

# Topology optimisation methodologies for thermal and structural design

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## Abstract

The paper proposes a topology optimisation methodology based on optimality criteria to solve structural and thermal problems. The results of its application to the optimal design of the brackets linking the Meteor & Debris Protection System panels to the main structure of the ISS module Columbus are shown.

*Keywords: topology optimisation, optimality criteria, Von Mises stress, heat flux.*

## 1 Introduction

The description of many physical problems leads to partial differential equations which have to be integrated under initial or boundary conditions [1]. In many cases it is possible to replace the problem of integrating a differential equation set by the equivalent problem of seeking a function which gives a minimum value to a variational functional. General rules for deriving natural variational principles from linear differential equations are described in Mikhlin [2-3]. If a closed domain  $\Omega \in R^3$  with a smooth boundary  $\Gamma$  is considered, a physical problem can be described by a linear system of differential equations:

$$A(u) = Lu + b = 0 \quad (1)$$

in which  $L$  is a self-adjoint linear differential operator and  $u$  is a function defined in the closed domain  $\Omega$  leading to the definition of the variational principle:

$$\Pi = \int_{\Omega} \left[ \frac{1}{2} u^T Lu + u^T b \right] d\Omega \quad (2)$$

By including the boundary conditions defined for the unknown function  $u$ :

$$B(u) = 0 \quad \text{on } \Gamma \quad (3)$$



the variational principle (2) can be transformed in a natural one:

$$\bar{\Pi} = \int_{\Omega} u^T A(u) d\Omega + \int_{\Gamma} u^T B(u) d\Gamma \quad (4)$$

The requirement for stationarity leads to:

$$\delta \bar{\Pi} = \int_{\Omega} \delta u^T A(u) d\Omega + \int_{\Gamma} \delta u^T B(u) d\Gamma = 0 \quad (5)$$

Eqn (5) has to be true for any variations  $\delta u$  and implies the existence of a function  $u_0$  able to satisfy contemporary eqns (1) and (3). The solution of the problem leads to the minimum value of the natural variational principle:

$$\bar{\Pi}(u_0) = -\frac{1}{2} \int_{\Omega} u_0^T L u_0 d\Omega \quad (6)$$

In a series of problems of mathematical physics the magnitude of the functional (4) is proportional to the potential energy of the system. In such cases the search for a function which gives a minimum value to the variational functional (4) is equivalent to the principle of minimum potential energy.

## 2 General topology optimisation problem layout

The introduction of a continuous variable  $\eta$  depending on position to describe the effectiveness of the material in the domain  $\Omega$  leads to the formulation of the topology optimisation problem:

$$\begin{aligned} & \underset{\eta}{\text{maximise}} \quad \min_{u \in V} \bar{\Pi}(\eta, u) \\ & \text{subject to} \quad \int_{\Omega} \eta d\Omega \leq \bar{V} \\ & \quad \quad \quad 0 < \eta_{\min} \leq \eta \leq \eta_{\max} < \infty \\ & \quad \quad \quad \eta \in L^{\infty}(\Omega) \end{aligned} \quad (7)$$

where  $\eta_{\max}$  and  $\eta_{\min}$  are the upper and lower limits (side constraints) of the effectiveness variable,  $\bar{V}$  is the volume of effective material in the optimal layout and  $u$  is the state function describing the physical problem. The optimisation problem described by eqn (7) is a min-max problem for a concave-convex functional  $\bar{\Pi}(\eta, u)$ . The functional and the constraint set satisfy conditions for the existence of a saddle point, thus proving the existence of solutions [4].



The introduction of eqn (6) in eqn (7) leads to the optimisation problem:

$$\begin{aligned} \underset{\eta}{\text{minimise}} \quad & -\bar{\Pi}(\eta, u_0) = \frac{1}{2} \int_{\Omega} u_0^T L u_0 \eta \, d\Omega \\ \text{subject to} \quad & \int_{\Omega} \eta \, d\Omega \leq \bar{V} \\ & 0 < \eta_{\min} \leq \eta \leq \eta_{\max} < \infty \\ & \eta \in L^{\infty}(\Omega) \end{aligned} \tag{8}$$

The volume of the final structure  $\bar{V}$  as well as the minimum value of the variational principle  $\bar{\Pi}(\eta, u_0)$  depend linearly on the variable  $\eta$ . The existence of solutions has already been proved and a relaxation method or the introduction of materials with a micro structure is not required [4].

