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Modelling of frost formation over a flat plate

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Abstract

This paper models the frost formation process on a cold surface exposed to warm moist air flow. The model employs a one dimensional transient formulation based upon the local volume averaging technique. This model enables predicting the frost temperature, density and thickness distribution along the plate as well as the void fraction. Available experimental data was used to validate the model. Also numerical experiments were realized to establish the best initial values of the diffusivity and initial radius of the ice crystals.

1 Introduction

Thermal processes in which heat is transfered to or from a cold surface are important in a variety of applications in refrigeration, compressores blades, heat pumps and heat recovery systems, etc. As the frost layer grows, the convective heat transfer is affected in part because the insulating effect of the frost thickness. This can adversely affect the performance of cooling coils and plate freezers in domestic and industrial refrigeration equipments. Depending on exposure time and physical circunstances, frost forming on cold heat exchanger surfaces may enhance or reduce heat fluxes while airflow rates and pressure drops may remain nearly constant or sometimes change drastically. Defrosting is essential once the performance degradation reaches a selected level. In spite of the large number of refrigeration, heat pump, and heat recovery applications, designing heat exchanger surfaces to accomodate frost growth and selecting defrost cycles have been, mostly dependent on test results of specific equipments. Although this experimental effort is essential for the efficient operation of heat exchangers under frosting conditions, it is not enough for the design purposes of such equipments.

The frost formation process is complex because the frost properties vary continuously during the development of the frost layer and also due to the continuous change both temporal and spatial of the air-frost interface temperature. As the interface temperature changes, the partial pressure of water vapour at the surface also changes, leading to changing the thermal and diffusion boundany layers and eventually affecting the heat transfer and frost growth rates.Trammell, Canterbury and Killgore[1] studied experimentally the heat transfer from humid air to a horizontal flat plate held at sub-freezing temperatures. Padki, Sherif and Nelson[2] proposed a simple method for modelling the frost formation phenomenon in different geometries. The method predicts the heat transfer and frost growth rates, frost thickness and surface temperature as functions of time and position. The method utilizes known convectives heat transfer correlations for different geometries and the Lewis anology to determine the convective mass transfer and the enthalpy transfer coefficients. Model results are compared with existing experimental data and good agreement is obtained. Sami and Duong[3] presented a numerical model based on the molecular diffusion of water vapour, the energy, and mass balances as well as the equations of state. Numerical results revealed that the proposed model reliably predicted frost growth parameters such as frost thickness and density and compared well with existing experimental data as well as other analytical models.

Ostin and Andersson[4] studied exprimentally the frost on parallel horizontal plates facing an air stream at varying temperatures, relative humidities and air velocities. Both the surface temperature of the plate and the relative humidity of the air stream are found to have important effects for the frost thickness. The density of frost is found to increase with relative humidity and air velocity. Later Mao, Besant and Rezkallah[5] presented measurements and correlations for frost formation on a flat plate. Typical measurements of frost thickness, mass concentration and heat flux are presented as a function of distance from the leading edge, humidity ratio of the inlet air, cold plate temperatures, inlet velocity (or Reynolds number), and time. Sherif, Raju, Padki and Chan[6] presented a semi-empirical transient method for modelling frost formation on a flat plate under forced convection conditions. The model is based on empirical correlations for predicting the frost thermal conductivity and density. Model results were compared with existing experimental data and with other theoretical models showing good agreement. Tao, Besant and Mao[7] studied the characteristics of frost growth on a flat plate during the early growth period while Tao, Besant and Rezkallah[8] proposed a mathematical model for predicting the growth of frost on a flat plate. They simulated the frost deposition on a cold surface exposed to moist air flow using one dimensional transient formulation based upon the local volume averaging technique.

The objetive of the present study is to elaborate a computational code for predicting the frost formation on a cold flat plate. The initial entry parameters were optimized experimentally and the results validated with experiments.

