The effect of the wave radiation on the second-order steady loads for arbitrary bodies

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Abstract

Marine structures are usually designed to operate in a wave environment. Structural loading of the body surface under the water and unsteady motions of the body are two of the principal resulting problems. A body in waves is also acted upon by steady forces and moments due to the reflection of the wave energy and as a consequence, the body starts drifting, depending on the rigidity of the constraint. The drift loads on marine structures are important in designing mooring or dynamic positioning devices. The most general way to compute these steady loads is the application of the pressure integration around the wetted surface of the body. An exact solution for the computation of second-order drift forces and moments for arbitrary bodies are presented using a non-singular boundary integral equation method (NBIEM) and the application of B-spline to model the wetted body surface in wave. The influences of radiation velocity potentials on drift loads are investigated for different types of bodies.
Keywords: drift forces, wave radiation, second-order steady force, wave-body interactions, arbitrary bodies, spherical structures, boundary integral methods.

1 Introduction

Marine structures are usually designed to operate in a wave environment. Structural loading of the body surface under the water and unsteady motions of the body are two of the principal resulting problems. When the characteristic body dimension is comparable to the wave length, the potential effects dominate. The presence of the body alters the pattern of wave propagation in the vicinity of the structure and causes wave scattering. The body may also oscillate and cause the radiation of waves if the constraints are not sufficiently rigid. As a consequence,

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the body experiences reacting forces from the surrounding fluid and constraints. For this study, it is assumed that the fluid is homogeneous and incompressible, and the effect of the surface tension is neglected.

The viscosity of the fluid and the irrotational flow are important in determining the wave-induced loads and motions of the body. If the amplitude of the wave is small in comparison with the characteristic length of the body, the effects of the flow separation due to the fluid viscosity may be neglected. As a consequence, the fluid can be considered inviscid, and the whole fluid flow can be characterized by a scalar function called the velocity potential. This function is a solution of the Laplace equation and is subjected to nonlinear boundary conditions on variable surfaces. The nonlinearity of the boundary conditions precludes solutions without further simplifications. If it is assumed that the steepness of the wave is small, the free surface boundary condition can be linearized and applied on the undisturbed free surface of the fluid. Further simplification is obtained by considering small motions of the body.

In linearized wave theory, the problem is reduced to an analysis of the loads and motions of the body in plane progressive sinusoidal waves. It is assumed that the motions of the body are steady, and all of the transient effects are removed. A combination of two independent classical problems is considered in order to find the body-induced motions and loads in time harmonic waves. The first is the radiation problem, where the body undergoes prescribed oscillatory motions in otherwise calm fluid. The other is the diffraction problem, where the body is held fixed in the incident wave field and determines its influence over the incident wave.

A body in waves is also acted upon by steady force and moment due to the reflection of the wave energy. As a consequence, the body starts drifting, depending on the rigidity of the constraint. These are called the drift force and moment and are important in designing mooring or dynamic positioning devices. These steady loads can be computed in two ways: either from the far field method and the application of the conservation of momentum in the fluid, or from the near field method and the application of the pressure integration around the wetted surface of the body.A detailed descriptions of both method may be found in Ogilvie [7] and Pinkster and Oortmerssen [8].

The boundary integral equation method is applied to obtain the first order velocity potentials through the solution of diffraction and radiation problems. These two boundary value problems are of elliptical types that lead to integral equations with singular kernels. If the surfaces satisfy the Liaponov conditions, the kernels of the integral equations possess weak singularities (Pogorzelski [9]). The limit of an integral with weak singular integrand exists at the singular point and is independent of the shape of the exclusion zone. The integral equation with a weakly singular kernel function satisfies Fredholm theorems and hence is non-singular. Fredholm equations of the second kind can be approximated in a straightforward way by means of Gaussian quadrature formulas in which the integral equations are replaced by a weighted sum of values of the integrand evaluated at certain points. The application of the Gaussian quadrature in solution of integral equations without attention to singular behavior of the integral kernels may result in erroneous solutions. Therefore, it is necessary to regularize the kernel of the integrals and make the integral equations amenable to solution directly by the Gaussian quadrature formulas.

