Paper from: *Hydraulic Information Management*, CA Brebbia and WR Blain (Editors).
ISBN 1-85312-912-7

Modeling and sensitivity analysis of aquifer parameters for subsidence due to pumping-injecting water

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Abstract

In the present paper, aquifer modeling and analysis of parameter sensitivity have been conducted. First, a linear elastic solution has been introduced for modeling and sensitivity analysis of subsidence due to aquifer artificial discharge and recharge. The analytical modeling corresponds to boundary and initial conditions for discharge-recharge activity in a single and two aquifers respectively. The boundary condition comprises linear and periodic changes with time in strains that are induced by fluctuations of hydraulic head. The confined and saturated aquifer system is assumed to behave like linear poroelastic material. Second, sensitivity of aquifer parameters has been analyzed for cases of single and double wells installed within aquifers. Differences of parameter sensitivity related to pumping-injecting activity and aquifer property have been compared and analyzed in a unit column of an idealized compressible semi-pervious layer for one-dimensional land subsidence due to groundwater withdrawal and rejection. Results of sensitivity analysis of aquifer parameters, therefore, can be applied to guidance of pumping-injecting water from and into aquifers so that the potential risk of land subsidence due to artificial recharge and discharge can be reduced.

1 Introduction

Technology of artificial pumping-injecting water within aquifers is widely applied in the United States for management of water resources [1]. Pumping-injecting water, however, induces fluctuations of hydraulic head in aquifer systems. Changes in hydraulic head can in turn cause land subsidence. For confined aquifer systems, this is because changes in hydraulic head cause

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confined aquifer systems, this is because changes in hydraulic head cause changes in pore water pressure that are directly related to changes in effective stress on an aquifer's skeletal frame through the principle of effective stress [2]. Fluctuations of effective stress control the deformation of an aquifer system [3]. Due to groundwater withdrawal or injection, the accumulated compression of an aquifer system eventually manifests itself at the land surface as land subsidence [4] and [5]. In the present paper, aquifer modeling and sensitivity analysis of aquifer parameters are conducted to estimate the potential risk of land subsidence due to pumping- injecting water into or from a single and double wells installed on opposite sides of a compressible confining bed.

An analytical solution in terms of displacement for the one-dimensional problem [6] is introduced in this paper for specified boundary and initial conditions. At the boundaries, an assumption of both periodic and linear variations of hydraulic head or water pressure with respect to time is made. Based on the principle of effective stress, the boundary conditions are related to boundary strain of the soil skeleton. The periodic component of hydraulic head fluctuation is meant to simulate the situation of pumping and/or injecting water into an aquifer at regular intervals. At the same time, the average mean of the fluctuating hydraulic head is assumed to decrease or increase linearly with time or to be constant. Thus, the case of the long-term recharge into the two aquifers can be less than, more than or equal to the overall long-term discharge from the aquifers.

2 A linear analytic solution

2.1 A sandwich conceptual model and boundary loadings

A conceptual model of two confined aquifers with two assumed wells for withdrawal and injection of water is drawn in Figure 1:

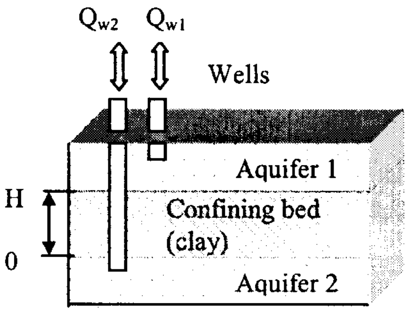

Figure 1. A conceptual model

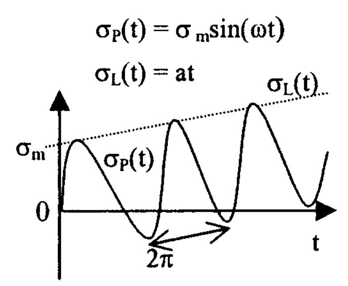

Figure 2. Loadings $\sigma_P(t)$ and $\sigma_L(t)$

In Figure 1, one well is located in the top aquifer (sand) and the other is located in the bottom aquifer (sand). Both wells pump and inject water from or