2 Formulation of the model

The process of frost formation shown in figure 1. is complex due to the fact that it involves simultaneous transfer of heat and mass under continuously varying thermal properties and air-frost interface conditions. To simplify the analysis it is usually considered that the process of frost formation can be conveniently divided into three periodes; one dimensional crystal growth period, three dimensional growth period and finally the period of quasi-static growth. In the present work a two stage model was used with a transition time to establish the change from the first to the second stage. The second stage of the present model accounts for both the three dimensional and quasi-static stages mentioned above.

a) The stage of one-dimensional crystal growth can be subdivided into two sub-stages:

- The nucleation stage, this stage stars from $t = 0$ to the moment $t = t_{tr1}$ when water condenses on nucleation sites of the plate coalese to form bigger droplets of more uniform size and then start to freeze. This sub-stage is not modelled here and its data is obtained from the literature.
- One dimensional growth period (from $t = t_{tr1}$ to $t = t_{tr2}$), during this period the ice crystals grow in the normal direction to the cold surface. This sub-stage is modelled and the initial conditions are given by the first sub-stage.

b) The stage of quasi-static growth: $t \geq t_{tr2}$.

This period involves the three dimensional growth period of branching ice crystals and then followed by the steady growth.

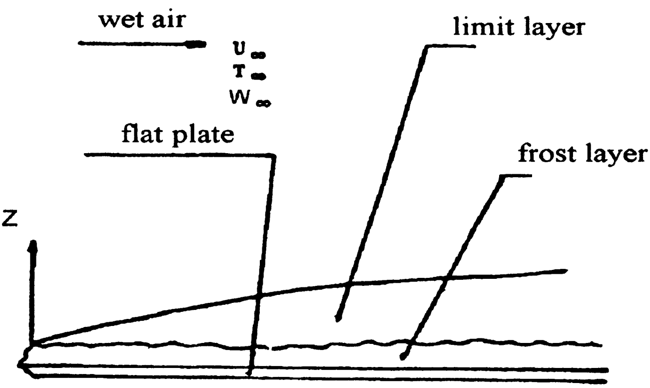

Fig 1. process of frost formation

Considering the formulation of the first stage one can write.

a) The energy equation

$$\rho^*_\beta cp^*_\beta d^* \frac{\partial T^*}{\partial t^*} = k^*_\beta d^* \frac{\partial^2 T^*}{\partial z^{*2}} + 2k^*_\beta \frac{\partial T^*}{\partial z^*}\frac{\partial d^*}{\partial z^*} - 4h^*\left(T^* - T^*_\gamma\right) + 2\rho^*_\beta h^*_{sg} \frac{\partial d^*}{\partial t^*} \quad (1)$$

b) The diffusion equation

$$\rho^*_\beta \frac{\partial d^*}{\partial t^*} = 2hm^*\left(w_\gamma - w\right) \quad (2)$$

The boundary conditions: at $z = \delta_f$

$$\frac{d\delta_f^*}{dt^*} = \frac{hm^*}{\rho_\beta^*}\left[w_\infty - w\left(z^* = \delta_f^*, t^*\right)\right] \tag{3}$$

$$\frac{\partial T^*\left(z^* = \delta_f^*, t^*\right)}{\partial z^*} = \frac{h^*}{\rho_\beta^*}\left[T_\infty^* - T^*\left(z^* = \delta_f^*, t^*\right)\right] + \frac{\rho_\beta^* h_{sg}^*}{k_\beta^*}\frac{d\delta_f^*}{dt^*} \tag{4}$$

and at $z = 0$

$$T^*\left(z^* = \delta_f^*, t^*\right) = T_c^* \quad (5); \qquad \frac{\partial d^*\left(z^* = 0, t^*\right)}{\partial z^*} = 0 \tag{6}$$

The initial conditions:

$$\delta_f^*\left(t = t_{tr1}\right) = \delta_{fo}^* \quad (7); \quad d^*\left(z^*, t^* = t_{tr1}\right) = d_o^* \quad (8); \quad T^*\left(z^*, t^* = t_{tr1}\right) = T_c^* \quad (9)$$

To formulate the second stage, we consider that the frost is a porous medium, the solid and the gas are in local equilibrium, the convective effects are negligible, within frost layer, the gas phase is thermodynamically ideal, compression work and viscous dissipation in the β and γ phases are negligible.