A weakly singular integral may be regularized by subtracting a proper function from the integrand so that the integral tends to zero at the singular points. Then, the exact integration of that function adds to the integral equation. Based on this idea, Landweber and Macagno [1] introduced a numerical scheme for solving the irrotational flow about ship forms. Following the procedure adopted by Landweber and Macagno [1] in the treatment of the singularity of the kernels of the integral equation in potential flow, Mousavizadegan et al. [5, 6] extended a numerical technique for solving the radiation and diffraction problem of submerged and floating bodies in waves in the case of infinite fluid depth. A comparison of the method with the flat panel method was presented by Mousavizadegan and Rahman [4] to show the effectiveness of this higher-order method. Different types of the geometry modeling, either implicitly or explicitly, can be applied to solution of the integral equation. Therefore, the method may be categorized as an independent geometry modeling method. This is shown by Mousavizadegan and Rahman [3] using different representation of the body surface geometry of various structures in the cases of finite and infinite fluid depth. There is no need to approximate the velocity potential distributions around the body surface. The order of the velocity potential corresponds to the number of the Gaussian points. These imply that the method is a higher-order method with an arbitrary degree.

One of the important features of the presented method is the ability to use the exact geometry in computations. If the Gaussian quadrature points, the normal at those points and the area element around those points are all computed accurately, it will enhance the accuracy of computations. The body surface may be modeled differently with various basis functions. All types of parametric expressions of body surface can be applied in solving the associated integral equations. This makes the method independent from geometrical modeling. There are only a few special geometries that have exact implicit and explicit expressions for their surfaces. In general, it is necessary to construct approximations to the body surfaces in an explicit manner.

The advent and first development of interactive computer graphics has initiated several curve and surface representation techniques. B-spline curves and surfaces are powerful tools in curve fitting and curve fairing techniques. The successful use of B-spline functions in representing and modifying the surface of the marine structures, such as a ship's hull, initiated the idea of the application of B-spline panels in boundary integral equation methods.

2 Mathematical formulations

It is assumed that the fluid is incompressible and the motion is irrotational. The fluid flow field can be defined by a velocity potential that is expressed in finite amplitude wave theory by a series in power of ε in the form

$$\Phi = \varepsilon\Phi_1 + \varepsilon^2\Phi_2 + \cdots + \varepsilon^n\Phi_n + \cdots ,$$

where ε is the perturbation parameter that is proportional to the wave slop (wave height to wave length) and $\varepsilon^n \Phi_n$ is the nth-order velocity potential. If the analysis is approximated up to the second-order, it can be written that

$$\Phi = \Phi_\ell + \Phi_q = \Re\left\{\phi_\ell e^{-i\omega t} + \phi_q e^{-i2\omega t}\right\},$$

where Φ_ℓ and Φ_q are the linear and the quadratic diffraction velocity potentials, ω is the radian frequency and t denotes the time. The terms ϕ_ℓ and ϕ_q are the linear and quadratic time-independent velocity potentials. The motion of the fluid is subjected to the Laplace equation in fluid domains, a free-surface kinematic boundary condition and a free-surface dynamic boundary condition. The fluid flow field is also subjected to a bottom condition that indicates no flux of mass through the bottom of the fluid, a radiation condition at large distance from the body and a body surface boundary condition.