The optimisation problem described by eqn (8) can be solved by using an optimality criteria approach [5]. The Lagrangian function of the optimisation problem described by eqn (8) is given by:

$$L(\eta, \lambda) = \frac{1}{2} \int_{\Omega} u_0^T L u_0 \eta \, d\Omega - \lambda \left[ \int_{\Omega} \eta \, d\Omega - \bar{V} \right] \tag{9}$$

where  $\lambda$  is the Lagrange multiplier of the volume constraint and the side constraints concerning the design variable  $\eta$  have been temporarily neglected. The Kuhn-Tucker necessary conditions for optimality are:

$$\begin{aligned} \frac{\partial L}{\partial \lambda} &= \int_{\Omega} \eta \, d\Omega - \bar{V} = 0 \\ \frac{\partial L}{\partial \eta} &= \frac{1}{2} \int_{\Omega} u_0^T L u_0 \, d\Omega - \lambda \int_{\Omega} d\Omega = 0 \\ \lambda &\geq 0 \end{aligned} \tag{10}$$

The Lagrange multiplier  $\lambda$  for the optimal solution can be evaluated as:

$$\lambda = \frac{\frac{1}{2} \int_{\Omega} u_0^T L u_0 \, d\Omega}{\int_{\Omega} d\Omega} = \bar{e} \tag{11}$$

where  $\bar{e}$  is the average potential energy density in the design domain. Therefore, the optimal topology is characterised by a uniform distribution of the potential energy density as already shown by Venkayya [5] and Rossow and Taylor [6] in



the field of structural optimisation. If a linear static structural problem is considered, eqn (11) transforms in:

$$\lambda = \frac{\frac{1}{2} \int_{\Omega} \varepsilon^T(u) D \varepsilon(u) d\Omega}{\int_{\Omega} d\Omega} = \bar{e} \quad (12)$$

where  $\varepsilon$  is the strain field depending on the displacement distribution  $u$  and  $D$  is the constitutive matrix for a linear elastic material. If a steady-state heat conduction problem is considered, eqn (11) transforms in:

$$\lambda = \frac{\frac{1}{2} \int_{\Omega} \nabla^T(\varphi_0) k \nabla(\varphi_0) d\Omega}{\int_{\Omega} d\Omega} = \bar{e} \quad (13)$$

where  $\varphi_0$  is the temperature field,  $\nabla^T(\varphi_0)$  is the temperature gradient and  $k$  is the material conductivity.

Even if in the construction of the optimisation problem the function  $\eta$  does not make reference to the properties of the base material, for the continuum problem it can be linked to them by a simple relationship given by:

$$\eta = \frac{k}{k_0} \quad (14)$$

if the heat conduction problem is considered, where  $k$  is a continuous conductivity distribution over the design domain and  $k_0$  is the base material conductivity, or by:

$$\eta = \frac{E}{E_0} \quad (15)$$

if a linear static structural problem is considered, where  $E$  is a continuous Young modulus distribution over the design domain and  $E_0$  is the base material Young modulus. The relationship between the function  $\eta$  and the base material properties introduced by eqn (14) and (15) transfers the role of design variable to the continuous distribution of the material property  $k$  or  $E$ , respectively. The volume of the final structure  $\bar{V}$  and the minimum value of the variational functional keep depending linearly on the design variables, the solution to the optimisation problem keeps existing [4] and no filter stabilisation or perimeter control method is required to reach the convergence.



The optimality conditions described by eqns (12) and (13) have to be satisfied by updating the continuous function  $\eta$  through a resizing procedure. If a discrete design domain is considered, the following resizing rules can be used:

$$E_i^{new} = E_i^{old} \frac{e_i}{e} \quad (16)$$

$$k_i^{new} = k_i^{old} \frac{e_i}{e} \quad (17)$$

where  $E_i^{new}$ ,  $E_i^{old}$ ,  $k_i^{new}$ ,  $k_i^{old}$  and  $e_i$  are the new and the old value of the Young modulus of element  $i$ , the new and the old value of the material conductivity of element  $i$  and the strain or ‘thermal’ energy density of element  $i$ , respectively. The application of the resizing rules described by eqns (16) and (17) corresponds to find the point wise optimal distribution of the material characteristics for a given fixed strain or temperature field, respectively. If the structure would have been determinate, the resizing rules above described would have led to the identification of the optimal configuration in one step. Otherwise, the resizing rules affect the global behaviour of the structure and an iterative process is required until convergence is reached.