Using the average volumetric technique one can write:

a) The energy equation

$$\rho^* cp^* \frac{\partial T^*}{\partial t} + \Delta h_{sub}^* \dot{m}^* = \nabla.\left(k_{eff}^* \nabla T^*\right) = \frac{\partial}{\partial z^*}\left(k_{eff}^* \frac{\partial T^*}{\partial z^*}\right) \tag{10}$$

b) The continuity equation for the frost phase β

$$\frac{\partial \varepsilon_\beta}{\partial t^*} + \frac{\dot{m}^*}{\rho_\beta^*} = 0 \tag{11}$$

c) The diffusion equation the gas-vapor phase

$$\frac{\partial}{\partial t}\left(\varepsilon_\gamma \rho_v^*\right) - \dot{m}^* = \nabla.\left[D_{eff}^* \nabla \rho_v^*\right] = \frac{\partial}{\partial z^*}\left(D_{eff}^* \frac{\partial}{\partial z^*}\left(\rho_v^*\right)\right) \tag{12}$$

The boundary conditions: on the frost layer

$$\rho_{of}^* hm^*\left[w_\infty - w\left(z = \delta_f, t\right)\right] = D_{veffs}^* \frac{\partial \rho_v^*\left(z = \delta_f, t\right)}{\partial z^*} + \rho_f^* \frac{d\delta_f^*}{dt^*} \tag{13}$$

$$h^*\left[T_\infty^* - T^*\left(z = \delta_f, t\right)\right] = k_{eff}^* \frac{\partial T^*\left(z = \delta_f, t\right)}{\partial z^*} + h_{sg}^* \rho_f^* \frac{d\delta_f^*}{dt^*} \tag{14}$$

$$\frac{\partial \varepsilon_\beta\left(z = \delta_f, t\right)}{\partial z^*} = 0 \tag{15}$$

and on the surface of the flat plate

$$T^*\left(z = 0, t\right) = T_c^* \quad (16); \qquad \frac{\partial \varepsilon_\beta\left(z = 0, t\right)}{\partial z^*} = 0 \tag{17}$$

In order to be able to simulate the process of frost formation, it is necessary to determine the general thermophysical properties of the different phases and the transfer coefficients by using experimental and empirical correlations available in the literature[1,7,11].

3 Numerical Solution and Discussion of the Results

The finite difference approximate forms of equations (1) and (2), (10) - (12) are derived using the implicit scheme with the upwind difference for the time derivative, the central difference for internal nodes, and the backward or forward difference for the boundary nodes. A downwind first-order difference scheme is used for the frost growth rate equations (3) and (13) to achieve stability. The relaxation iteration scheme is used to solve the difference equations. The results are considered to be convergent when the deviation of any variable from the last iterated value is within 10^{-5}. The grid size in the frost domain is fixed as 15. For each time step the spatial coordinates are updated based on a new boundary position δ_f . The spatial distribution of each variable at the last time step is also updated to fit the new coordinate system, using polynomial spline interpolation method.

