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Nonlinear interaction of surface and internal waves with very large floating or submerged structures

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Abstract

This paper describes interaction of surface/internal water waves with floating/submerged platforms by considering nonlinearity of fluid motion and flexibility of oscillating structures. The model represents a multilayer fluid system interacting with horizontally very large and elastic thin-plates, where the set of governing equations is derived by vertical integration in each fluid layer with nonlinear boundary conditions satisfied on the interfaces. Numerical computation is performed for surface/internal long waves and floating/submerged plate oscillations in the vertical section. Resonated pressures beneath a floating plate on the sea surface in a two-layer system are obtained in forced oscillation cases by taking into account different order of wave height to water depth ratios. Surface and internal waves interacting with a floating or submerged plate structure are also calculated in free oscillation cases.

1 Introduction

When density stratification is well developed, a very large floating structure may generate internal waves that change the water environment including temperature and salinity distributions not only under the structure but also in the surrounding areas, which results in different force acting on the structure compared to that in the corresponding single-density flow. Here, interaction of motion between surface/internal water waves and floating/submerged flexible structures is studied by considering nonlinearity of fluid motion. It is conceivable that floating structures on the sea surface are used for airports, large bridges, storage facilities, residential spaces, etc., while flexible structures under the sea are for example submerged elastic plates able to decrease wave height and man-made gathering places for fish.

Specifically, a nonlinear model is utilized to investigate interaction of a multilayer fluid system with elastic structures assumed to be horizontally large thin-plates oscillating flexibly on/below the sea surface. A set of time-dependent and fully nonlinear equations obtained by Kakinuma [1] is applied, i.e. a vertically integrated type model of each fluid layer under the variational principle that satisfies nonlinear boundary conditions on every interface including the sea surface and bottom. It should be noted that this model is applicable to strongly nonlinear and strongly dispersive surface/internal waves if we use enough number of vertical distribution functions of velocity potential. It is assumed that no fluid mixes with another having a different density, which leads boundary faces to be of discontinuity, and a plate can accordingly be put in between two different fluids. Thus fluid motion in contact with elastic thin-plates can be studied.

The numerical model presented here represent two-layer fluid systems in the vertical section. Effects of up to the first-order scale in vertical length are taken into account. Resonance of internal waves with forced oscillation of a thin-plate floating on the sea surface is examined by changing relative frequency or wavelength of surface oscillation. In free oscillation, an initial profile of a floating thin-plate or the sea surface is given, after which motion of the interface or a submerged thin-plate is investigated by changing flexural rigidity of the structure.

2 Governing equations for surface and internal waves interacting with thin-plate oscillation

2.1 Stratified fluids and thin-plates

In this section, a set of governing equations which describes surface and internal waves interacting with oscillation of horizontally large thin-plates is derived in exactly the same way we previously reported in [1].

As illustrated in Fig. 1, inviscid and incompressible fluids are represented as i ($i = 1, 2, \cdots, I$) from top to bottom, where density ρ_i is constant in each layer. None of the fluids mix even in motion, and frictional force is assumed to be zero on every interface. No unstable phenomenon such as vortex generation or wave breaking occurs on any interface, whose profile is assumed to be a single-valued function of $\boldsymbol{x}$.

The i-layer is sandwiched between two elastic thin-plates. The elevation of its lower interface is represented by $z = \eta_{i,0}(\boldsymbol{x}, t)$, and the pressure there is $p_{i,0}(\boldsymbol{x}, t)$, while the elevation of the upper interface is $z = \eta_{i,1}(\boldsymbol{x}, t)$, where the pressure is $p_{i,1}(\boldsymbol{x}, t)$. Pressure on any point of a plate surface is assumed to be equal to that at this point of the fluid in contact with the plate surface.

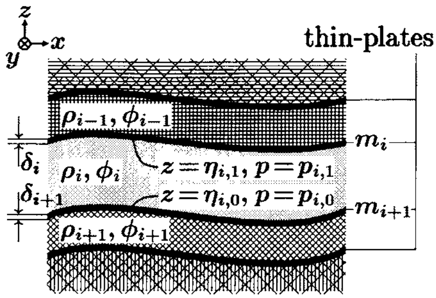


Figure 1: Diagram of a multilayer fluid system with thin-plates.

