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Computation of the stress intensity factors for repaired cracks with bonded composite patch in mode I and mixed mode

B.Bachir Bouiadjra , M.Belhouari , B.Serier and T.Achour .

LECM, Department of Mechanical Engineering, University of Sidi Bel Abbes, Sidi Bel Abbes 22000, Algeria

Abstract

In this study, the finite element method is used to analyse the behaviour of repaired cracks with bonded composite patches in mode I and mixed mode by computing the stress intensity factors at the crack tip. The effects of the patch size and the adhesive properties on the stress intensity factors variation were highlighted. The plot of the stress intensity factors according to the crack length in mode I, shows that the stress intensity factor exhibits an asymptotic behaviour as the crack length increases. In mixed mode, the obtained results show that the Mode I stress intensity factor is more affected by the presence of the patch than that of mode II.

1 Introduction

Externally bonded composite patches have proved to be an effective method of repairing cracks or defects in aircraft structures. Considerable researches have been performed to develop the technology of bonded composite repairs in aircraft structures. Alan Baker pioneered these researches at the aeronautical and maritime research laboratory for the Royal Australian Air force [1-7]. The bonded patch offers many advantages over a mechanically fastened doubler, which include improved fatigue behaviour, reduced corrosion and easy conformance to complex aerodynamic contours [1,2]. The determination of the stress intensity factors at the crack tip is one of the possible means to analyse the performance of the bonded composite repairs. It is known that the finite element method gives, with a great accuracy, the stress intensity factors at the crack tip. Among the authors whom used this method for computing the SIF in the case of repaired cracks, we can quote Ting et al [8], Callinan et al [9], Jones and Chiu [10] and Turaga and Ripudaman [11]. However, the most of these studies are limited to the case of mode I opening of the crack. In this paper, the behaviour of repaired cracks in aluminium thin plates with graphite/epoxy patch repairs (which have very successfully used by the Australian team [12]) is investigated by the finite element method in mode I and mixed mode. The stress intensity factors are calculated using the finite element code Franc2D/L developed at Kanasas University under the direction of Professor Swenson. It was shown by various authors that in practice the parameters influencing the performances of the bonded composite repairs are the patch and the adhesive properties. For that, the effects of adhesive shear modulus, adhesive thickness, patch thickness and patch width on the variations of the stress intensity factor of repaired cracks were examined.

2 Geometrical model

2.1 Mode I

Consider a thin elastic aluminium plate having the following dimensions: height H_p= 150mm, width W_p = 150 mm and thickness e_p = 1.3 mm. The elastic properties of the material are: Young modulus E_p = 72000 MPa and Poissson ratio ν_p=0.33. . The plate is subjected to uniaxial tensile load giving a remote stress state of σ= 140 MPa. A central crack of length 2a parallel to the loading axis is supposed to exist in the plate. The crack is repaired with a bonded Graphite/Epoxy patch having dimensions: width W_r and thickness e_r. The material properties of the patch are: Young's modulus E_{r1} =172370 MPa, E_{r2} = 10340 MPa, shear modulus G_r= 4825 MPa, and Poisson's ratios ν_{r1}=0.3 ν_{r2}=0.18. The adhesive properties are: shear modulus G_a and Thickness e_a. The plane stress conditions are assumed. Since the geometry and loading is symmetric, only one half needs to be analysed. Figure 1 shows the geometrical model for the mode I.

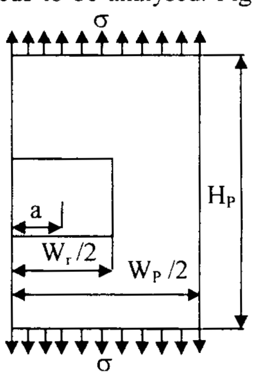

Figure 1: Geometrical model of the half of the patched structure for the mode I opening of The crack

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2.2 Mixed mode

To study the behaviour of repaired cracks in mixed mode, let consider a thin elastic aluminium plate of dimensions H_p= 150 mm, W_p= 150 mm and e_p=1.3 mm, with an inclined edge crack of length a=25 mm. Loading conditions are the same ones as previously. The crack is repaired with a graphite/epoxy patch having dimensions: W_r= 76 mm and e_r= 1.3 mm. the adhesive properties are : G_a= 500 MPa and e_a= 0.13 mm. Figure 2 represents the geometrical model of mixed mode.