In order to study the effects of the variation of the parameters of the problem, numerical experiments were realized to optimize the grid size used in the numerical simulations. Figure 2 shows the variation of the time step on the time-wise variation of the thickness of the frost layer. As can be seen a step of 0,5 second seems to be good enough. Figure 3 shows the effect of varying the grid size. Four numerical tests were realized for n_z=5,10.15 and 20. $n_z = 5$ gave very inacurate results as compared with the experiments $n_z = 20$ gave unstable behaviour and hence was rejected. $n_z = 15$ seems to give good results and compares well with available experimental results[5,8]. Hence the value of $n_z = 15$ and a time step of 0,5 were used in the numerical simulations. Another parameter which is important in the numerical calculations is the initial value of the radius of the ice column r_o. The value of r_o was varied from $50x10^{-6}$ m to $55x10^{-6}$ m. The results indicate that a value of $52x10^{-6}$ m gives good results as compared with the experimental data as shown in figure 4 and 5. The value of the parameter α was also verified experimentally from 0,3 to 0,5 and the obtained results were compared with available data as shown in figure 6. Analysing the comparative results one can certify that a value of $\alpha = 0.4$ is good enough and hence this value was used in the numerical calculations.

Figure 7 shows the predicted variation of the frost thickness as a function of time and its comparison with experimental data. As can be verified although the agreement in the general trends is good, the diferences for the small and large operational times is noticiable. This can be attributed to the precision of the measurements at the initial stages. Variations of frost density with time is shown in figure 8 showing reasonable trend from 1000 to 7000seconds with a difference of 10%. From the initial stage up to 500 seconds both the trend and differences are very differents. This is due to the initial conditions imposed in the model and also the inaccurate available experimental data. on the other hand we wish to mention that although the differences during the fist instants are big, they seem to have insignificant practical effects. The authors are aware of the necessity of realizing very accurate non-interferring experimental measusurements. The variation of the dimensionless temperature of the frost surface as function of time is shown in figure 9. As can be seen the temperature profile changes smoothly. Figure 10 shows the temperature profiles across the frost porous layer for different time intervals. As can be verified the temperature increases with the increase of time. The rate of mass deposit across the frost porous layer is shown in figure 11 for various time intervals. As can be seen the results are similar to other authors findings. The variation of the void fraction across the frost porous layer is shown in figure 12 for different time intervals indicating the same trends obtained by other authors.

Conclusions.

One can conclude that the model and the numerical code seem to be able to predict the frost properties reasonably well as verified by comparisons with available experimental data. Also as a result of the numerical experiments one can also conclude that a good value of the radius of the ice column r_o is $52x10^{-6}$ m. and of the parameter α is 0.4 .

Acknowledgements.
The authors wish to thank the CAPES for the financial support.

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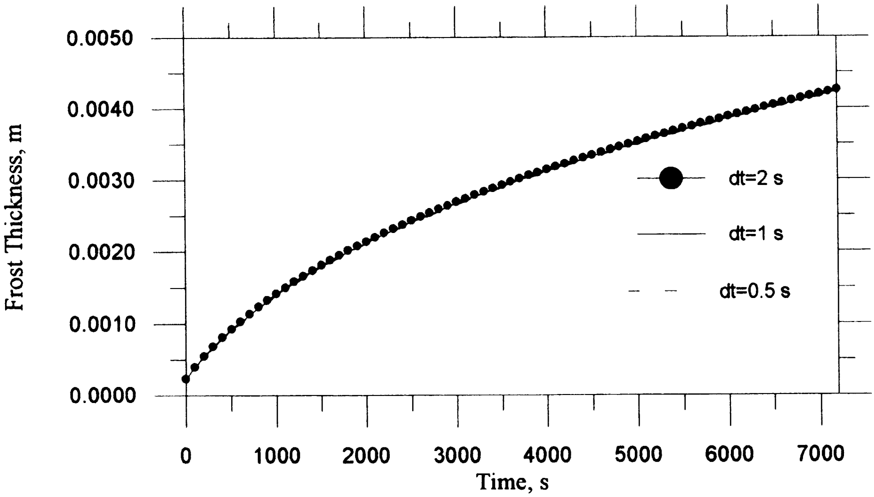

Figure 2. Optimization of the time step Δt.