The thin-plate touching the upper interface of the i-layer is called the i-plate, whose density and vertical width are m_i and δ_i, respectively. Both these values of each plate are taken as constant for simplicity. The structures are assumed to be horizontally very large with our attention directed to their central areas.

2.2 Equations of motion for elastic thin-plates

The i-plate is assumed to consist of a homogeneous and elastic body whose scale in horizontal length is much larger than that in thickness. According to the classical theory, plate motion is determined by

$$m_i\delta_i\frac{\partial^2\eta_{i,1}}{\partial t^2} + B_i\nabla^2\nabla^2\eta_{i,1} + m_i g\delta_i + p_{i-1,0} - p_{i,1} = 0, \tag{1}$$

where B_i is flexural rigidity of the i-plate, g is the gravitational acceleration, and $\nabla = (\partial/\partial x, \partial/\partial y)$, i.e. a partial differential operator in the horizontal plane. In eqn (1) difference of curvature is ignored between the lower and neutral surfaces of the i-plate.

2.3 Equations for surface and internal waves

2.3.1 Functional in variational problems

Fluid motion is assumed to be irrotational, resulting in existence of velocity potential ϕ_i defined by $\boldsymbol{u}_i = \nabla\phi_i$ and $w_i = \partial\phi_i/\partial z$, where $\boldsymbol{u}_i$ and w_i are the horizontal vector and vertical scalar components of velocity at each point in the i-layer, respectively.

Elevation of one of two interfaces belonging to the i-layer, $z = \eta_{i,1-j}$ ($j = 0$ or 1), and pressure on the opposite interface, $p_{i,j}$, are treated as known variables; consequently the unknown variables are velocity potential ϕ_i and interface elevation $\eta_{i,j}$ such that Lagrangian in the i-layer, $\mathcal{F}_i[\phi_i, \eta_{i,j}]$, is defined as eqn (2) by modifying that of Luke [2], i.e.

$$\mathcal{F}_i[\phi_i, \eta_{i,j}] = \int_{t_0}^{t_1} \iint_A \int_{\eta_{i,0}}^{\eta_{i,1}} \left\{ \frac{\partial \phi_i}{\partial t} + \frac{1}{2}(\nabla \phi_i)^2 + \frac{1}{2}\left(\frac{\partial \phi_i}{\partial z}\right)^2 \right.$$
$$\left. + gz + \frac{p_{i,j} + P_i + W_i}{\rho_i} \right\} dz\ dA\ dt,$$
$$P_i = \sum_{k=1}^{i-1}\{(\rho_i - \rho_k) g h_k\}, \quad W_i = \sum_{k=1}^{i}(-m_k g \delta_k), \tag{2}$$

where $(\nabla\phi_i)^2 \equiv |\nabla\phi_i|^2$, and $h_i(\boldsymbol{x})$ is still water depth in the i-layer. Note that both the values of P_i and W_i are constant in each layer.

2.3.2 Vertical distribution functions

The velocity potential ϕ_i is expressed as a sum of vertical distribution functions z^α multiplied by their weights $f_{i,\alpha}$, i.e.

$$\phi_i(\boldsymbol{x}, z, t) \equiv \sum_{\alpha=0}^{N-1}\{z^\alpha \cdot f_{i,\alpha}(\boldsymbol{x}, t)\} = f_{i,\alpha} \cdot z^\alpha$$
$$= f_{i,0} \cdot 1 + f_{i,1} \cdot z + f_{i,2} \cdot z^2 + \cdots + f_{i,N-1} \cdot z^{N-1}. \tag{3}$$

