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A software package for analysis of circular corrugated horns in their operative environment

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Abstract

A hybrid method for the analysis of the interaction between circular corrugated horns and conducting structures usually present in their operative environment is proposed. The method is based on the application of a particular formulation of the equivalence principle that subdivides the original problem into an exterior and an interior one. The former is formulated by using a moment method based on a Combined Field Integral Equation (CFIE), the latter by employing the Mode Matching technique. The Green's function in the kernel of the CFIE for the formulation of the exterior problem is modified to take into account the presence of conducting structures placed nearby the corrugated horn. Numerical results showing the level of field on the horn opening due to unwanted interactions in a dual Gregorian antenna system are reported.

1 Introduction

Circular corrugated horns are often used as feeders in modern reflector high-performance antenna systems because of their very good characteristics in term of high copolar pattern symmetry [1-2], low-level of cross polarization [3] and good impedance matching [4]. Because of these attractive features, corrugated horns have been extensively studied and many efforts have been directed towards the development of efficient methods for their analysis and design [4].

One of the most used analysis method is the Mode-Matching (MM) [5] which provides a very accurate model of the field inside the horn, at the expense of computational cost. Among the advantages of the MM, its great flexibility is probably the most important since it allows to analyze horns with corrugations

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having arbitrarily varying depth and width. This feature is particularly important for horns used in space applications where excitation of higher order modes in multimode horns as well as matching the feeder to the beam forming network must be achieved using only a suitable design of the corrugations shape. In fact, for this kind of application is desirable to avoid using dielectric materials.

Because this method is suitable for the analysis of the interior part only of the horn, many efforts have been focused towards the hybridization of MM with integral formulations which can take into account also the external geometry of the horn [6-9]. However, no effort has been done so far to include in the analysis interactions between the horn and perfectly conducting structures often present in its operative environment. This aspect of the analysis is particularly important when the horn is used as feeder in a reflector antenna system and parabolic reflectors and struts constitute an important part of the horn operative environment.

Aim of this work is to present a software package for the analysis and design of circular corrugated horns that can also evaluate the level of unwanted coupling between the horn and nearby conducting structures with characteristic dimensions large with respect to the wavelength. The package implements a hybrid method that is an extension of that presented in [8-9] for the analysis and optimization of circular corrugated horns radiating in free space. The effects of nearby structures on the horn performance have been estimated by using a perturbative approach in conjunction with the Uniform Theory of Diffraction (UTD) [10]. The UTD is employed to modify the kernel of the integral equation used to formulate the exterior part of the problem.

This approach has the advantage of requiring a limited number of unknowns and it provides a first order correction to results obtained considering the horn isolated. On the other hand, its validity is limited to the case of perfectly conducting objects with dimensions and distance from the horn of the same order of wavelength. Indeed, this is not a limitation for the analysis of reflector antenna systems using corrugated horns as feeders.

The paper is organized as follows: the analysis method implemented in the software package is described in details in Section 2; both the rigorous formulation and the perturbative approach actually implemented are considered. Numerical data relative to a dual Gregorian Rotatable Coverage satellite antenna are presented in Section 3. Finally, conclusions are drawn in Section 4.

2 Analysis Method

The longitudinal section of a transverse corrugated circular horn and a metallic object placed nearby is sketched in Fig. 1. The analysis of this problem is carried on using a two-step technique. In the first step, a particular formulation of the equivalence theorem [11,12] is used to subdivide the original problem into two separate but coupled problems.

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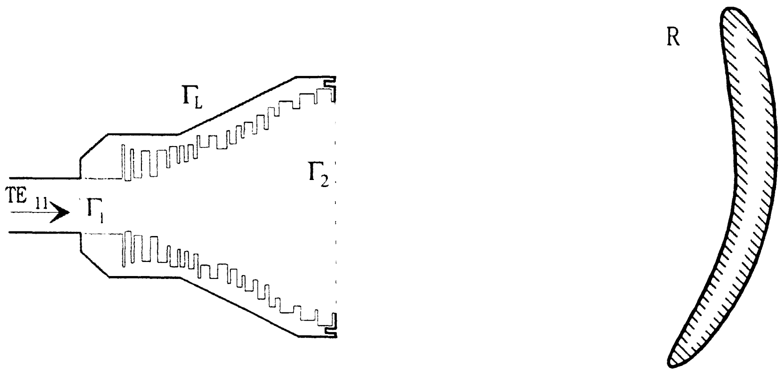