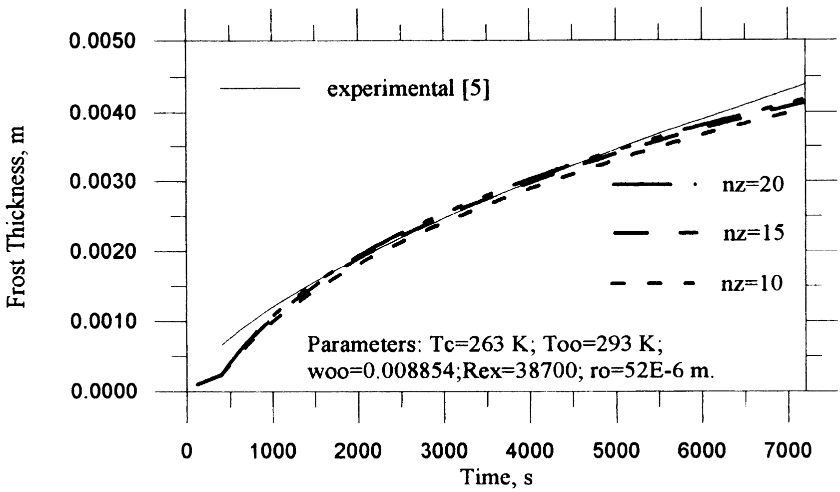

Figure 3. Optimization of the space increment Δz.

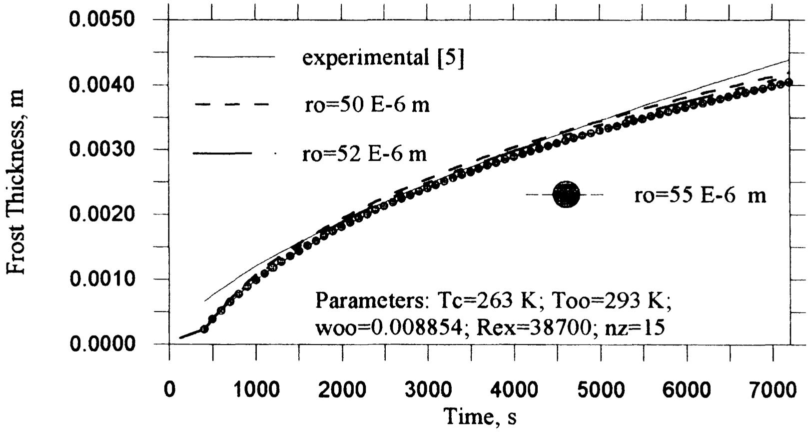

Figure 4. Effect of varying the initial ice column radius on the frost thickness.

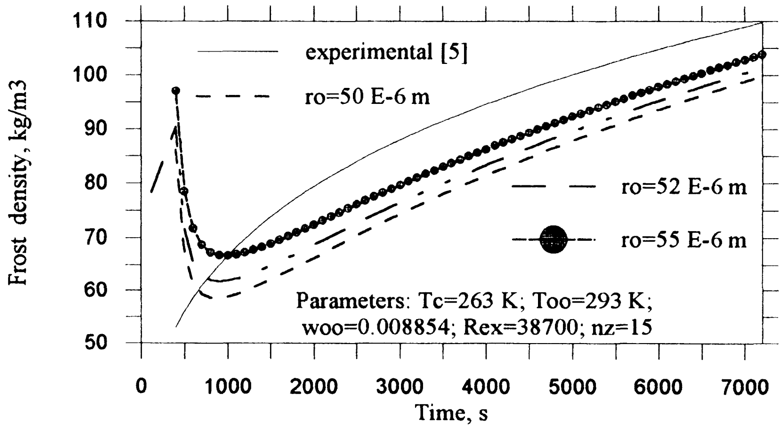

Figure 5. Effect of varying the initial ice column radius on the frost density.

