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Heat transfer at the outer surface of a rotating cylinder in the presence of axial flows

R. Smyth & P. Zurita

Department of Mechanical and Process Engineering, University of Sheffield, P.O. Box 600, Mappin Street, Sheffield, S1 4DU, UK

ABSTRACT

A numerical analysis has been made of forced convection heat transfer for axial flow of air along the outer surface of a rotating cylinder. The analysis was two dimensional with an allowance for a swirling component of velocity around the cylinder.The results show that an exponential law correlates the numerical data well with an exponent of 0.8, such as is obtained for turbulent flow past a flat plate. The results were compared with that available for cross flow of air but it is shown that for axial flow, rotation never becomes dominant enough to make the Nusselt number independent of the blowing Reynolds number.

INTRODUCTION

The heat transfer at the outer surface of a rotating cylinder, both in a still environment or in cross flow, has been studied by various authors for different flow regimes. In these analyses it was found that there were two extreme conditions: either (a) the cross flow, or (b) the rotational effect, was dominant. The cross flow extreme gave Nusselt numbers close to the values known for a stationary cylinder. The other extreme for a rotating cylinder immersed in still air also gave known values of Nusselt numbers. Kays [1] found this latter fact to occur when the tangential surface velocity of the cylinder becomes equal to twice the cross flow velocity. It was suggested that this was due to the fact of the two stagnation points coinciding and a continuous rotating boundary layer being developed making the heat transfer independent of the cross flow velocity. More recently, Shimada et al [2] arrived at the same conclusion but extended the range of the Reynolds numbers below the values studied by Kays [1] and included in the analysis, the effects of free and mixed convection.

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The case of the axial flow over a rotating cylinder appears not to have been studied as extensively and there is a lack of information about how the heat transfer depends on the different parameters involved. The flow pattern will be a swirling flow around the cylinder with a varying pitch depending upon the velocity ratio ξ. Hence the heat transfer mechanism is always of the same form. Again, when the axial effect is dominant, the heat transfer should be similar to that for a stationary cylinder. For this case the heat transfer coefficients $\bar{h}$ predicted for the turbulent flow past a flat plate are usually taken and are accurate for large radii of curvature of cylinder. For small radii of curvature of cylinder, the sizes of the eddies in turbulent flow could be of the order of the cylinder diameter. Then the heat transfer mechanism would be more similar to that for cross flow but with smaller Nusselt numbers. This effect has been confirmed by Mueller [3] for axial flow along thin wires.

When rotation of the cylinder becomes the dominant mechanism, it could be expected that the heat transfer would not depend on the axial flow rate, with the average heat transfer coefficient being constant along the cylinder. However, it will be shown that within the range of blowing and rotational Reynolds numbers Re_D considered, the heat transfer coefficient $\bar{h}$ still varies along the cylinder which does not happen in the cross flow situation because air around the cylinder is being continuously replaced by a fresh cross flow of air.

The following calculation procedure was made for 5 cm diameter cylinders, with lengths up to 30 cms. The rotational speeds varied from 450 to 4,500 rpm and the axial velocity from 1.5 to 15 m/s. This gave rotational Reynolds numbers Re_D of 3,000 to 30,000 and blowing Reynolds numbers Re_L of up to 250,000.

COMPUTATION MODEL

The computational model solves the conservation equations of mass, momentum and energy by the well established method of Patankar [4]. Since for low rotational speeds the centrifugal force is enough to throw off the closest particles to the cylinder and replace them by new ones, as explained by Anderson and Saunders [5], it was decided that the air viscosity would be modelled using the turbulent kinetic energy/turbulence dissipation, turbulence modelling procedure. The calculation domain is bounded by the cylindrical wall (at a constant temperature of 340K), the upstream axial section (with a uniform axial velocity), the downstream axial section (with $\partial / \partial x$ of all main variables zero) and a section with undisturbed conditions at large radius (with $\partial / \partial r$ of all main variables zero). This latter section was located at 5.15 cm measured from the wall. All the results shown later were obtained using this computational model. The configuration was two-dimensional in a cylindrical co-ordinate system, even though the three components of velocity had to be calculated.

Although buoyancy effects were neglected, they could be important for the lower range of blowing velocities. For cross flow, its effects are negligible for Reynolds numbers higher than 1000. The case of rotation only has not been computed because free convection is the only mechanism of motion outside the boundary layer. Also, it was observed that it was more difficult to converge the solutions for the cases in which rotation was highly dominant.