The total force and moment vector acting upon the body due to the wave propagation is obtained by

$$\mathbf{F} = \iiint_{S_B} P(\mathbf{x}, t)\ \mathbf{n}\ ds$$
$$\mathbf{M} = \iiint_{S_B} P(\mathbf{x}, t)\ (\mathbf{x} \times \mathbf{n})\ ds \tag{1}$$

where ds is the elemental surface area and S_B is the wetted surface of the body. This surface S_B is displaced and rotated with respect to a equilibrium condition that is specified with S_{BM}. The term $\mathbf{x}$ is the position vector of a point with respect to the coordinate $Oxyz$ that is an inertial reference frame system on the undisturbed water surface, $\mathbf{x} = x\hat{i} + y\hat{j} + z\hat{k}$. The position vector with respect to a body fixed reference coordinate system $Ox'y'z'$ is $\mathbf{x}' = x'\hat{i} + y'\hat{j} + z'\hat{k}$. It is taken into account that two coordinate systems are coincided with each other at $t = 0$. Transformation between two coordinate system may be given in the form:

$$\mathbf{x} = \mathbf{x}' + \boldsymbol{\xi} + \boldsymbol{\alpha} \times \mathbf{x}' + H\ \mathbf{x}' + O(\varepsilon^3) \tag{2}$$

where $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$ is the translation and $\boldsymbol{\alpha} = (\xi_4, \xi_5, \xi_6)$ is the rotation of the body reference coordinate system with respect to the inertial reference coordinate systems at a time t. The matrix H is the second-order transformation matrix.

$$H = -\frac{1}{2}\begin{bmatrix} \xi_5^2 + \xi_6^2 & 0 & 0 \\ -2\xi_4\xi_5 & \xi_4^2 + \xi_6^2 & 0 \\ -2\xi_4\xi_6 & -2\xi_5\xi_6 & \xi_4^2 + \xi_5^2 \end{bmatrix} \tag{3}$$

The unit normal vector in inertial reference frame system is denoted by $\mathbf{n}$. The transformation of the unit normal vector between two coordinate systems is

$$\mathbf{n} = \mathbf{n}' + \boldsymbol{\alpha} \times \mathbf{n}' + H\ \mathbf{n}' + O(\varepsilon^3) \tag{4}$$

where $\mathbf{n}'$ is the unit normal vector in body reference frame system. The term denoted by ε is the perturbation parameter.

The pressure P can be obtained from Bernoulli's equation. For equilibrium condition and wetted surface S_{BM}, it can be written that

$$P = -\rho g z - \varepsilon\rho \frac{\partial \Phi_\ell}{\partial t} - \varepsilon^2 \rho \left\{ \frac{\partial \Phi_q}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \Phi_\ell}{\partial x}\right)^2 + \left(\frac{\partial \Phi_\ell}{\partial y}\right)^2 + \left(\frac{\partial \Phi_\ell}{\partial z}\right)^2 \right] \right\} + O(\varepsilon^3) \tag{5}$$

The first term on the right hand side of (5) is hydrostatic pressure and the rest are the hydrodynamic pressures due to the first and second order potentials. The pressure at exact wetted surface S_B can be expressed using the Taylor series expansion with respect to S_{BM}.

$$P|_{S_B} = P|_{S_{BM}} + (\mathbf{x} - \mathbf{x}') \cdot \nabla P + \cdots \tag{6}$$

Substituting (4) in (5) and using (2), the pressure on the exact wetted surface of the body is

$$P = -\rho g z' - \varepsilon[\rho g(\xi_3 + y'\xi_4 - x'\xi_5)] - \varepsilon^2[\rho g(H\mathbf{x}' \cdot \nabla z)] - \varepsilon\rho \frac{\partial \Phi_\ell}{\partial t} - \varepsilon^2\left[\rho(\boldsymbol{\xi} + \boldsymbol{\alpha} \times \mathbf{x}') \cdot \nabla\left(\frac{\partial \Phi_\ell}{\partial t}\right)\right] - \varepsilon^2 \rho \left\{ \frac{\partial \Phi_q}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \Phi_\ell}{\partial x}\right)^2 + \left(\frac{\partial \Phi_\ell}{\partial y}\right)^2 + \left(\frac{\partial \Phi_\ell}{\partial z}\right)^2 \right] \right\} + O(\varepsilon^3) \tag{7}$$

The forces and moments acting on the body may be obtained by the integration over S_{BM} and taking into account (3) and (5) by considering an adjustment. The integration over S_B is to be carried out up to the water surface $z = \eta$.