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into each aquifer independently of each other at pumping-injecting rates Q_{w1} and Q_{w2}. In Figure 2, changes in hydraulic head or pore water pressure induce loading stresses σ_L and σ_P that change linearly and periodically with time at the two interfaces between aquifers (sand) and the hydraulic separator (clay). Namely periodic changes in effective stress σ_P (= $\sigma_m \sin\omega t$) are combined with linear changes in effective stress σ_L (= at) at boundaries. Subscripts L and P denote linear and periodic stresses. The parameter a in units of kN/s-m^2 is related to the slope of the linear loading function long-term rise or fall of hydraulic head, and σ_m in units of kN/m^2 (kPa) associates with amplitude of periodic rise and fall of hydraulic head. t is time and ω (= $2\pi f$) is angular frequency. The frequency f is introduced for periodic fluctuations of hydraulic head due to pumping-injecting water either from or into the two confined aquifers as shown in Figure 1.

2.2 Linear elastic constitutive relationship

If one assumes that the compressible hydraulic separator behaves as linear poroelastic material, a linear stress-strain relation under linear plus periodic stress at the aquifers -confining interfaces is given in the following form:

$$\varepsilon_1(t) = \varepsilon_{L1}(t) + \varepsilon_{P1}(t) = \sigma_{L1}(t)/E_{L1} + \sigma_{P1}(t)/E_{P1} \tag{1a}$$

$$\varepsilon_2(t) = \varepsilon_{L2}(t - t_i) + \varepsilon_{P2}(t - t_i) = \sigma_{L2}(t - t_i)/E_{L2} + \sigma_{P2}(t - t_i)/E_{p2}, \tag{1b}$$

where strains $\varepsilon_1(t)$ and $\varepsilon_2(t)$ are at the top and bottom boundaries of the hydraulic separator (the clay layer) due to loading stresses $\sigma_P(t)$ and $\sigma_L(t)$ shown in Figure 2. Variables t and t_i represent time and initial time that denotes the delayed time of pumping-injecting activity in the second or lower aquifer. Subscripts 1 and 2 of loading stresses respectively stand for upper and lower interfaces of aquifers and the hydraulic separator (the clay layer) shown in Figure 1. Parameters E_P and E_L as shown in figure (3a) and (3b) denote Young's modulus of the confining layer at the interfaces under periodic and linear loadings respectively as shown in Figures (3a) and (3b).

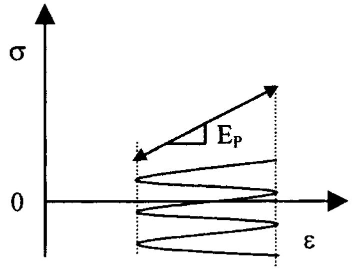

Figure 3a. Elastic Modulus E_P

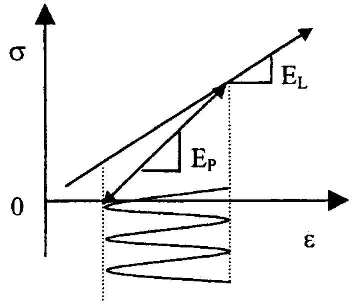

Figure 3b. Elastic Moduli E_P and E_L

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Substituting the linear and sinusoidal loading stresses $\sigma_P(t)$ and $\sigma_L(t)$ into eqns (1a) and (1b), then one has eqns (1a) and (1b) in the alternative forms:

$$\varepsilon_1(t) = a_1 t/E_{L1} + \sigma_{m1}\sin(\omega_1 t)/E_{P1} \tag{2a}$$

$$\varepsilon_2(t) = a_2(t - t_i)/E_{L2} + \sigma_{m2}\sin[\omega_2(t - t_i)]/E_{P2}. \tag{2b}$$

The cases of long-term recharge larger than ($a < 0$), less than ($a > 0$) and equal to ($a = 0$) long-term discharge within an aquifer system can be analyzed. If one assumes the compressible clay is linear poroelastic, one can find 1) that for the case of $a > 0$ (i.e., hydraulic head decreases), E_L is less than E_P than since Young's modulus E_P represents the slope of the loading-unloading curve of consolidation, and 2) that for the case of $0 \geq a$ (i.e., hydraulic head increases), E_L equals E_P since both E_L and E_P are the same and equal the slopes of the loading-unloading curve of consolidation.