A uniform grid size was used in the axial direction. In the radial direction, a varying grid size was used: a region close to the cylinder surface with a fine grid and a second region outside this fine region, with an expanding grid. Figure 1 shows the local value of the heat transfer coefficient at the end of the cylinder for increasing number of nodes for the case with Ω =2000 rpm, u=5 m/s. The maximum number of grid nodes was 100x50 nodes, but grid independence was achieved with a 100x30 grid, it being found that the number of nodes in the axial direction was the most critical.

DISCUSSION OF THE RESULTS

Sixty-six different cases were computed, with the blowing velocity varying between 1.5 and 15 m/s and a rotational speed in the range 450 up to 4500 rpm. Additionally, the case of 10,000 rpm was solved for the whole range of axial velocities in order to study the behaviour of highly dominant rotational speeds, but this discussion will not be extended to this high rotational speed because no calculations were made between 4500 and 10,000 rpm. For each case computed, the axial distribution of heat transfer coefficient along the cylinder was determined. The average heat transfer coefficient $\bar{h}$ was then determined for various lengths of cylinder.

To present the results in a dimensionless form, it would appear that the characteristic length should be the diameter of the cylinder for rotation of the cylinder, whereas the length of cylinder should be chosen for axial flow. On the other hand, as the heat transfer is the product of both these effects, the characteristic length to include in the Nusselt number should be carefully chosen. If the axial flow were dominant a Nusselt number based on the diameter only would not be enough to give a complete description of the heat transfer. The converse of this situation would also apply. As the average Nusselt number for a cylinder of length L is required, it was decided to base the Nusselt number on the length L. If the diameter had been included instead, L would only have appeared in the blowing Reynolds number. Such a

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description is clearly insufficient because the average heat transfer coefficient increases as V is increased, but falls when the length is increased.

In Figure 2 the Nusselt number $\overline{Nu_L}$ was plotted against the blowing Reynolds number Re_L for a typical set of intermediate values of velocities (1500 rpm and 5 m/s). It can be seen that the curve is almost a straight line, with only a little curvature at low values of Re_L. As this curvature is small, an equation of the form $\overline{Nu_L} = \alpha . Re_L^n$ correlates the results well.

In Figures 3 and 4 the results are given for the whole range of blowing velocities and for the two extreme rotational speeds (450 rpm in Figure 3 and 4500 rpm in Figure 4). The straight lines obtained are almost parallel for the whole range of tangential and axial velocities. The slope of the lines is almost 0.8. This index is the same as that for the case of turbulent flow past a flat plate, for which $\overline{Nu_L} = 0.036 Re_L^{0.8} Pr^{1/3}$. This fact would suggest that the development of the boundary layer is similar for both cases. The slope $n=0.8$ for the case of the flat plate was obtained using a measured velocity profile for the buffer sublayer and an analogy between the diffusions of momentum and heat. So it would seem as if a similar profile (u_+ versus $y_+^{1/5}$) could also represent correctly the change in velocity through the boundary layer of the rotating cylinder, though with different coefficients α depending on the rotating velocities. An important difference is that for a stationary flat plate, Re_L is enough to provide a description of the heat transfer. However in the case of the rotating cylinder, it was observed (Figures 3 and 4) that different curves of $\overline{Nu_L}$ versus Re_L were obtained for different blowing velocities, and it would appear that another dimensionless parameter including the blowing velocity, is required. This parameter is ξ, the velocity ratio, and measures the relative importance of blowing and rotation. The inverse of ξ can be interpreted as the pitch of the swirl flow.

The effects of axial flow and rotation are now considered separately, paying special attention to the extremes in which either rotation or blowing is dominant.

Effects of Rotation

The first effect of rotation is obviously to increase the Nusselt number due to the mass transfer resulting from the centrifugal forces on the air and subsequent higher level of turbulence. Additionally, as rotation becomes dominant the decrease in the average heat transfer coefficient along the cylinder is reduced (profiles of n are more flat). These effects show an increase in the slope n, Figure 5, for an axial velocity of $u=1.5$ m/s and different rotational speeds. Figure 6 shows how the best index n fitting the equation $\overline{Nu_L} = \alpha . Re_L^n$ increases with Re_D. Theoretically, if rotation were strongly

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dominant n would be constant along the cylinder and so the slope of $\overline{Nu_L}$ versus Re_L would be unity. For the range of velocities and rotational speeds considered, the highest slope obtained was less than 0.9. However, it seems unlikely from the results, that rotation could ever become that dominant. The rotational motion, even when strongly dominant, would never remove the heated air except by means of the turbulent mechanism associated with the centrifugal force as previously explained. The blowing is much more efficient, even with low velocities, and so there will always be a fall in the average heat transfer coefficient as the length of the cylinder increases.