2.3.3 Euler-Lagrange equations under variational principle

Equation (3) is substituted into eqn (2) and the Lagrangian is vertically integrated, after which the variational principle is applied to yield a set of Euler-Lagrange equations on $f_{i,\alpha}$ and $\eta_{i,j}$ ($\alpha = 0,\ 1,\ 2,\ \cdots,\ N-1$) as

$$\eta_{i,1}^\alpha \frac{\partial \eta_{i,1}}{\partial t} - \eta_{i,0}^\alpha \frac{\partial \eta_{i,0}}{\partial t} + \nabla(Q_i[\alpha+\beta]\nabla f_{i,\beta}) - R_i[\alpha,\beta] f_{i,\beta} = 0, \tag{4}$$

$$\eta_{i,j}^\beta \frac{\partial f_{i,\beta}}{\partial t} + \frac{1}{2}\eta_{i,j}^{\gamma+\beta}\nabla f_{i,\gamma}\nabla f_{i,\beta} + \frac{1}{2}S_{i,j}[\gamma,\beta] f_{i,\gamma} f_{i,\beta}$$
$$+ g\eta_{i,j} + (p_{i,j} + P_i + W_i)/\rho_i = 0, \tag{5}$$

where $Q_i[\alpha]$, $R_i[\alpha,\beta]$ and $S_{i,j}[\alpha,\beta]$ are functions of $\eta_{i,e}$ ($e = 0$ or 1), i.e.

$$Q_i[\alpha] = \frac{1}{\alpha+1}(\eta_{i,1}^{\alpha+1} - \eta_{i,0}^{\alpha+1}), \tag{6}$$

$$R_i[\alpha,\beta] = \begin{cases} \dfrac{\alpha\beta}{\alpha+\beta-1}\,(\eta_{i,1}^{\alpha+\beta-1} - \eta_{i,0}^{\alpha+\beta-1}) & (\alpha\beta \neq 0) \\ 0 \quad (\alpha\beta = 0) \end{cases}, \tag{7}$$

$$S_{i,j}[\alpha,\beta] = \begin{cases} \alpha\beta\ \eta_{i,j}^{\alpha+\beta-2} & (\alpha\beta \neq 0) \\ 0 \quad (\alpha\beta = 0) \end{cases}. \tag{8}$$

It should be noted that the presented set of equations are theoretically applicable to strongly nonlinear and strongly dispersive surface/internal waves using enough number of vertical distribution functions.

2.3.4 Assumption for long waves

Every layer is assumed to be in a shallow-water region and the following nondimensional variables are introduced:

$$x^* = \frac{x}{\ell},\ y^* = \frac{y}{\ell},\ z^* = \frac{z}{h},\ t^* = \frac{\sqrt{gh}}{\ell}t,\ \nabla^* = \ell\nabla,$$
$$\frac{\partial}{\partial t^*} = \left(\frac{\partial}{\partial t}\right)^* = \frac{\ell}{\sqrt{gh}}\frac{\partial}{\partial t},\ \eta^*_{i,e} = \frac{\eta_{i,e}}{h},\ f^*_{i,\alpha} = \frac{h^n f_{i,n}}{\sqrt{gh}\ell}\left(\frac{h}{H}\right)^n\Big|_{n=\alpha},$$
$$B^*_i = \frac{B_i}{\rho g \ell^4},\ p^*_{i,e} = \frac{p_{i,e}}{\rho g h},\ P^*_i = \frac{P_i}{\rho g h},\ m^*_i = \frac{m_i}{\rho},\ \delta^*_i = \frac{\delta_i}{H},\ W^*_i = \frac{W_i}{\rho g H}, \quad (9)$$

where order of every nondimensional variable with the mark of '$*$' is assumed $O(1)$. With these variables we can expand $\boldsymbol{u}_i$ and w_i into such series as

$$\frac{\boldsymbol{u}_i}{\sqrt{gh}} = k^*_{\boldsymbol{u}_i,0}\cdot 1 + k^*_{\boldsymbol{u}_i,1}\varepsilon z^* + k^*_{\boldsymbol{u}_i,2}\varepsilon^2 z^{*2} + \cdots + k^*_{\boldsymbol{u}_i,N-1}\varepsilon^{N-1}z^{*N-1},$$
$$\frac{w_i}{\sqrt{gh}} = \sigma\{k^*_{w_i,1}\cdot 1 + k^*_{w_i,2}\varepsilon z^* + \cdots + k^*_{w_i,N-1}\varepsilon^{N-2}z^{*N-2}\}, \quad (10)$$

where $\varepsilon = H/h$ and $\sigma = h/\ell$, i.e. representative ratios of wave height to water depth and water depth to wavelength, respectively, under the assumption that $O(\varepsilon) = O(\sigma^2)$.