$$\eta = \eta_0 + \varepsilon\eta_1 + \varepsilon^2\eta_2 + \cdots \tag{8}$$

where η is the instantaneous elevation of the water free surface and $\eta_0,\ \eta_1 \& \eta_2$ are the zeroth-, the first- and the second-order free surface elevations, respectively. The integration over S_{BM} is to be carried out up to the water surface $z = 0$ that is correspond to $z = \xi_3 + y\xi_4 - x\xi_5$ on S_B. Therefore, it can be written that:

$$\mathbf{F} = \iint_{S_{BM}} P(\mathbf{x}, t)\, \mathbf{n}\, ds + \iint_{\Delta S} P(\mathbf{x}, t)\, \mathbf{n}\, ds$$
$$\mathbf{M} = \iint_{S_{BM}} P(\mathbf{x}, t)\, (\mathbf{x} \times \mathbf{n})\, ds + \iint_{\Delta S} P(\mathbf{x}, t)\, (\mathbf{x} \times \mathbf{n})\, ds. \tag{9}$$

The integration over a thin layer ΔS can be given up to second order in the form

$$\iint_{\Delta S} P(\mathbf{x}, t)\ \mathbf{n}\ ds = \oint_{C_M} d\ell \int_0^{\varepsilon[\eta_1-(\xi_3+y\xi_4-x\xi_5)]} \mathbf{n}(x, y, 0)\ dz' \Bigg\{$$

$$\varepsilon[-\rho g z' - \rho g(\xi_3 + y'\xi_4 - x'\xi_5)] - \varepsilon\rho\left(\frac{\partial \Phi_\ell}{\partial t}\right)_{z=0}\Bigg\} + O(\varepsilon^3)$$

$$= \frac{1}{2}\rho g \varepsilon^2 \oint_{C_M} [\eta_1 - (\xi_3 + y\xi_4 - x\xi_5)]^2 \mathbf{n}(x, y, 0)\ d\ell + O(\varepsilon^3) \qquad (10)$$

That is a point force around the intersection of the body with water surface, C_M. The moment due to this point force can be given in the following form.

$$\iint_{\Delta S} P(\mathbf{x}, t)\ (\mathbf{x} \times \mathbf{n})\ ds = \frac{1}{2}\rho g \varepsilon^2$$

$$\oint_{C_M} [\eta_1 - (\xi_3 + y\xi_4 - x\xi_5)]^2 (\mathbf{x} \times \mathbf{n})|_{z=0}\ d\ell + O(\varepsilon^3) \qquad (11)$$

The total second-order forces and moments on a floating body is obtained by integrating of (7) over S_{BM} and adding with (10) and (11).

There is also a mean second-order wave force arising from

i - quadratic term in the Bernoulli equation and according to the general identity of functions of complex variable;

$$\Re(Ue^{i\omega t})\Re(Ve^{i\omega t}) = \frac{1}{2}\Re(UVe^{i2\omega t} + UV^*),$$

ii - a mean average force due to the variation of the first order dynamic pressure.

The total time average forces and moments up to second order according to Ogilvie [7] are:

$$\mathbf{F_d} = -\rho \iint_{S_{BM}} \overline{\left[\frac{1}{2}|\nabla\Phi_\ell|^2 + (\boldsymbol{\xi_\ell} + \boldsymbol{\alpha_\ell} \times \mathbf{x}) \cdot \nabla\Phi_{\ell_t}\right]\mathbf{n}ds}$$

$$+ \oint_{C_M} \overline{[\eta_1 - (\xi_3 + y\xi_4 - x\xi_5)]^2 \mathbf{n}}\ d\ell$$

$$- \rho g A_{wp} \overline{\alpha_{\ell_3}(x_f \alpha_{\ell_1} + y_f \alpha_{\ell_2})\nabla z} + O(\varepsilon^3) \qquad (12)$$