Lastly, when rotation is dominant the Nusselt number depends little on the blowing velocity, being dependent on the axial flow Re_L only through the length L. For the highest speed (4500 rpm) it can be seen in Figure 4 that the curves are different for different blowing velocities but only because the blowing velocity increases Re_L without changing $\overline{Nu_L}$. The values at the same length have also a common $\overline{Nu_L}$ (see for example the last point of each line located at 30 cm in all cases). This fact does not occur with a low rotational speed (see Figure 3 for 450 rpm).

Effect of Blowing

The effect of blowing should be studied, not only through Re_L, but with the velocity ratio ξ also. As blowing is increased the curves get closer, so that when blowing becomes dominant the Nusselt number distribution along the cylinder depends only on Re_L, as is the case for a flat plate. This effect is shown both in Figures 3 and 4 for 450 and 4500 rpm, respectively. In these Figures it can be seen that the curves corresponding to 10 and 15 m/s are much closer than those for 5 and 10 m/s, though the increment in the velocity is the same for both cases. This effect is more important for 450 rpm than for 4500 rpm, probably because in this case blowing is more dominant.

CONCLUSIONS

The main conclusions derived from the analysis are:

1. The decrease in the Nusselt number along the cylinder approximately follows the blowing Reynolds number with an exponential law of exponent $n=0.8$.
2. Rotation slightly increases the exponent n as does the axial velocity.
3. Different axial distributions of Nusselt number were obtained for different axial velocities. A velocity ratio ξ takes account of the relative importance of rotation and blowing.
4. When blowing becomes dominant the curves tend to coincide, giving the Nusselt number independent on ξ.
5. When rotation becomes dominant the Nusselt number still depends on the blowing Reynolds number.

NOMENCLATURE

D	diameter of cylinder (m)
h	local heat transfer coefficient (W/m^2 K)
$\bar{h}$	average heat transfer coefficient (W/m^2 K)
L	length of cylinder (m)
$\overline{Nu_L}$	average Nusselt number $= \bar{h}L/k$
Pr	Prandtl number
r	radial direction
Re_D	rotational Reynolds number $= VD/\upsilon$
Re_L	blowing Reynolds number $= uL/\upsilon$
u	uniform axial velocity (m/s)
u_+	dimensionless velocity
V	peripheral velocity $= \pi\Omega D/60$ (m/s)
x	axial direction
y_+	dimensionless distance
Ω	rotational speed (rpm)
ξ	velocity ratio $=2V/u$
υ	kinematic viscosity (m^2/s)
α, n	constants and functions used to correlate results

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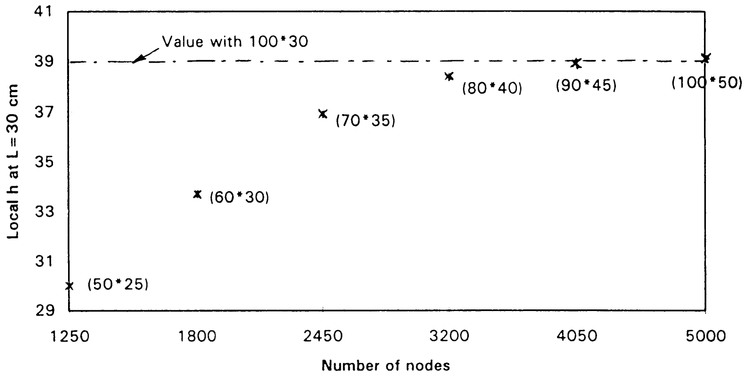

Figure 1: Optimization of grid nodes

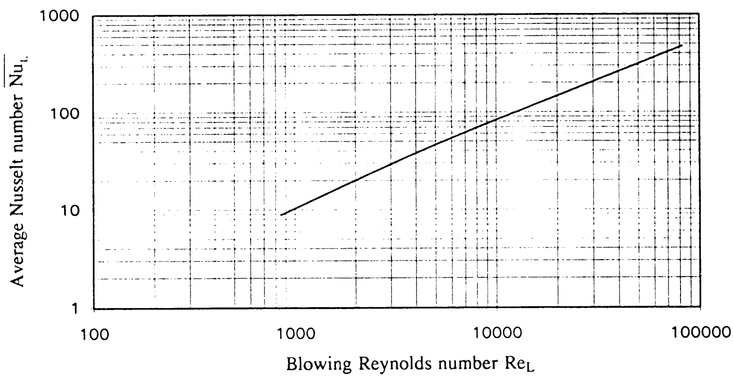

Figure 2: Nusselt number variation with blowing Reynolds numbers for typical case of u=5 m/s and Ω=1500