By substituting eqn (9) into eqn (1), we obtain

$$\varepsilon\sigma^2 m^*_i\delta^*_i\frac{\partial^2\eta^*_{i,1}}{\partial t^{*2}} + B^*_i\nabla^{*2}\nabla^{*2}\eta^*_{i,1} + \varepsilon m^*_i\delta^*_i + p^*_{i-1,0} - p^*_{i,1} = 0. \quad (11)$$

Otherwise, by substituting eqn (9) into eqn (4) to represent fluid motion, we obtain

$$\eta^*_{i,1}{}^{\alpha}\frac{\partial\eta^*_{i,1}}{\partial t^*} - \eta^*_{i,0}{}^{\alpha}\frac{\partial\eta^*_{i,0}}{\partial t^*} + \varepsilon^\beta\nabla^*(Q^*_i[\alpha+\beta]\nabla^* f^*_{i,\beta}) - \frac{\varepsilon^\beta}{\sigma^2}R^*_i[\alpha,\beta]f^*_{i,\beta} = 0, \quad (12)$$

or, for each α,

$$\alpha = 0:$$
$$\frac{\partial\eta^*_{i,1}}{\partial t^*} - \frac{\partial\eta^*_{i,0}}{\partial t^*} + \nabla^*\{(\eta^*_{i,1} - \eta^*_{i,0})\nabla^* f^*_{i,0}\} + \varepsilon\frac{1}{2}\nabla^*\left\{({\eta^*_{i,1}}^2 - {\eta^*_{i,0}}^2)\nabla^* f^*_{i,1}\right\}$$
$$+\varepsilon^2\frac{1}{3}\nabla^*\left\{({\eta^*_{i,1}}^3 - {\eta^*_{i,0}}^3)\nabla^* f^*_{i,2}\right\} + O(\varepsilon^3) = 0,$$
$$\alpha = 1:$$
$$\eta^*_{i,1}\frac{\partial\eta^*_{i,1}}{\partial t^*} - \eta^*_{i,0}\frac{\partial\eta^*_{i,0}}{\partial t^*} + \nabla^*\left\{\frac{1}{2}({\eta^*_{i,1}}^2 - {\eta^*_{i,0}}^2)\nabla^* f^*_{i,0}\right\}$$
$$+\varepsilon\frac{1}{3}\nabla^*\left\{({\eta^*_{i,1}}^3 - {\eta^*_{i,0}}^3)\nabla^* f^*_{i,1}\right\} + \varepsilon^2\frac{1}{4}\nabla^*\left\{({\eta^*_{i,1}}^4 - {\eta^*_{i,0}}^4)\nabla^* f^*_{i,2}\right\}$$
$$-\frac{\varepsilon}{\sigma^2}(\eta^*_{i,1} - \eta^*_{i,0})f_{i,1} - \frac{\varepsilon^2}{\sigma^2}({\eta^*_{i,1}}^2 - {\eta^*_{i,0}}^2)f_{i,2}$$