$$\mathbf{M_d} = -\rho \iint_{S_{BM}} \overline{\left[\frac{1}{2}|\nabla\Phi_\ell|^2 + (\boldsymbol{\xi_\ell} + \boldsymbol{\alpha_\ell} \times \mathbf{x}) \cdot \nabla\Phi_{\ell_t}\right](\mathbf{x} \times \mathbf{n})ds}$$

$$+ \oint_{C_M} \overline{[\eta_1 - (\xi_3 + y\xi_4 - x\xi_5)]^2 (\mathbf{x} \times \mathbf{n})}\ d\ell$$

$$- \rho g A_{wp} \overline{\alpha_{\ell_3}(x_f \alpha_{\ell_1} + y_f \alpha_{\ell_2})(\mathbf{x} \times \nabla z)} + O(\varepsilon^3) \qquad (13)$$

The first integral in both formulae is distributed forces and moments around the mean wetted surface of the body due to the velocity of the fluid particles and the variation of the first-order hydrodynamic pressure. The second integral in both formulae is due to the variation of the fluid surface at the water-line of the body. These are the contribution of the first-order velocity potential to the second order steady force. There is no mean forces and moments due to the second-order potential. The knowledge of the first-order potential is enough to compute these steady second-order effects.

A combination of two independent classical problems is considered in order to find the first-order velocity potentials due to the body-induced motions and loads in time harmonic waves. The first is the radiation problem, where the body undergoes prescribed oscillatory motions in otherwise calm fluid. The other is the diffraction problem, where the body is held fixed in the incident wave field and determines its influence over the incident wave. The radiation and diffraction problems are subjected to the Laplace equation in the fluid domain, and a set of boundary conditions. The boundaries of the fluid consist of the free surface, a fixed bottom and the immersed surface of the body. There are two sets of boundary conditions that must be held on the surfaces that confine the fluid. The first set is the kinematic boundary conditions. These indicate that the normal velocity of a fluid particle just near a point of these surfaces should be equal to the normal velocity of that point. The other set is the dynamic boundary conditions. The modern theories of water waves and the motion of bodies in waves can be found in Rahman [10].

The boundary integral equation method is applied to obtained all components of the velocity potential of a body in regular wave. The techniques of Landweber and Macagno [1] are applied to modify the kernels of the integral equations associated with the motion of bodies in time-harmonic waves. The modified integral equations are nonsingular and amenable to solution directly by the Gaussian quadrature formula. The collocation method is applied to form systems of algebraic equations. The collocation points and the integral points coincide with each other. The system of linear algebraic equations may be presented in matrix form as

$$[A]\{x\} = \{B\}, \tag{14}$$

where the vector $\{x\}$ is the unknown velocity potentials around the body. The term denoted by $[A]$ is the coefficient matrix. It is formed by the integration of the normal derivative of Green's function on each integration point. The term denoted by B is a vector that its elements are obtain through the application of the body surface boundary condition for the diffraction and radiation velocity potentials. A complete description of the method can be found in Mousavizadegan [2].

3 Results and discussion

The method is applied to several bodies but we only mentioned the result for a submerged sphere in regular wave. The body is modeled by B-spline as given in Fig. 1. The figure shows the distribution of the quadrature points around a quarter sphere.

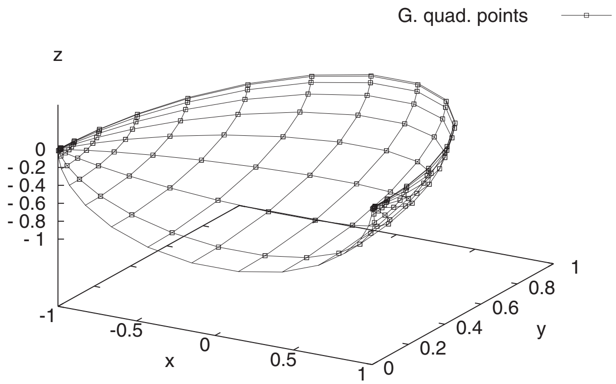

Figure 1: Distribution of the Gaussian quadrature points around the surface of a sphere.