Fig. 1 – Geometry of the problem

The two problems are sketched in Fig. 2. The first (Fig. 2a) is the interior problem consisting of the horn with a metallic cover on its opening Γ_2, excited by the TE_{11} mode through its throat and the equivalent surface magnetic current densities $\mathbf{M}^{int}$ impressed on the interior side of Γ_2. The second, exterior problem (Fig. 2b) is comprised of the equivalent surface magnetic current densities $\mathbf{M}^{ext}$ on the exterior side of the cover Γ_2, radiating in presence of the completely metallized horn and the nearby conducting object *R*.

The two problems are coupled through the equivalent magnetic current densities $\mathbf{M}^{int}$ and $\mathbf{M}^{ext}$:

$$\mathbf{M}^{ext} = \mathbf{E} \times \mathbf{i}_n = \mathbf{M}, \qquad \text{at } \Gamma_2, \tag{1.a}$$

$$\mathbf{M}^{int} = \mathbf{E} \times (-\mathbf{i}_n) = -\mathbf{M}^{ext} = -\mathbf{M}, \qquad \text{at } \Gamma_2, \tag{1.b}$$

with $\mathbf{E}$ total electric field and $\mathbf{i}_n$ outward normal unit vector at the horn opening Γ_2. The opposite sign of the interior and exterior equivalent magnetic currents automatically guarantees the appropriate continuity condition of the electric field for the interior and exterior problems at Γ_2.

The second step of the solution procedure consists in matching the solutions in the two regions at the horn aperture (metal surface Γ_2 of Fig. 2). Because the continuity of the electric field is automatically guaranteed by the choice (1.b) of the equivalent magnetic currents, only the continuity of the magnetic field must be explicit enforced:

$$\mathbf{i}_n \times \mathbf{H}^{ext} = \mathbf{i}_n \times \mathbf{H}^{horn}, \qquad \text{at } \Gamma_2. \tag{2}$$

$\mathbf{H}^{ext}$ and $\mathbf{H}^{horn}$ in equation (2) are the total magnetic field in the exterior and interior region, respectively. The total magnetic field in the interior region can be expressed as the superposition of two contributions:

$$\mathbf{H}^{horn} = \mathbf{H}^{inc} + \mathbf{H}^{int}(-\mathbf{M}), \qquad \text{at } \Gamma_2, \tag{3}$$

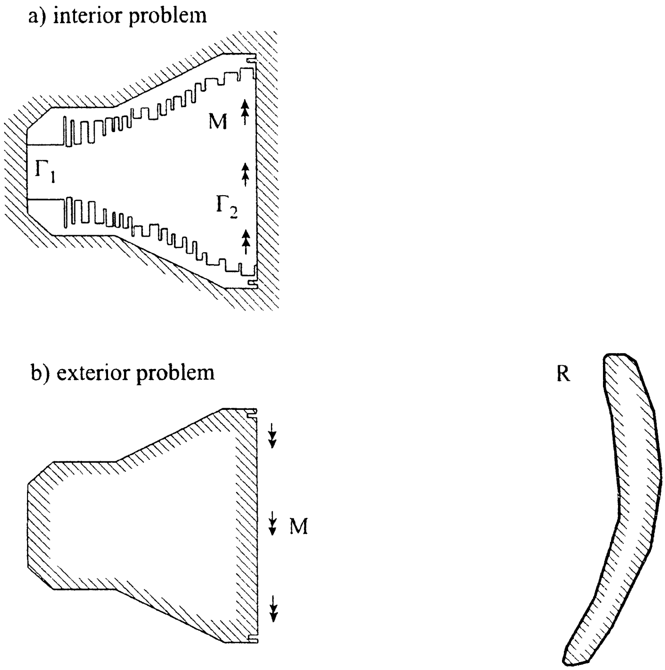

Fig. 2 – The original problem is subdivided into an interior (a) and an exterior one (b).