2.3 An analytical solution in terms of displacement

If one assumes that the hydraulic separator between the two aquifers has linear poroelastic behavior as shown in Figure 3, that the fluctuation of hydraulic head within each aquifer approximately changes as a periodic function in Figure 2, and that the average mean of hydraulic head changes linearly; that the effective stress principle holds; that total stress within the boundary layers does not change very much with time, then a simplified governing equation written in terms of displacement u of the skeletal frame [7] is applicable for the one-dimensional deformation. Based on loading at both the upper and lower boundary and initial conditions in the conceptual model as shown in Figure 1, one can write the analytical solution in terms of displacement u as a function of non-dimensional variables T and Z with two parts [8]:

$$u(Z,T) = u_L(Z,T) + u_P(Z,T). \tag{3}$$

The variables Z and T are defined as normalized space ($Z = z/H$) and non-dimensional time factor ($T = tc_v/H^2$) where z is the coordinate in the vertical direction. The first part of the solution (3) stands for the displacement u_L in response to the linear loading stress at boundaries and the second part u_P is related to the periodic loading stress at the boundaries. Both u_L and u_P are found in following expressions:

$$u_L(Z,T) = 2H\left\{\int_0^T \left[\frac{\sigma_{1L}(\tau)}{E_{1L}}\sum_{n=1}^{\infty}(-1)^{(n-1)}\sin MZ e^{-M^2(T-\tau)} + \frac{\sigma_{2L}(\tau - T_i)}{E_{2L}}\sum_{n=1}^{\infty}(-1)^{(n-1)}\sin M(1-Z)e^{-M^2(T-\tau)}\right]d\tau\right\} \tag{4a}$$

$$u_P(Z,T) = -2H\left\{\int_0^T \left[\frac{\sigma_{1P}(\tau)}{E_{1P}}\sum_{n=1}^{\infty}(-1)^{(n-1)}\sin MZ e^{-M^2(T-\tau)} + \frac{\sigma_{2P}(\tau - T_0)}{E_{2P}}\sum_{n=1}^{\infty}(-1)^{(n-1)}\sin M(1-Z)e^{-M^2(T-\tau)}\right]d\tau\right\} \tag{4b}$$

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where c_v (= $K/m_v\gamma_w$ = K/S_s) is the consolidation coefficient of the hydraulic separator and is assumed to be a constant though conductivity K and coefficient of volume compressibility m_v (= 1/E) are not constant and change individually with increase of depth [9]. S_s and H stand for storage coefficient and thickness of the clay layer in Figure 1. In eqns (4a) and (4b), dimensionless variable τ stands for a non-dimensional integral variable respect with time ($\tau = t'c_v/H^2$). The dimensionless time factor T within the integration is not a variable, but dimensionless time factor τ (related to time t') is an integral variable. The term T_i is initial dimensionless time factor T at $t = t_i$ and represents the time delayed in the second aquifer. M is defined by $(2n-1)\pi/2$ and n is a summation integer. Further sensitivity analysis of aquifer parameter can be conducted using software such as MathCad or Maple.

3. Sensitivity analysis of aquifer parameters

3.1 Analysis of parameter sensitivity for two wells

If one assumes that two wells are installed into both the upper and lower aquifers as shown in Figure 1, from the solution (3), the total cumulated deformation can be calculated from the difference u(Z, T) - u(Z_0, T), where Z_0 can be any elevation at which no displacement (or no vertical movement) is assumed to be occurring, say, a point that serves as a datum. For example, one can choose a convenient datum to lie at the base of the clay bed, namely at z = 0. Solving eqn (3) for Z at the specified Z_0 (= 0) and then subtracting the result from an independent solution of eqn (3) for nonzero Z yields the total cumulated displacement between the two elevations. This subtraction process simply translates the origin of the zero-displacement coordinate to a datum of interest. For example, if one chooses Z_0 = 0 and Z = 1.0 (i.e., z = H), then the total cumulative displacement of the clay layer between Z = 0 to Z = 1.0 can be:

$$\Delta u = u(1,T) - u(0,T) = \Delta u_L(T) + \Delta u_P(T), \tag{5}$$

where Δu_L and Δu_P at $T = T_a$ are defined by:

$$\Delta u_L = \frac{2H^3}{c_v}\int_0^{T_a}\{[\frac{a_1\tau}{E_{L1}} + \frac{a_2(\tau - T_i)}{E_{L2}}]\sum_{n=1}^{\infty} e^{-M^2(T-\tau)}\}d\tau \tag{6a}$$

$$\Delta u_P = \frac{2H^3}{c_v}\int_0^{T_a}\{[\frac{\sigma_{m1}\sin(\varpi\tau)}{E_{p1}} + \frac{\sigma_{m2}\sin\varpi(\tau - T_i)}{E_{p2}}]\sum_{n=1}^{\infty} e^{-M^2(T-\tau)}\}d\tau, \tag{6b}$$

where ϖ is dimensionless factor of angular frequency ($\omega H^2/c_v$), T_a is a constant time factor, and subscripts L and P represent the linear and periodic loading individually, ϖT_i denotes the difference of pumping-injecting phase between the two aquifers. If $a_2 = a_1 > 0$ in eqn (6a) and $\varpi T_i = 0$ in eqn (6b), which means 1) that the linear loading in the upper aquifer increases in the same rate as that in

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the lower aquifer, and 2) that there is no pumping-injecting phase lag between the two aquifers, the maximum Δu_L and Δu_P can be found from (6a) and (6b). If the following assumptions are applied: $E_{L1} = E_{L2}$, $E_{P1} = E_{P2}$, $\sigma_{m1} = \sigma_{m2}$ and $\omega_1 = \omega_2$, the maximum Δu_{Lmax} and Δu_{Pmax} at $T = T_a$ becomes:

$$\Delta u_{Lmax} = \frac{4aH^3}{E_L c_v} \int_0^{T_a} \tau \sum_{n=1}^{\infty} e^{-M^2(T-\tau)} d\tau \tag{7a}$$

$$\Delta u_{Pmax} = \frac{4\sigma_m H^3}{E_p c_v} \int_0^{T_a} \sin(\varpi\tau) \sum_{n=1}^{\infty} e^{-M^2(T-\tau)} \} d\tau . \tag{7b}$$

In contrast, if $a_2 = -a_1$ in eqn (6a) and $\varpi T_i = \pi$, eqns (6a) and (6b) reach their minimum value Δu_{Lmin} and Δu_{Pmin} (= 0). If one defines the normalized maximum displacement as:

$$\Delta u^*_{Lmax} = [\Delta u_{Lmax}(p) / \Delta u_{Lmax}(p_0)] - 1 \tag{8a}$$

$$\Delta u^*_{Pmax} = [\Delta u_{Pmax}(p) / \Delta u_{Pmax}(p_0)] - 1 , \tag{8b}$$

where p and p_0 denote arbitrary and initial values of an aquifer parameter for sensitivity analysis. In the present paper, p represents parameters for both soil property (e.g., E, S_s, K and c_v) and pumping-injecting parameters (i.e., a, σ_m and ω). A set of initial values of parameter p_0 applied in the present paper is given in Table 1.

Table 1. A set of values used for initial parameters p_0

H_0 (m)	ω_0 ($=2\pi f$)	a_0 (kPa/day)	σ_{m0} (kPa)	E_{L0} (kPa)	E_{p0} (kPa)	K_0 (m/day)	c_{v0} (m^2/day)
50	0.017	10^{-3}	100	10^3	10^2	10^{-5}	10^{-3}