$$
\begin{aligned}
&-\frac{\varepsilon^3}{\sigma^2}({\eta^*_{i,1}}^3-{\eta^*_{i,0}}^3)f_{i,3}+O(\varepsilon^3)=0,\\
&\alpha=2:\\
&{\eta^*_{i,1}}^2\frac{\partial\eta^*_{i,1}}{\partial t^*}-{\eta^*_{i,0}}^2\frac{\partial\eta^*_{i,0}}{\partial t^*}+\frac{1}{3}\nabla^*\left\{({\eta^*_{i,1}}^3-{\eta^*_{i,0}}^3)\nabla^* f^*_{i,0}\right\}\\
&\quad+\varepsilon\frac{1}{4}\nabla^*\left\{({\eta^*_{i,1}}^4-{\eta^*_{i,0}}^4)\nabla^* f^*_{i,1}\right\}+\varepsilon^2\frac{1}{5}\nabla^*\left\{({\eta^*_{i,1}}^5-{\eta^*_{i,0}}^5)\nabla^* f^*_{i,2}\right\}\\
&\quad-\frac{\varepsilon}{\sigma^2}({\eta^*_{i,1}}^2-{\eta^*_{i,0}}^2)f^*_{i,1}-\frac{\varepsilon^2}{\sigma^2}\frac{4}{3}({\eta^*_{i,1}}^3-{\eta^*_{i,0}}^3)f^*_{i,2}\\
&\quad-\frac{\varepsilon^3}{\sigma^2}\frac{3}{2}({\eta^*_{i,1}}^4-{\eta^*_{i,0}}^4)f_{i,3}+O(\varepsilon^3)=0,\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,
\end{aligned}
\tag{13}
$$

while, by substituting eqn (9) into eqn (5), we get

$$
\begin{aligned}
&\varepsilon^\beta{\eta^*_{i,j}}^\beta\frac{\partial f^*_{i,\beta}}{\partial t^*}+\varepsilon^{\gamma+\beta}\frac{1}{2}{\eta^*_{i,j}}^{\gamma+\beta}\nabla^* f^*_{i,\gamma}\nabla^* f^*_{i,\beta}+\frac{\varepsilon^{\gamma+\beta}}{\sigma^2}\frac{1}{2}S^*_{i,j}[\gamma,\beta]f^*_{i,\gamma}f^*_{i,\beta}\\
&\quad+\eta^*_{i,j}+p^*_{i,j}+P^*_i+\varepsilon W^*_i=0,
\end{aligned}
\tag{14}
$$

or

$$
\begin{aligned}
&\frac{\partial f^*_{i,0}}{\partial t^*}+\varepsilon\eta_{i,j}\frac{\partial f^*_{i,1}}{\partial t^*}+\varepsilon^2{\eta^*_{i,j}}^2\frac{\partial f^*_{i,2}}{\partial t^*}+\frac{1}{2}\nabla^* f^*_{i,0}\nabla^* f^*_{i,0}+\varepsilon\eta_{i,j}\nabla^* f^*_{i,0}\nabla^* f^*_{i,1}\\
&\quad+\varepsilon^2{\eta^*_{i,j}}^2\nabla^* f^*_{i,0}\nabla^* f^*_{i,2}+\varepsilon^2\frac{1}{2}{\eta^*_{i,j}}^2\nabla^* f^*_{i,1}\nabla^* f^*_{i,1}+\frac{\varepsilon^2}{\sigma^2}\frac{1}{2}f^*_{i,1}f^*_{i,1}\\
&\quad+\frac{\varepsilon^3}{\sigma^2}2\eta^*_{i,j}f^*_{i,1}f^*_{i,2}+\eta^*_{i,j}+p^*_{i,j}+P^*_i+\varepsilon W^*_i+O(\varepsilon^3)=0.
\end{aligned}
\tag{15}
$$

By taking j as 0 or 1 in eqn (14), the relation between dimensional pressures at the upper and lower surfaces of the i-layer is expressed by

$$
\begin{aligned}
p_{i,0}&=p_{i,1}+\rho_i g(\eta_{i,1}-\eta_{i,0})\\
&\quad+\rho_i\left\{(\eta^\beta_{i,1}-\eta^\beta_{i,0})\frac{\partial f_{i,\beta}}{\partial t}+\frac{1}{2}(\eta^{\gamma+\beta}_{i,1}-\eta^{\gamma+\beta}_{i,0})\nabla f_{i,\gamma}\nabla f_{i,\beta}\right.\\
&\quad\left.+\frac{1}{2}(S_{i,1}[\gamma,\beta]-S_{i,0}[\gamma,\beta])f_{i,\gamma}f_{i,\beta}\right\},
\end{aligned}
\tag{16}
$$

accordingly influence of not only hydrostatic but also hydrodynamic pressures can be considered dependently on the degrees of interface displacement those we take into account when $N\geq 2$.