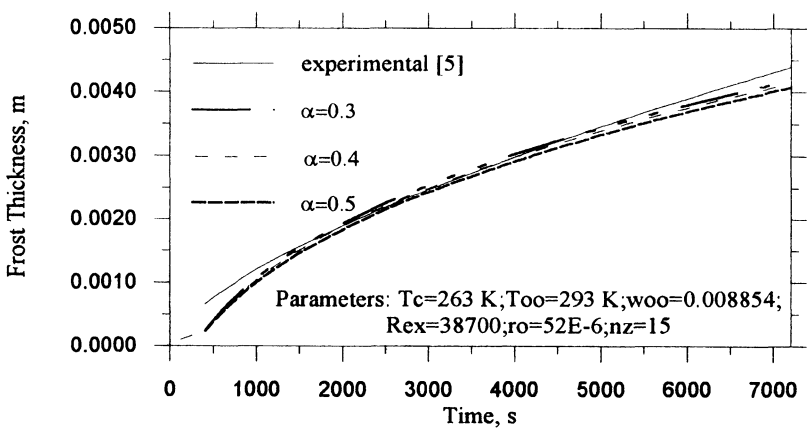

Figure 6. Effect of varying parameter α on the frost thickness.

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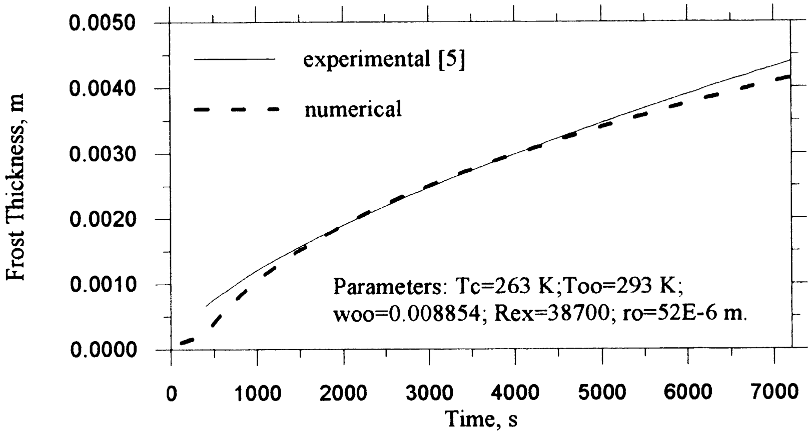

Figure 7. Numerical Predictions and experimental comparison of the frost thickness.

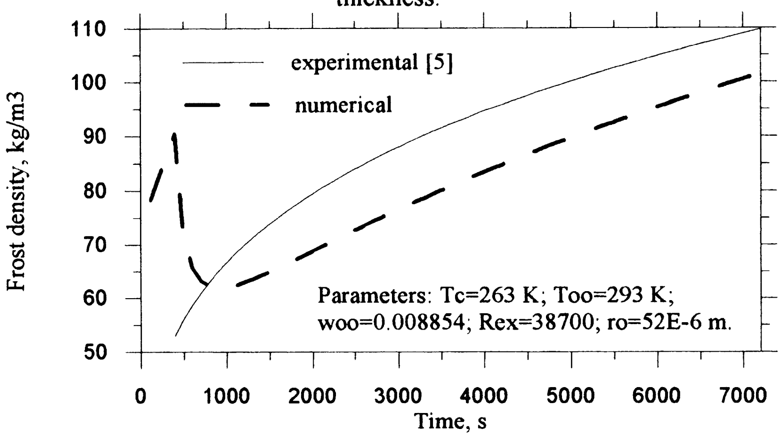

Figure 8. Numerical Predictions and experimental comparison of the frost density.