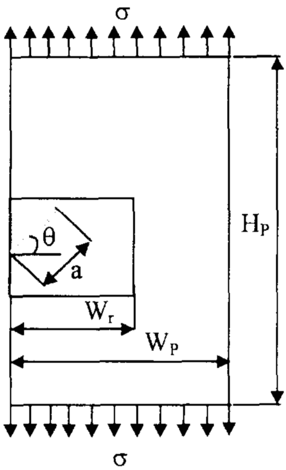


Figure 2: Geometrical model of the patched structure for the mixed mode

3 Finite element modelling

Finite element analysis of the configurations of figure 1 and figure 2 is done using the finite element code Franc2D/L developed at Kansas University [13]. A layered structure is actually a three dimensional structure. A three-dimensional finite element or mathematical modelling of such a structure will involve several degrees of complexity. In this study simplifying assumptions are made which still allow us to capture the essential features of the response. These assumptions include:

Each layer is considered as an individual two-dimensional structure under a state of plane-stress.

Individual layers can be connected with adhesive bonds.

It is assumed that the adhesive layer is homogeneous, linear elastic and isotropic.

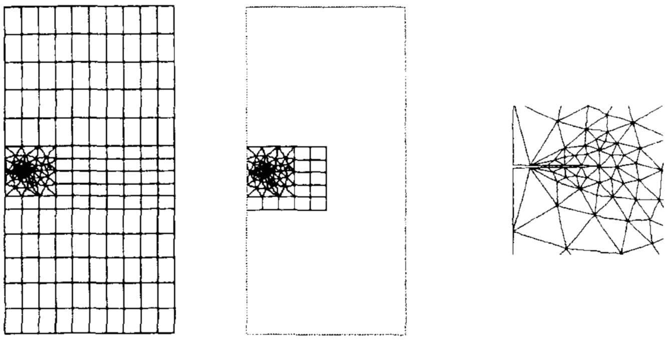
The adhesive is assumed to deform only in shear and this deformation is uniform throughout the adhesive thickness.

The surface shear transmitted through the adhesive is assumed to act as surface traction on the adherends.

The shear stresses in the adhesive are given by:

$$\tau = \frac{G_a}{e_a}(u_1 - u_2) \tag{1}$$

Where: u displacements in layers 1 and 2 (plate and patch)
The adhesive forces are obtained by using the adhesive shear stresses as surface tractions on the layer and integrating. Since the surface tractions are proportional to the relative displacement of the two layers, the adhesive force can be expressed in term of nodal displacements of the top and bottom layer. This gives a stiffness matrix for the adhesive elements.
The total structure (plate and patch) is meshed using standard eight noded serendipity elements with quadratic shape functions. These elements perform well for elastic analysis and have the advantage that the stress singularity at the crack tip can be incorporated in the solution by moving the eight nodes to the quarter-point locations [14]. Figure 3 shows typical mesh model of the plate, the patch and near the crack tip.
The stress intensity factors, which govern the fracture process in LEFM context, are calculated using modified crack closure techniques.



a) b) c)

Figure 3: Typical mesh model: a) of the plate b) of the patch c) near the crack tip

4 Analysis and Results

4.1 Stress Intensity factor in mode I

4.1.1 Comparison between patched and unpatched crack

Figure 4, shows the variation of the mode I stress intensity factor variation (K_I) with respect to the crack length for patched and unpatched crack. It is shown that the patch repair highly decreases the stress intensity factor. This is because the patch carries the loads as the crack growth. The maximum reduction of K_I amounts to 80%. Jones and Chiu [10] give a value of 60% for the Boron/Epoxy patch. It means that stiffer patch materials are more desirable. It can be seen in figure 4 that the stress intensity factor, for patched crack, exhibits an asymptotic behaviour as the crack length increases. Several investigators among those of the Australian team [8,10] announced this tendency. The asymptotic value of the stress intensity factor can be approximated by the formulae [15]:

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$$K_{\infty} = \sigma_0 \sqrt{\pi\lambda} \tag{2}$$

Where

$$\pi\lambda = \sqrt{\frac{E_p e_p}{\beta\left(1 + \frac{E_p e_p}{E_r e_r}\right)}} \tag{3}$$

$$\sigma_0 = \frac{\sigma E_p e_p}{\left(E_p e_p + E_r e_r\right)} \tag{4}$$

and

$$\beta = \frac{\left(\frac{e_a}{G_a} + \frac{e_r}{3G_r} + \frac{e_p}{3G_p}\right)}{\left(\frac{e_a}{G_a} + \frac{3e_r}{8G_r} + \frac{3e_p}{8G_p}\right)^2} \tag{5}$$

One can note that the asymptotic value of K_I depends on the plate, the patch and the adhesive properties. In the following, the effects of the adhesive shear modulus, the adhesive thickness, the patch thickness and the patch width on the variation of the stress intensity factor are analysed. The last parameter does not appear in equations 2,3,4 and 5.