Hereafter, it is assumed that $O(\varepsilon)\ll 1$, and terms up to the first order, i.e. $O(\varepsilon)$, are taken into account. In eqns (11) – (15), let $j=1$ and eliminate $p^*_{i,1}$ to obtain the following equations on dimensional variables:

$$
\frac{\partial\eta_{i,1}}{\partial t}-\frac{\partial\eta_{i,0}}{\partial t}+\nabla\{(\eta_{i,1}-\eta_{i,0})\nabla f_{i,0}\}+\frac{1}{2}\nabla\{(\eta^2_{i,1}-\eta^2_{i,0})\nabla f_{i,1}\}=0,
\tag{17}
$$

$$\eta_{i,1}\frac{\partial \eta_{i,1}}{\partial t} - \eta_{i,0}\frac{\partial \eta_{i,0}}{\partial t} + \frac{1}{2}\nabla\left\{({\eta_{i,1}}^2 - {\eta_{i,0}}^2)\nabla f_{i,0}\right\}$$
$$+\frac{1}{3}\nabla\left\{({\eta_{i,1}}^3 - {\eta_{i,0}}^3)\nabla f_{i,1}\right\} - (\eta_{i,1} - \eta_{i,0})f_{i,1} = 0, \qquad (18)$$

$$\frac{\partial f_{i,0}}{\partial t} + \eta_{i,j}\frac{\partial f_{i,1}}{\partial t} + \frac{1}{2}\nabla f_{i,0}\nabla f_{i,0} + \eta_{i,j}\nabla f_{i,0}\nabla f_{i,1} + \frac{1}{2}f_{i,1}f_{i,1}$$
$$+g\eta_{i,1} + (B_i\nabla^2\nabla^2\eta_{i,1} + p_{i-1,0} + P_i + W_{i-1})/\rho_i = 0, \qquad (19)$$

where the inertia term of the thin-plate equation is relatively ignored, q.m. in Takagi [3]. According to eqn (16), the relation between the two pressures is

$$p_{i,0} = p_{i,1} + \rho_i(\eta_{i,1} - \eta_{i,0})\left(g + \partial f_{i,1}/\partial t + \nabla f_{i,0}\nabla f_{i,1}\right). \qquad (20)$$

3 Numerical calculation

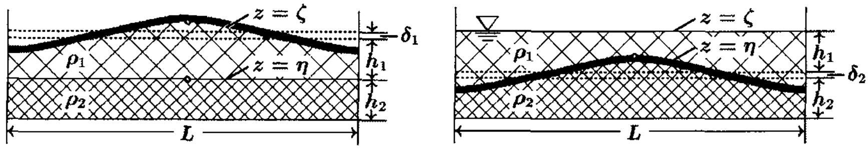
3.1 Internal waves generated by oscillation of a floating thin-plate

3.1.1 Forces oscillation

Figure 2 shows two-layer fluid systems with a thin-plate in the vertical two dimensions. Elevations of the surface and interface are represented by $z = \eta_{1,1} = \zeta$ and $z = \eta_{1,0} = \eta$, respectively. Equations (17)–(19) are solved using a finite-difference method we presented in [4]. It is assumed that every floating structure is horizontally large, hence the same phenomenon appears periodically along the x-axis, such that the ends of the calculation domain are smoothly connected together. The atmospheric pressure is set at zero. The initial value of velocity potential ϕ_i is equal to zero everywhere.