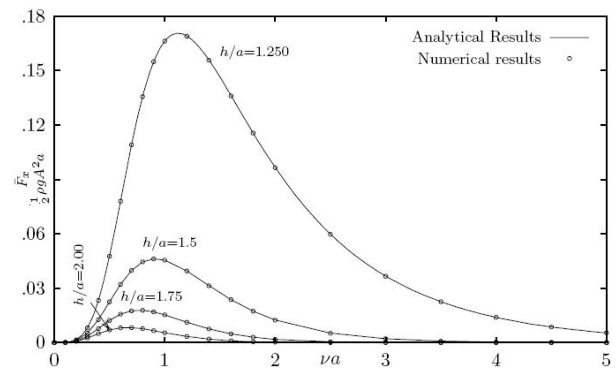

Figure 2: Drift force on a sphere due to the diffraction velocity potential.

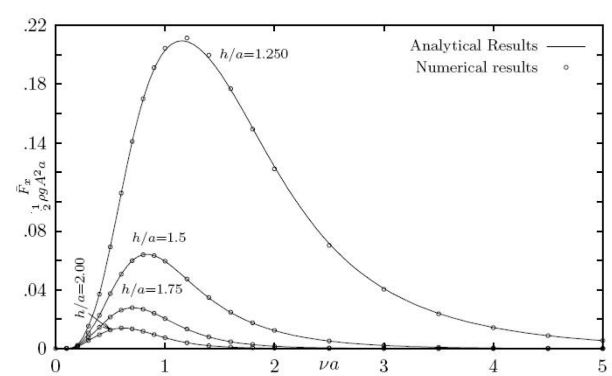

Figure 3: Total drift force at different immersion depths and $Z_G/a = 0.1$.

The numerical results for the non-dimensional horizontal drift force in different immersion depths are shown in Figs. 2 and 3. The first figure shows the drift force coefficients when the effect of the motion is neglected and the second one depicts the total drift force coefficient due to the total velocity potentials. The center of the mass is set to be $0.1a$ under the center of the sphere. The analytical solutions are also shown to provide a comparison between them. The analytical and numerical solutions comply with each other.

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References

[1] L. Landweber and M. Macagno, *Irrotational flow about ship forms*, Iowa Institute of Hydraulic Research, The university of Iowa, Iowa city, Iowa (1969), no. IIHR Report No. 123.

[2] S. H. Mousavizadegan, *Analytical and numerical approaches to determine the second-order forces in wave-body interactions*, Ph.D. thesis, Dalhousie University, September 2005.

[3] S. H. Mousavizadegan and M. Rahman, *An independent geometry modeling method in wave-body interactions*, Third International Conference on Fluid structure Interaction, LaCrona, Spain (2005).

[4] S. H. Mousavizadegan and M. Rahman, *Numerical method in wave-body interactions*, J. Appl. Math. & Computing (JAMC) **17** (2005), no. 1 - 2, 73–91.

[5] S. H. Mousavizadegan, M. Rahman, and M. G. Satish, *Radiation and diffraction of a submerged sphere*, Proceedings of Fluid Structure Interactions II, Cadiz, Spain (2003), 13–22.

[6] S. H. Mousavizadegan, M. Rahman, and M. G. Satish, *Motion of floating bodies in time-harmonic waves*, Proceeding of Advances in Fluid Mechanics V, Lisbon, Portugal (2004), 181–190.

[7] T. F. Ogilvie, *Second-order hydrodynamc effects on ocean platfoms*, The Proceedings : International workshop on ship and platform motions: University of California (1985), 205–264.

[8] J. A. Pinkster and G. Van Oortmerson, *Computation of the first and second order wave forces on oscilating bodies in regular waves*, The Proceedings : Second international conference on numerical ship hydrodynamics, University of California (1977), 136–156.

[9] W. Pogorzelski, *Integral equations and their applications*, vol. 1, Pergamon Press, Oxford, 1966.

[10] M. Rahman, *Water waves: Relating modern theory to advanced engineering applications*, IMA Monograph Series, No. 3, Oxford University Press, Oxford, 1994.