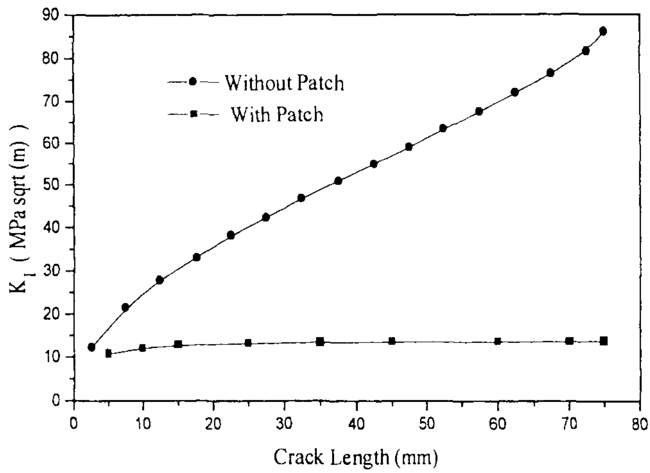

Figure 4: Comparison of the stress intensity factor between patched and unpatched cracks.
(G_a = 50 Mpa, e_a =0.13 mm, W_r = 75 mm, e_r=1.3 mm)

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4.1.2 Effect of the adhesive shear modulus

It is known that the adhesives of better qualities are characterised by weak shear modulus, which make it possible to attenuate the stresses transmitted to the adhesive. In the case of repaired cracks, the objective is to transmit the maximum of the stresses to the adhesive and consequently to the patch in order to reduce them at the crack tip. Theoretically, it is thus preferable to use adhesives with high shear modulus (adhesive of bad qualities) for repairing cracks or defects. In figure 5, the plot of the variation of the stress intensity factor according to the crack length for various values of shear modulus of the adhesive confirms what was advanced previously. Indeed, the stress intensity factor decreases as the shear modulus of the adhesive increases, but the decrement of K_I according to G_a tends to be cancelled as G_a increases indefinitely.Actually, an increase of the adhesive shear modulus reduces the adhesive strength which can generate the adhesion failure. Consequently, the choice of the adhesive (characterised by its shear modulus) for repairing cracks must be optimised in order to allow the transmission of the stresses towards the patch and to avoid the adhesive failure due to the increase of the stresses in the adhesive layer.

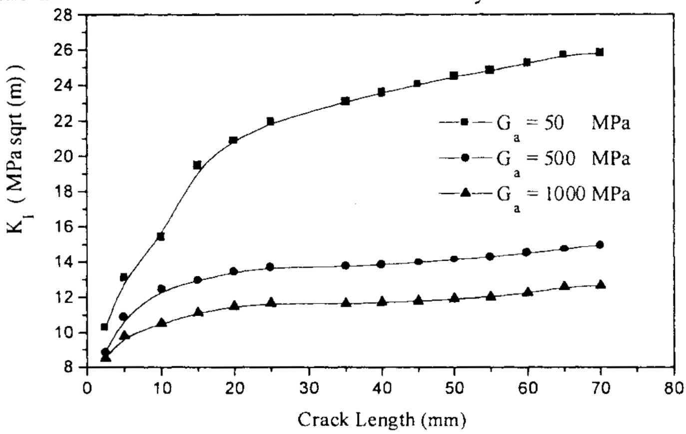

Figure 5: Effect of the adhesive shear modulus on the mode I stress intensity factor variation (e_a =0.13 mm, W_r = 75 mm, e_r=1.3 mm)

4.1.3 Effect of the adhesive thickness

Figure 6 shows the variation of the stress intensity factors as a function of the crack length for various values of the adhesive thickness (e_a). It can be seen that a reduction in the adhesive thickness decreases the stress intensity factor, it means that lower adhesive thickness is desirable for repairing crack. This effect was highlighted by Turaga and Ripudaman [11]. However, and by analogy with what was advanced in the study of the effect of the adhesive shear modulus, It proves that the choice of the adhesive thickness must be also optimised. The high thickness reinforces adhesion but attenuates the transfer of the loads towards the patch, which reduces the beneficial effects of the patch. On the other hand, lower thickness supports the transfer of the load towards the patch but increases the risk of the adhesive failure.