When a floating structure, as shown in Fig. 2 (a), oscillates with a constant period and a constant wavelength, it gives a forced oscillation to the fluid system. In Fig. 3, time variations of pressure at the middle point on the structure, p_{M} where $(x, z) = (L/2, \zeta)$, are shown when a continuous oscillation, $\zeta = a\sin(2\pi t/T)\cos(2\pi x/L)$, is given on the sea surface. The pressure p_{M} in each corresponding one-layer case is represented by a broken line. Resonance is excited in the cases of Fig. 3 (a). The calculations were stopped the moment the interface touched the seabed ($x = L/2$) when $T\sqrt{gh}/L = 0.8\tau$ and τ, or touched the structure ($x = 0$) when $T\sqrt{gh}/L = 1.3\tau$. If the terms up to the order of $O(\varepsilon)$ are considered, effects of linearly distributed $\boldsymbol{u}_i$ and constant w_i in the vertical direction are theoretically taken into account. Consequently the negative pressure is weaker than in the cases where only the terms of $O(1)$ are considered. When $T\sqrt{gh}/L = \tau$, and 1.3τ, phase shifts appear compared with the one-layer cases.

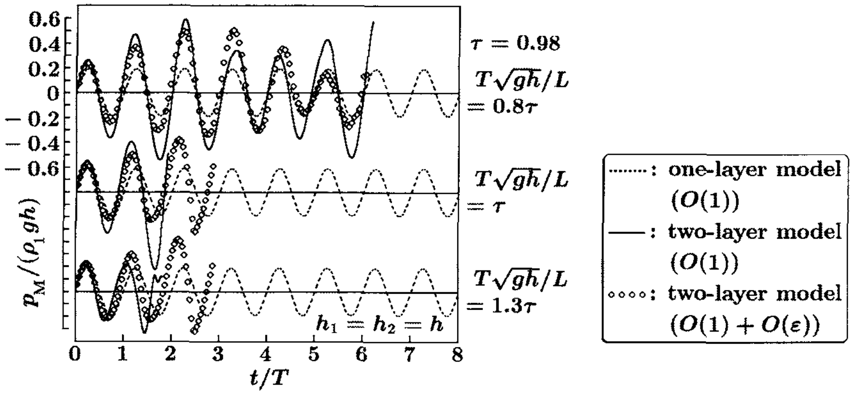
The cases shown in Fig. 3 (b), where the pressure does not increase so remarkably as in Fig. 3 (a), have no resonance phenomenon. There are several different modes. When $T\sqrt{gh}/L = 2\tau$, there is difference in



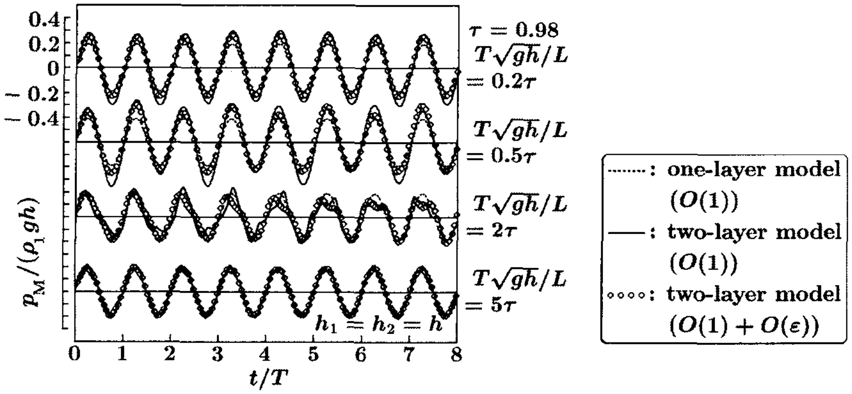
(a) A thin-plate oscillates on the sea surface.

(b) A thin-plate oscillates below the sea surface.

Figure 2: Definition sketches of two-layer fluid systems with a thin-plate structure.



(a) The interface resonates with the oscillation of the structure.



(b) The interface does not resonate with the oscillation of the structure.

Figure 3: Pressure beneath the thin-plate structure floating on the sea surface ($L/h = 10,\ a/h = 0.2,\ \rho_2/\rho_1 = 1.5,\ x/L = 0.5$).

phase of the higher frequency modes between the two cases of $O(1)$ and $(O(1) + O(\varepsilon))$. As the period is long enough, the amplitude of pressure in the two-layer fluid cases is almost equal to that in the one-layer cases.