Side constraints have not been taken into consideration in the definition of the Lagrangian function of the design optimisation problem. Their satisfaction has to be verified at each iteration and for each discrete element of the design domain during the updating process of the material properties. The requirement for a structure with the base material Young modulus  $E_0$  and material conductivity  $k_0$  requires the proper selection of the upper limit for the artificial variable  $\eta$ :

$$\eta_{max} = 1 \quad (18)$$

The requirement for a positive definite stiffness matrix of the design domain leads to the selection of a lower limit for the artificial variable  $\eta$  given by:

$$\eta_{min} = 10^{-4} \div 10^{-5} \quad (19)$$

The value of  $\eta_{min}$  is extremely low and allows to consider the elements with the corresponding value of Young modulus and material density as void.

The Lagrange multiplier of the volume constraint makes reference to the optimal structural configuration. It has not to be searched *a posteriori* in order to comply with the volume constraint. Instead, it can be calculated *a priori* in order to comply with the mean stress, displacement and stiffness constraints or the temperature gradient and heat flux constraints defined on the optimal solution. Therefore, the volume of the optimal solution usually unknown *a priori* is indirectly controlled by the imposition of a reference strain or ‘thermal’ energy density evaluated by taking into account the average strain or ‘thermal’ energy that should characterise the optimal solution. The imposed average strain or ‘thermal’ energy density will be called in the following as reference energy density,  $\bar{e}_{ref}$  and will be used instead of the average energy density  $\bar{e}$  in eqn (16)



and (17). For example, if the optimal solution would be characterised by a maximum allowable stress for the base material  $\sigma_{max}$  and a truss like structure is expected, the reference energy density can be evaluated as:

$$\bar{e}_{ref}(\sigma_{max}) = \frac{1}{2} \frac{\sigma_{max}^2}{E_0} \quad (20)$$

Otherwise, if the material has to be characterised by a prescribed heat flux  $q(\varphi_0) = k\nabla^T(\varphi_0)_{max}$  or temperature gradient  $\nabla^T(\varphi_0)_{max}$ , the reference energy density can be evaluated as:

$$\bar{e}_{ref}(\nabla^T(\varphi_0)_{max}) = \frac{1}{2} \nabla^T(\varphi_0)_{max} k \nabla(\varphi_0)_{max} \quad (21)$$

The procedure described to formulate a topology optimisation problem has already been implemented in the static structural field and has been applied by the author to solve benchmark [7] and real industrial structural problems [8].

### 3 Bracket design with thermal and structural requirements

The proposed optimization methodology has been used to identify the optimal structural configuration of the Meteor & Debris Protection System (MDPS) panels of the International Space Station (ISS) module Columbus (figure 1).

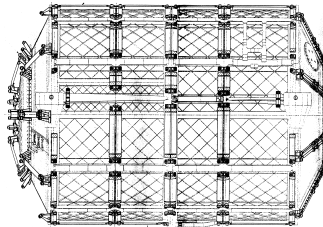


Figure 1: Design scheme of the ISS module Columbus.

The main structure of the module is shown in figure 1. It is characterised by a sheet metal cylindrical surface with longitudinal and circumferential T shape stiffening components. Several panels for meteor and debris protection purposes (figure 2) are linked to the stiffening elements by means of four brackets. The brackets have to be designed by taking into account several working conditions. The structural loads due to the launch accelerations in the Shuttle payload bay and the heat flow by conduction between the meteor and debris protection panel and the inside of the module have been taken into account in the present work.

The topology optimisation problem has been solved in two steps. In the first step a topology optimisation problem in the thermal field has been set up in order to identify the preferential way of heat flow by conduction between the external

panel and the inside of the module. In a following step, a topology optimisation problem in the structural field has been set up in order to identify the optimal material layout with maximum stress constraints.