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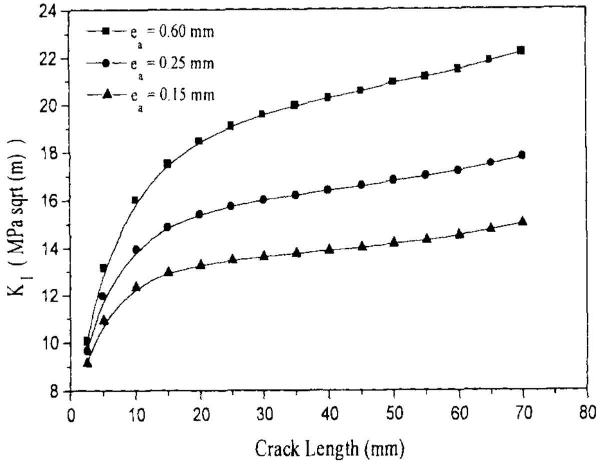

Figure 6: Effect of the adhesive thickness the mode I stress intensity factor variation (G_a =500 Mpa, W_r = 75 mm, e_r=1.3 mm)

4.1.4 Effect of the patch thickness

This effect is illustrated in figure 7 by the plot of the SIF variation according to the crack length for various patch thicknesses. It can be seen that the increase of the patch thickness reduces the stress intensity factor at the crack tip in a proportional way. Indeed, an increase of about 50% of the patch thickness reduces the stress intensity factor by the same order. That can able us to confirm that the choice of thicker patches makes it possible to increase their performances. For a better distribution of the stresses, it is preferable to use a multiple layers of bonded composite patches for repairing cracks.

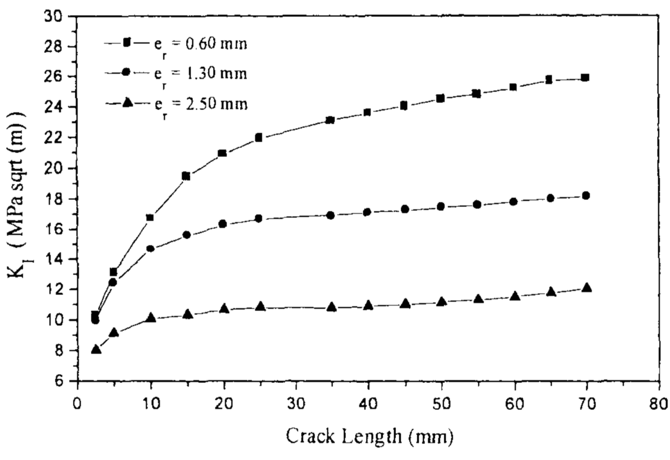

Figure 7: Effect of the patch thickness on the mode I stress intensity factor variation (Ga= 500 Mpa, e_a =0.13 mm, W_r = 75 mm)

4.1.5 Effect of the patch width

Let consider three values of the patch width such as W_r = 1/3 W_p, 1/2 W_p and W_p . Figure 8 represents the variation of the SIF variation versus the crack length for these values of the patch width. It's clearly seen that the stress intensity factor is slightly affected by the variation of the patch width. However, It can be seen that the patch width increase stabilises more the stress intensity factor and widened its asymptotic behaviour. It is preferable to use patch of width definitely greater than the crack length. It should be noted that the stress intensity factor starts increasing again as the crack tip reaches the edge of the patch. This tendency was announced by Turaga and Ripudaman [11].