Substituting eqns (7a) and (7b) into eqns (8a) and (8b), one can express Δu_{Lmax} and Δu_{Pmax} further in the forms below:

$$\Delta u^*_{Lmax} = [\frac{aH^3}{E_L c_v} \int_0^{T_a} \tau \sum_{n=1}^{\infty} e^{-M^2(T-\tau)} d\tau / \frac{a_0 H_0^3}{E_{L0} c_{v0}} \int_0^{T_a} \tau_0 \sum_{n=1}^{\infty} e^{-M^2(T_0-\tau_0)} d\tau_0] - 1 \tag{9a}$$

$$\Delta u^*_{Pmax} = [\frac{\sigma_m H^3}{E_p c_v} \int_0^{T_a} \sin(\varpi\tau) \sum_{n=1}^{\infty} e^{-M^2(T-\tau)} d\tau / \frac{\sigma_{m0} H_0^3}{E_{P0} c_{v0}} \int_0^{T_a} \sin(\varpi_0 \tau_0) \sum_{n=1}^{\infty} e^{-M^2(T_0-\tau_0)} d\tau_0] - 1 , \tag{9b}$$

where $T_0 = tc_{v0}/H_0^2$. Relations of the normalized deformation u^* of the compressible layer versus aquifer parameters p for t = 100 days and t = 500 days are plotted in Figures 4a and 4b.

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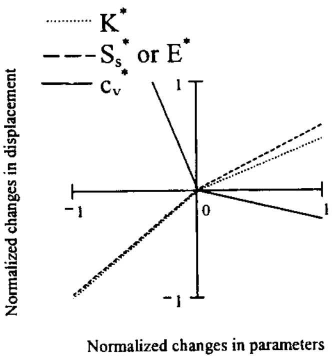

Figure 4a. Δu^*_{max} vs. K^*, c_v^* and E^* or S_s with t = 100 days

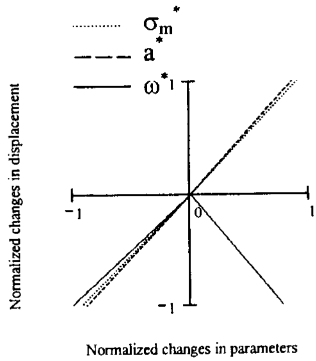

Figure 4b. Δu^*_{max} vs. a^*, σ_m^* and ω^* with t = 100 days

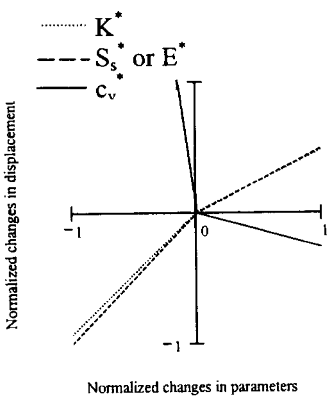

Figure 5a. Δu^*_{max} vs. K^*, c_v^* and E^* or S_s with t = 500 days

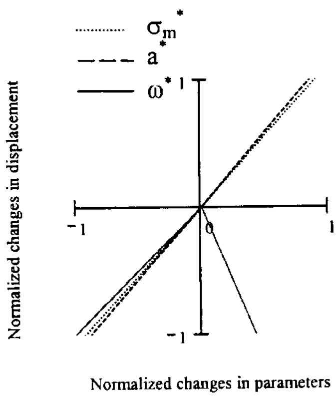

Figure 5b. Δu^*_{max} vs. a^*, σ_m^* and ω^* with t = 500 days

From the curves shown in Figures 4a and 5a, one nay note the following facts related to the parameters of soil properties are observed:

1. When $-1.0 \leq p^* < 0$ where p^* can be either K^* or c_v^*, the normalized displacement u^* indicates the same sensitivity to parameters K^* and c_v^*. u^* is most sensitive to Young's modulus E^* or the storage coefficient S_s^*, when one compares the curve slope of u^* - E^* (or u^* - S_s^*) to those of u^* - K^* and u^* - c_v^*.

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2. When $0 \leq p^* \leq 1.0$, the normalized displacement u^* shows less sensitivity to both K and c_v if the curve slopes for $0 \leq p^* \leq 1.0$ are compared to those for $1.0 \leq p^* < 0$. In contrast, when $0 \leq p^* \leq 1.0$ u^* seems least sensitive to E^* or S_s^*, if the curve slopes of u^* - K^* and u^* - c_v^* are compared to the curve slope of u^* - E^* (or u^* - S_s^*).
3. When $-1.0 \leq p^* < 0$, if one compares Figure 4a for t = 100 days to Figure 5a for t = 500 days, sensitivity of normalized Young's modulus E^* (or S_s^*) to u^* is a function of time and increases with time during the given period.