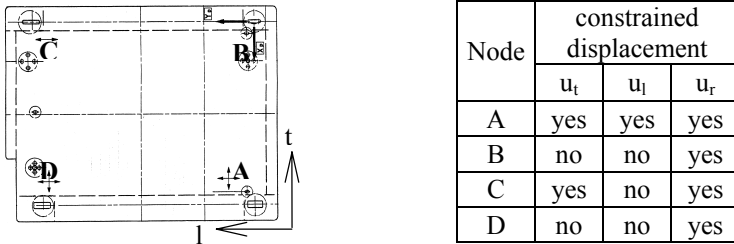


Figure 2: Geometry and boundary conditions of a MDPS panel.

### 3.1 Thermal topology optimisation problem

The first step of the bracket design process has been carried out by setting up a topology optimisation problem in the thermal field. The solution of the thermal optimisation problem allows to identify the optimal material layout in the design domain which maximise the heat flow by conduction between the external panel and the inside of the module with a prescribed temperature gradient.

The calculation of the temperature field and of the conductive heat flux between the external panel and the inside of the module has been carried out by taking into consideration a portion of the stiffening element the bracket is connected to. The discrete model (MDPS panel, design domain, stiffening element and external wall of the module) for the thermal topology optimisation problem and a detail of the design domain are shown in figure 3.

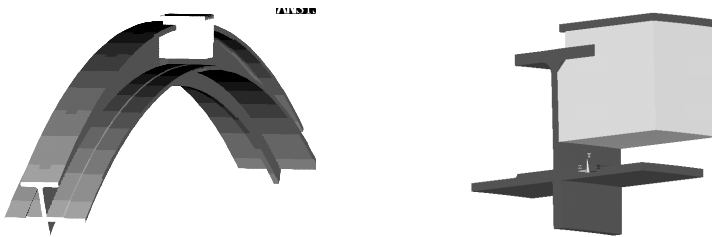


Figure 3: Numerical model for the thermal topology optimisation problem with a detail of the design domain.

A temperature of  $+150^{\circ}\text{C}$  has been imposed on the MDPS panel and a temperature of  $+25^{\circ}\text{C}$  has been imposed on the internal surface of the main body of the module. A Ti6Al4V alloy with a  $k=7.6$  W/mK thermal conductivity has been taken into account as base material for the bracket. The solution of the thermal optimisation problem has been carried out by taking into account only the material conductivity properties of the bracket and of the module stiffening elements. Radiation has been neglected due to the presence of radiation

insulating cover interposed between the MDPS panels and the main body of the module.

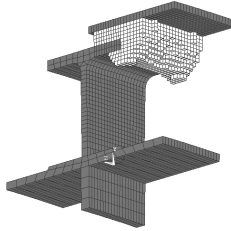


Figure 4: Optimal solution of the thermal topology optimisation problem.

The optimisation problem has been solved by imposing several different temperature gradients and the fully connected optimal material layout characterised by the lowest uniformly distributed temperature gradient has been selected in order to define the design domain for the structural optimisation problem. The selected optimal solution is shown in figure 4 together with a portion of the circumferential module stiffening element.

### 3.2 Structural topology optimisation problem

The discrete model for structural topology optimisation is shown in figure 5. The design domain for the structural topology optimisation problem has been obtained by removing from the initial design domain shown in figure 3 the material volume identified by solving the thermal topology optimisation problem previously described. The displacement constraints of the protection panel are described in figure 2. The connection between the panel and the bracket is designed in order to avoid the presence of a stress field due to material thermal expansion. Symmetry constraints have been taken into account.

Several different loading conditions should be considered in order to take into account the correct and incorrect exploitation of the external protection panels. The bracket design problem has been set up by taking into account only the forces and moments due to the accelerations supported by the external panel during the launch phase (table 1).

Table 1: Force and moment peak values for bracket design.

$F_r$ [N]	$F_l$ [N]	$F_t$ [N]	$M_l$ [Nmm]	$M_t$ [Nmm]
$\pm 3831$	$\pm 6000$	$\pm 6000$	$\pm 97066$	$\pm 182700$

Every combination of these loads defines a different independent loading condition to be analysed ( $2^5$  different loading conditions). In order to reduce the computational time for the solution of the optimisation problem, only five independent loading conditions have been analysed characterised by the application of each load alone.



