results and discussion

The numerical results have been obtained using the computer code HAMVAB[8] which is suitable for the solution of the diffraction and the radiation problems around a group of vertical axisymmetric bodies. The computations were carried out on a Silicon Graphics Workstation at the Laboratory for Ship and Marine Hydrodynamics, NTUA.

Figs. 4a and 4b display the wave exciting horizontal and vertical forces, and the overturning moment on the middle of an array of seven identical cylinders having their vertical axes all in a plane perpendicular to the wave incidence (Fig. 1). This configuration simulates the case of one cylinder placed in the centerline of a channel of width l. It has been shown that the inclusion of more image cylinders would not affect the accuracy of the results, which are plotted against the nondimensional wave number parameter ka. The results have been made nondimensional by the factors $rga^2(H/2)$ and $rga^3(H/2)$ for the forces and moments, respectively. The water depth has been kept constant, d/a = 30, whereas two different cylinder drafts have been examined, i.e. d/h = 2.5 (Fig. 4a) and d/h = 10.0 (Fig. 4b).

The ratio of channel width to cylinder diameter is parametrically introduced in the graphs. The solid line depicts results for the isolated body. It can be seen that the general features of the force and moment are similar to those of the isolated body with the exception of the occurrence of small peaks at the frequencies, corresponding to $kl/2\pi = 1, 2, ...$, where symmetric transverse resonant modes in the channel appear[2]. It seems that for deeper drafts the horizontal forces and overturning moments are more affected by the presence of channel walls than the vertical exciting forces (Fig. 4a). On the contrary for smaller cylinder sumergence, Fig. 4b, the vertical forces experience deviations from the single cylinder results due to wall confinement effects. Since the exciting loads are related directly to the radiation damping, this behaviour can be explained as follows : As the source of disturbance, i.e. the bottom of the cylinder, which produces the radiation damping by the heaving motion of the cylinder, gets less submerged, the wave effects because more pronounced.

The experimental results for the horizontal force and the pitching moment in the case of the fixed cylinder are plotted in Figs. 5 and 6 along with the respective numerical predictions. To give some insight into the relative importance of non-linear effects, the linearized and the total second-order force and moment are depicted in these figures, for the experimental wave heights. The numerical results are obtained using an exact second-order formulation[18] which accounts for the second-order diffraction potential. It seems that the inclusion of second-order effects improves the correlation between numerical predictions and experimental data in the whole frequency range. Finally, the experimental results for the heaving motion, plotted in Fig. 7, are in good correlation with the respective numerical ones.

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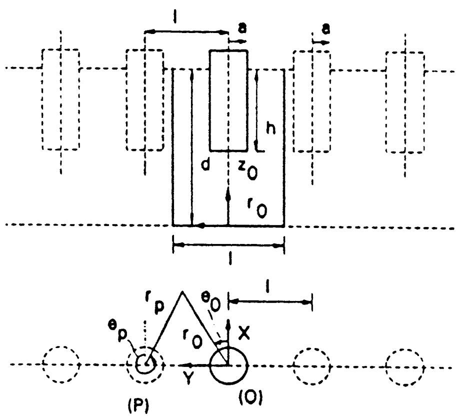

Figure 1: Array of vertical cylinders. Definitions

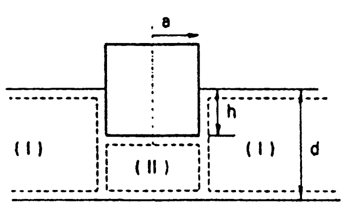

Figure 2: Vertical truncated cylinder. Flow discretization

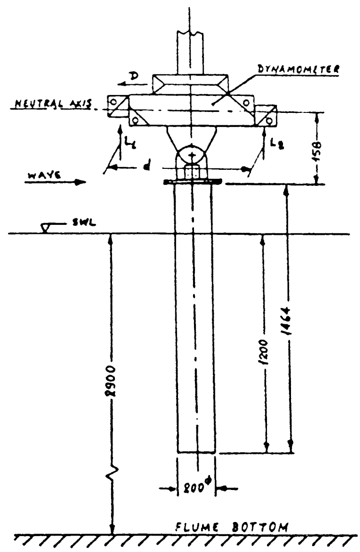

Figure 3: Model description and its mounting system

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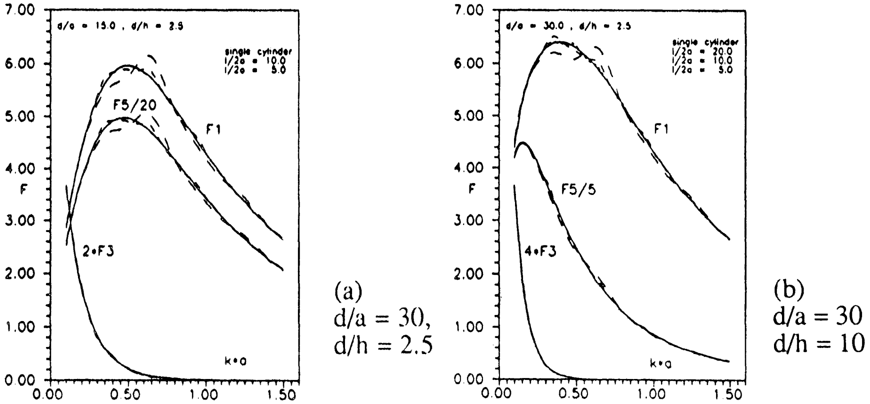

Figure 4: Nondimensional exciting forces and moment.

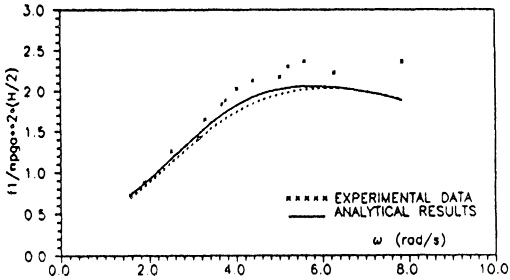

Figure 5: Correlation of dimensionless maximum horizontal forces on the model. (--- : linearized theory, ___ : exciting force up to second order, x : experiments)

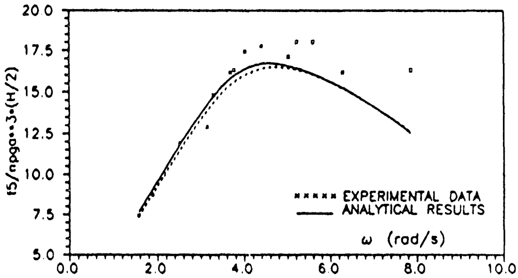

Figure 6: Correlation of dimensionless maximum overturning moment on model (---: linearized theory, ___: exciting moment up to second order, x: experiments)

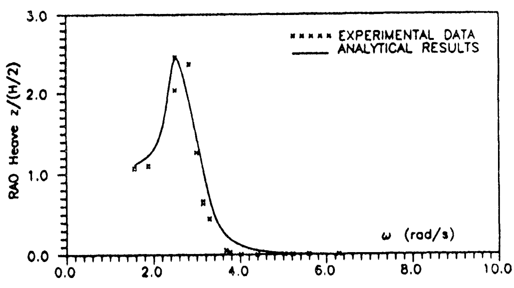

Figure 7: Correlation of measured and computed heave motions.