Furthermore from Figures 4b and 5b, one may note the following facts are related to pumping –injecting parameters:

1. When $-1 \leq p^* < 0$ where p^* can be either a^* or σ_m^* or ω^*, the normalized displacement u^* indicates the similar sensitivity to the normalized linear loading rate a^* and the normalized periodic loading amplitude σ_m^*. It is found that u^* is most sensitive to the normalized pumping-injecting angular frequency ω^* when one compares the curve slope of u^* - ω^* to those of u^* - a^* and u^* - σ_m^*.
2. When $0 \leq p^* \leq 1.0$ the normalized displacement u^* shows the same sensitivity to the normalized linear loading rate a^*, σ_m^* and ω^* when one compares the curves slopes for $0 \leq p^* \leq 1.0$ to those for $-1.0 \leq p^* < 0$.
3. When $0 \leq p^* \leq 1.0$, if one compares Figure 4b for t = 100 days to Figure 5b for t = 500 days, sensitivity of the normalized angular frequency ω^* to the u^* is a function of time and increases during the investigated period.

3.2 Analysis of parameter sensitivity for one well

If one assumes that only one well is installed instead of two wells as shown in Figure 1 (i.e., assuming that Well 2 is installed into the lower aquifer), the analytical solution (3) reduces to:

$$u(Z,T) = -2H\{\int_0^T [\frac{\sigma_{2L}(\tau)}{E_{2L}} + \frac{\sigma_{2p}(\tau)}{E_{2p}}]\sum_{n=1}^{\infty}(-1)^{(n-1)}\sin M(1-Z)e^{-M^2(T-\tau)}d\tau\}. \tag{10}$$

For convenience of sensitivity analysis of aquifer parameters, the normalized displacement u^* for the case of using a single pumping-injecting well at $T = T_a$ and $Z = 0$ is defined as:

$$u^* = [u(0, T_a, p)/u(0, T_a, p_0)] - 1. \tag{11}$$

The same set of values of aquifer parameters as listed in Table 1 can be applied for the sensitivity analysis for the case of single well. Applying eqns (9) and (10), one can have periodic and linear parts of the normalized displacement respectively in the following forms:

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$$u_L^* = [\frac{aH^3}{E_L c_v} \int_0^{T_a} \tau \sum_{n=1}^{\infty} e^{-M^2(T-\tau)} d\tau / \frac{a_0 H_0^3}{E_{L0} c_{v0}} \int_0^{T_a} \tau_0 \sum_{n=1}^{\infty} e^{-M^2(T_0-\tau_0)} d\tau_0] - 1 \quad (12a)$$

$$u_P^* = [\frac{\sigma_m H^3}{E_P c_v} \int_0^{T_a} \sin(\varpi\tau) \sum_{n=1}^{\infty} e^{-M^2(T-\tau)} d\tau / \frac{\sigma_{m0} H_0^3}{E_{P0} c_{v0}} \int_0^{T_a} \sin(\varpi_0 \tau_0) \sum_{n=1}^{\infty} e^{-M^2(T_0-\tau_0)} d\tau_0] - 1 \quad (12b)$$

One may note that u_L^* and u_P^* as indicated in eqns (12a) and (12b) have the same expressions as Δu^*_{Lmax} and Δu^*_{Pmax} given in eqns (9a) and (9b). This implies that the normalized displacement u^* induced by pumping-injecting water using a single well has the same sensitivity to aquifer parameters as the normalized maximum displacement Δu^*_{max} does for pumping-injecting water using two wells. In other words, one can conclude the relations of the changes in u^* versus changes in the normalized parameters p^* [$= (p - p_0)/p_0$] for using a single well will have the same pattern for using two wells as shown in Figures 4a, 4b and 5a and 5b.