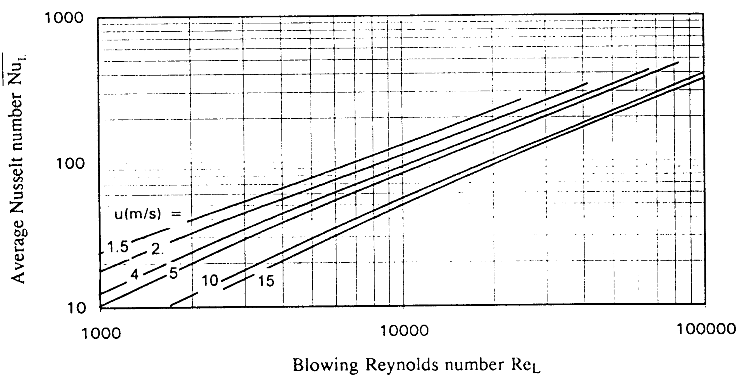

Figure 3: Effect of blowing Reynolds number on Nusselt numbers for minimum rotational speed Ω=450 rpm

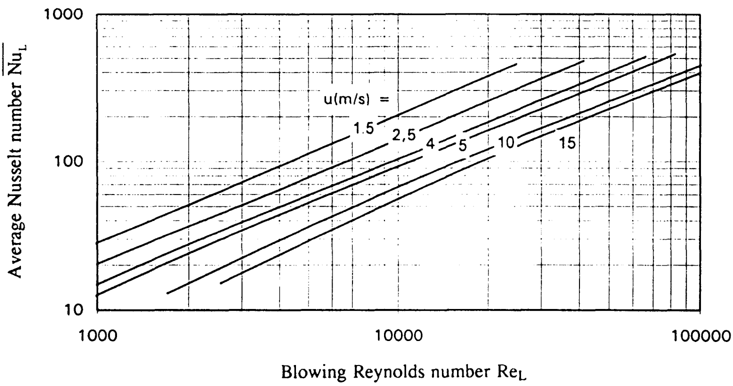

Figure 4: Effect of blowing Reynolds number on Nusselt numbers for maximum rotational speed Ω=4500 rpm

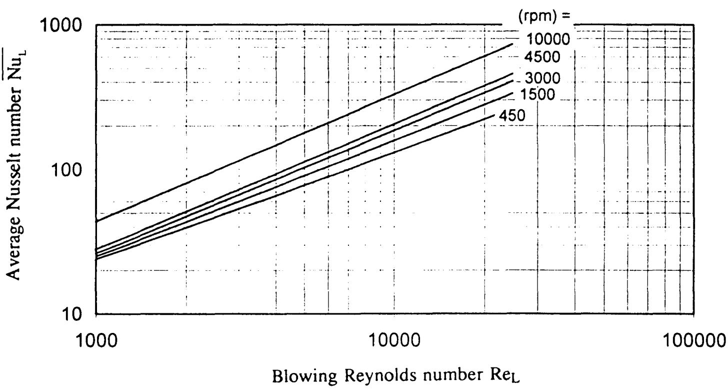

Figure 5: Effect of blowing Reynolds number on Nusselt numbers for minimum axial velocity u=1.5 m/s

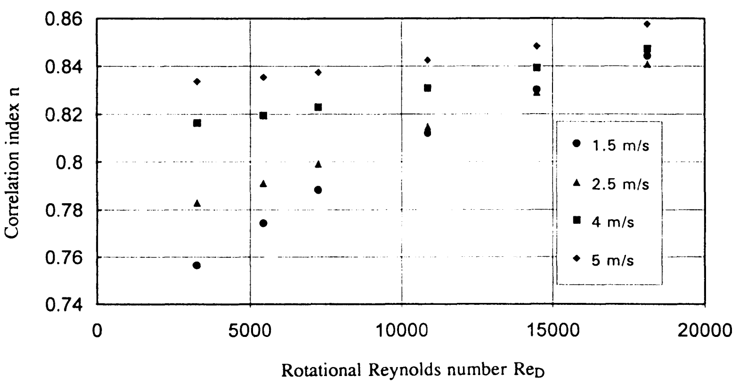

Figure 6: Variation of blowing Reynolds number index n with rotational Reynolds number