3.1.2 Free oscillation

A profile of a structure floating on the sea surface is taken as shown in Fig. 2 (a), and then the structure is released at the initial time. In this case the thin-plate structure interacts with the fluid motion and the system begins free oscillation. In Fig. 4, the elevation of the interface at the middle of the calculation domain, $\eta_{\mathrm{M}}(L/2, t)$, is shown when the initial profile of the structure is taken as $\zeta = -a_0 \cos(2\pi x/L)$. There are phase shifts in the lower frequency modes between the two cases of $O(1)$ and $(O(1) + O(\varepsilon))$, which are different phenomena from those in the forced oscillation cases.

3.2 Oscillation of a submerged thin-plate structure generated by surface waves

Oscillation of a thin-plate between two fluids as shown in Fig. 2 (b) is numerically simulated. Fig. 5 shows the upper face elevation of thin-plate at the middle of the calculation domain, $\eta_{\mathrm{M}}(L/2, t)$, when the sea surface is released after its initial profile is taken as $\zeta = -a_0 \cos(2\pi x/L)$. In this case the system also begins free oscillation, where the submerged elastic structure interacts with the upper and lower fluids. In consideration of $O(\varepsilon)$ phase shifts occur but are not so large as in the free oscillation cases where the thin-plate is floating on the sea surface. In the case where the flexural rigidity $B_1^* = B_1/(\rho_1 g L^4) = 0.1$, the trough levels of η_{M} are calculated as higher values with the terms of $O(\varepsilon)$ than except them.

4 Conclusions

A set of nonlinear equations governing the interaction between a multilayer fluid system and thin-plates was utilized to perform several computations of two-layer fluid systems with a horizontally very large floating or submerged elastic structure in the vertical section. Effects caused by the terms having the first order of wave height to water depth ratio appear as weaken negative pressure on a floating structure in forced oscillation, as well as phase shifts of higher frequency modes in forced oscillation with a floating structure or of lower frequency modes in free oscillation with a floating or submerged structure. In consideration of influence of higher order terms further work is required to clarify the generation mechanism of different frequency modes in oscillation.

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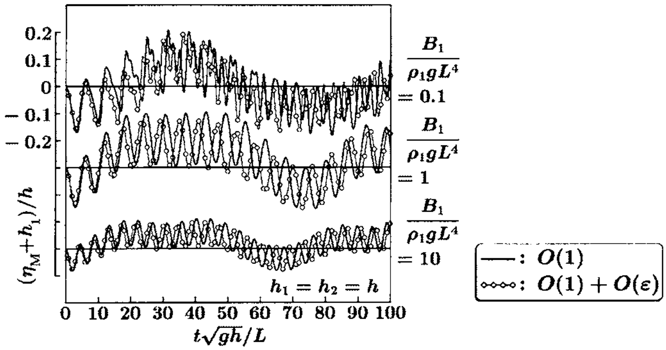

Figure 4: Elevation of the interface. The initial profile of the 1-plate on the sea surface is given ($L/h = 10$, $a_0/h = 0.2$, $\rho_2/\rho_1 = 1.025$, $x/L = 0.5$).

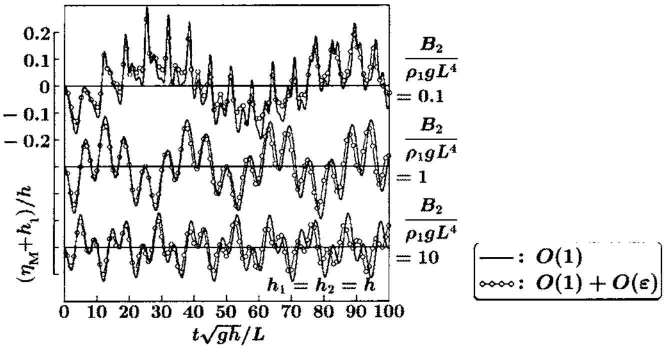

Figure 5: Displacement of the 2-plate. The initial profile of the sea surface is given ($L/h = 10$, $a_0/h = 0.2$, $\rho_2/\rho_1 = 1.025$, $x/L = 0.5$).

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