4 Summary and conclusions

In brief, the following can be summarized. First, a one-dimensional analytic model has been introduced for sensitivity analysis. The analytic modeling is based on conceptual, physical and mathematical models. The conceptual model is featured by an aquifer-clay-aquifer sandwich pattern. The mathematical model is built on specific boundary and initial conditions that are set according to sinusoidal and linear pumping-injecting activity at the interfaces of the hydraulic separator and two aquifers. The physical model is introduced with poroelasticity. Second, sensitivity analysis has been conducted for aquifer parameters of the aquifer systems with installed two wells. Sensitivity analysis of aquifer parameters has been conducted for the case of maximum deformation. Finally, the sensitivity analysis of aquifer parameter has been conducted for the case of a single well installed for pumping-injecting water.

From the results of analysis, the following conclusions can be drawn: 1) aquifer displacement illustrates approximately the same sensitivity to soil property parameters K^* and c_v^*, and higher sensitivity to the parameter E^* or S_s^*. This is especially true when $-1.0 \leq E^* < 0$ (or $-1.0 \leq S_s^* < 0$) in Figures 4a and 4b, 2) aquifer displacement shows approximately the same sensitivity to pumping-injecting parameters a^* and σ_m^*, and higher sensitivity to ω^* when $0 \leq \omega^* \leq 1.0$ in Figures 5a and 5b, 3) sensitivity of aquifer parameters, such as the soil property parameter E^* and the pumping-injecting parameter ω, to the aquifer deformation change with time (Figure 4 for t = 100 days versus Figure 5 for t = 500 days) and 4) the analysis results imply that aquifer deformation indicates the same sensitivity to aquifer parameters when a single or double pumping-injecting wells are applied.

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ISBN 1-85312-912-7

Acknowledgement

This research project is supported through the NASA-MSU MNMG program. The author is grateful to the support for the research.

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Nomenclature (Sample Symbols)

a, a_i $(i = 1, 2)$	Slopes of linear changes in mean hydraulic head within aquifers, $M/T^3.L$.
c_v	$(\equiv K/m_v\gamma_w)$ coefficient of consolidation, L^2/T.
E_i $(i = L, P)$	Young's modulus of clay under linear and periodic loadings M/T^2L
f, $f_i(i = 1, 2)$	Loading frequency at the upper and lower boundaries, $1/T$
H	Thickness of the clay layer, L
K	Hydraulic conductivity, L/T
m_v	Coefficient of volume compressibility of porous material, T^2L/M
M	$[\equiv(2n-1)\pi/2]$ where n is an integer
n	Porosity
p, p^*	Aquifer parameters
Q_{wi} $(i = 1, 2)$	Pumping rate in wells 1 and 2, L^3/T
S_s	Storage coefficient (1/L)
t	Time, T.
T	$(\equiv tc_v/H^2)$ a dimensionless time factor
T_i	$(\equiv t_ic_v/H^2)$ an initial dimensionless time factor
T_a	$(\equiv t_ac_v/H^2)$ a constant dimensionless time factor
u, u_i $(i = L, P)$	Displacement and its members, L
z	Spatial coordinate in vertical direction, L
Z	$(\equiv z/H)$ normalized coordinate value
Z_0	$(\equiv z_0/H)$ normalized coordinate at $z = z_0$
γ_w	Unit weight of water, MT^2/L^2
σ_i $(i = L, P)$	Effective stress related to linear and periodic loading, M/T^2L
σ_{mi} $(i = 1, 2)$	Amplitude of effective stress related to periodic loading, M/T^2L
ϖ_i $(i = 1, 2)$	$(\equiv\omega_iH^2/c_v)$ dimensionless angular frequency factor.
ω_i $(i = 1, 2)$	$(\equiv 2\pi f_i)$ angle frequency, $1/T$
Δ	Increment of a variable

Subscripts

0	The initial value of a variable
1, 2	Upper and lower boundaries
i	Constant with respect to time representing an initial state.
L	Linear changes in mean hydraulic head at boundaries.
P	Periodic changes in fluctuation of hydraulic head at boundaries.

Superscripts

*	Normalized variables.