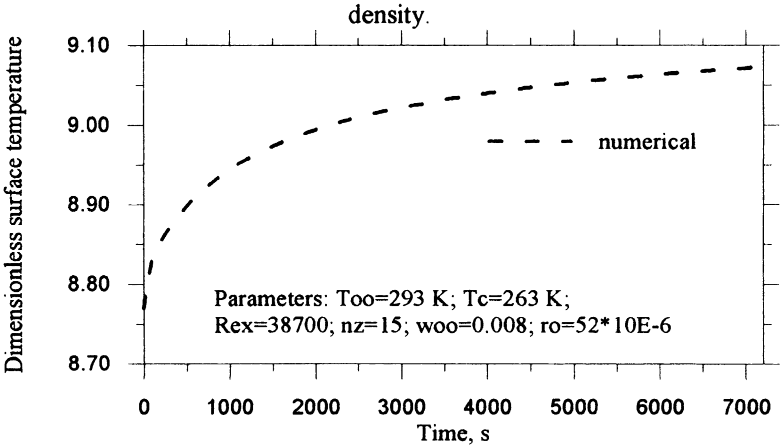

Figure 9. Variation of the surface temperature with time.

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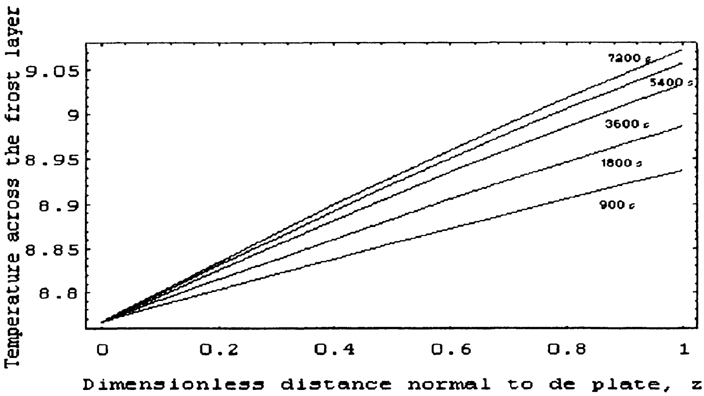

Figure 10. Variation of the frost temperaturre across to the frost layer

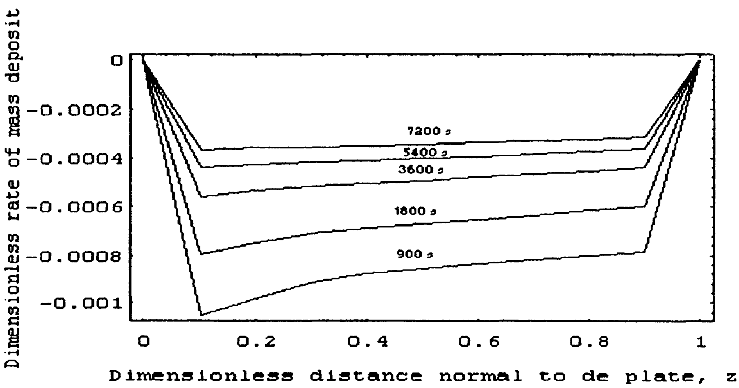

Figure 11. Variation of the rate of mass deposit across to the frost layer

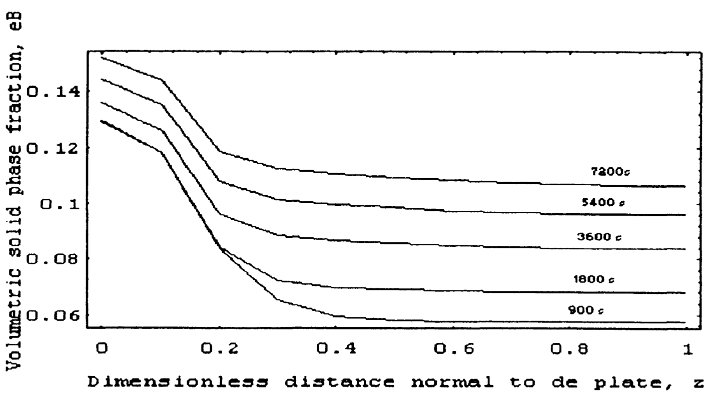

Figure 12. Variation of the volumetric solid phase fracction across to the frost layer