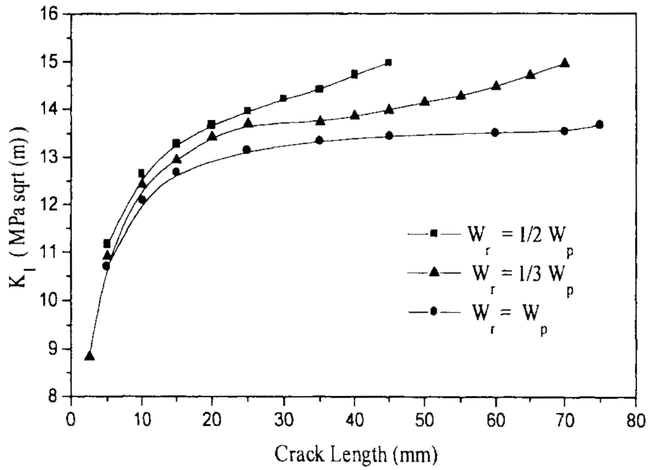

Figure 8: Effect of the patch width on the mode I stress intensity factor variation (Ga= 500 Mpa, ea =0.13 mm, e_r 1.3 mm)

4.2 Stress Intensity factor in mixed mode

In order to determine the effect of the patch presence on the performance of the bonded composite repair in mixed mode, the variation of the mode I and the mode II stress intensity factors as a function of the inclination of the crack (θ) for repaired and unrepaired crack are illustrated in figure 9 and 10. It is observed that the mode I stress intensity factor is largely reduced by the presence of the patch contrary to the mode II stress intensity factor. It means that the crack opening can be more attenuated than the crack sliding. This behaviour is due to the fact that the surface shear transmitted trough the adhesive acts as the surface traction on the adherend and it is known that the normal stresses are the causes of the crack opening.

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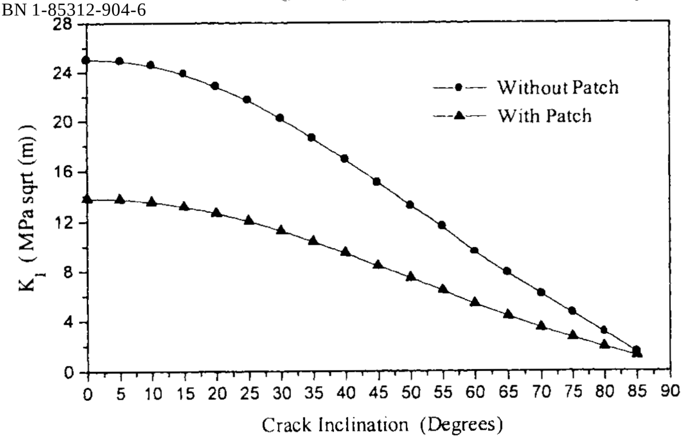

Figure 9: Comparison of the mode I stress intensity factor between patched and unpatched crack in mixed mode (a = 25 mm).

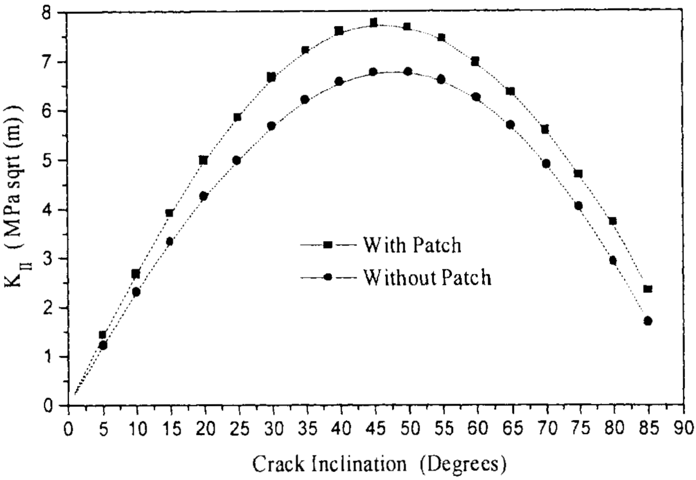

Figure 10: Comparison of the mode II stress intensity factor between patched and unpatched crack in mixed mode (a = 25 mm).

Conclusions

The results obtained in this study allow us to deduce the following conclusions:
-The mode I stress intensity factor is highly reduced by the presence of the patch.
-The stress intensity factor exhibits an asymptotic behaviour as the crack length increases.
-The choice of the adhesive properties for repairing crack, with the bonded composite. patch, must be optimised.
- The increase of the patch thickness reduces the stress intensity factor at the crack tip.

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- The choice of larger patches widened the asymptotic behaviour of the stress intensity factor.
- In the mixed mode the mode I stress intensity factor is more affected by the presence of the patch than that of mode II.

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