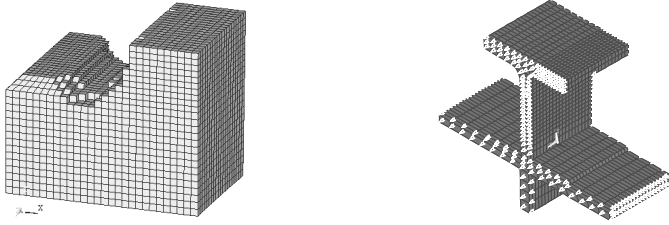


Figure 5: Design domain and symmetry constraints for the structural topology optimisation problem.



Figure 6: Optimal solution of the structural topology optimisation problem.

The topology optimisation problem has been solved by imposing a constraint on the maximum allowable Von Mises stress evaluated as the Ti6Al4V yield strength  $R_{ch}=800$  MPa divided by a safety factor  $S=2$ . The final optimal material layout is shown in figure 6.

### 3.3 Geometry extraction and design requirement verification

The optimal material layout obtained by solving the thermal and the structural topology optimisation problems has been used to identify a possible geometry of the bracket (figure 7).

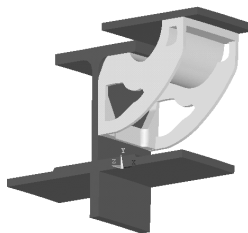


Figure 7: A possible geometrical model of the bracket.

The geometrical model of the bracket has been discretised and analysed in order to verify the stress constraint imposed in the structural topology optimisation problem and in order to calculate the final global conductivity of the overall system (panel, bracket, stiffening element).

The mass of the proposed bracket geometry is  $m=0.63$  kg. The constraint on the maximum Von Mises stress is verified with the exception of a few small areas where a stress concentration effect due to the geometry is present. The global conductivity of the system is  $C_S=0.089$  W/K. The Von Mises stress distribution in the worst loading condition and the heat flux are shown in figure 8.

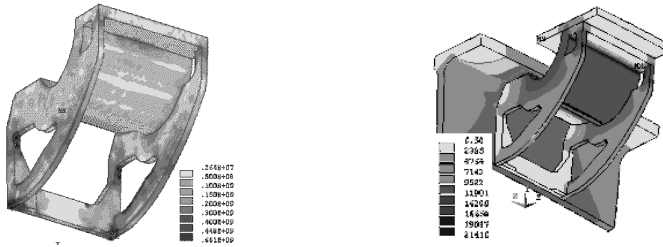


Figure 8: Von Mises stress distribution in the worst loading case and heat flow by conductivity in the proposed optimal geometry of the bracket.

## 4 Conclusions

The paper proposes a topology optimisation methodology based on optimality criteria to solve structural and thermal problems. It has been applied to search for an optimal design of a bracket with structural and thermal requirements showing the effectiveness of the proposed approach.

## References

- [1] Zienkiewicz O.C., Taylor R.L., The finite element method. McGraw-Hill, 1989.
- [2] Mikhlin S.G., Variational methods in mathematical physics. Pergamon Press, 1964.
- [3] Mikhlin S.G., The problems of the minimum of a quadratic functional. Holden-Day Inc., 1965.
- [4] Bendsoe M.P., Optimization of structural topology, shape, and material. Springer-Verlag, Heidelberg, 1995.
- [5] Venkayya V.B., Design of optimum structures. *Computers & Structures*, **1**, pp. 265-309, 1971.
- [6] Rossow M.P.J., Taylor J.E., A finite element method for the optimal design of variable thickness sheets. *AIAA Journal*, **11**(11), pp. 1566-1569, 1973.
- [7] Chiandussi G., Ibba A., Gaviglio I., Optimality criteria with a 'fixed' Lagrange multiplier for topology optimisation. Proc. V World Congress on Computational Mechanics (WCCM V), 2002.
- [8] Chiandussi G., Gaviglio I., Ibba A., Topology optimisation of an automotive component without final volume constraint specification. *Advances in Engineering Software*, to appear.