where $\mathbf{H}^{inc}$ is the magnetic field produced over the horn opening by the TE_{11} mode fed through the input port Γ_1, while $\mathbf{H}^{int}$ is the contribution due to the interior equivalent magnetic current distribution $-\mathbf{M}$. The magnetic field continuity condition yields the integrodifferential equation:

$$\mathbf{i}_n \times \mathbf{H}^{ext}(\mathbf{M}) + \mathbf{i}_n \times \mathbf{H}^{int}(\mathbf{M}) = \mathbf{i}_n \times \mathbf{H}^{inc}, \quad \text{at } \Gamma_2. \tag{4}$$

that can be solved for the unknown equivalent magnetic current. The application of the standard weighted residual method to equation (4) transforms that integrodifferential equation into the linear system of equations:

$$[\mathbf{Y}^{ext} + \mathbf{Y}^{int}]\,[\mathbf{V}] = [\mathbf{I}], \tag{5}$$

whose solution gives the equivalent magnetic current distribution $\mathbf{M}$. The knowledge of $\mathbf{M}$ immediately gives the actual electric field distribution over the horn aperture in presence of the conducting object (R in Fig. 1). In equation (5), $[\mathbf{V}]$ is the column vector of the unknown coefficients of the basis functions $\{\mathbf{M}_j, j = 1,N\}$ used to express the equivalent current distribution $\mathbf{M}$, $[\mathbf{I}]$ is the column vector of excitation associated to the field at Γ_2 due to the TE_{11} mode feeding the horn, $[\mathbf{Y}^{ext}]$ and $[\mathbf{Y}^{int}]$ account for the contributions to the magnetic field at Γ_2 coming from the exterior and interior magnetic currents, respectively.

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The choices of basis and weighting functions for the weighted residual solution of equation (4) as well as the expressions of the entries of the matrices [$\mathbf{Y}^{ext}$], [$\mathbf{Y}^{int}$], and the vector [**I**] have been discussed in [8-9] referring to the case of an isolated horn and will not repeated here. In the following subsection 2.1, the solution of the interior problem is briefly outlined with particular attention to the changes that need to be done with respect to the case of an isolated horn. Subsection 2.2 instead focuses on the perturbative approach used to treat the exterior problem and to compute the unwanted field at the horn opening due to interactions with nearby conducting objects.

2.1 Interior Problem: Mode Considerations

To evaluate the entries of both the [**I**] vector and the [$\mathbf{Y}^{int}$] matrix it is necessary to solve the interior problem. This can be efficiently accomplished once the Generalized Scattering Matrix (GSM) [**S**] of the corrugated horn is known. This matrix can be reckoned resorting to the well-known MM technique which accurately describes wave propagation through the corrugated horn. The MM technique is flexibile, accurate and efficient, therefore can be used for synthesis and optimization purpoese [7-9].

The circular corrugated horn can be thought of as comprised of a many cascaded discontinuity of radial-type. The scattering matrix of each discontinuity can be computed using an expansion in circular waveguide characteristic modes, and the GSM [**S**] of the whole horn is obtained by cascading the scattering matrices of each discontinuity.

Because of the rotational symmetry of the geometry and of the mode exciting the horn, in [8-9] the matrix [**S**] is computed using TE_{1m} and TM_{1m} modes only. However, when conducting objects are located nearby the horn, rotational symmetry of the geometry is generally destroyed and the matrix [**S**] must be computed taking into account all the possible TE_{nm} and TM_{nm} modes in a circular waveguides.

2.2 Exterior Problem: A Perturbative Approach

Evaluation of matrix [$\mathbf{Y}^{ext}$] j-th column entries requires the solution of the exterior problem (Fig. 2b). In particular, it is necessary to compute the magnetic field $\mathbf{H}^{ext}(\mathbf{M}_j)$ at Γ_2 due to the j-th basis function ($\mathbf{M}_j$) used for expanding **M**. The basis function $\mathbf{M}_j$ radiates in the presence of both the conducting object R and the perfectly conducting horn with short-circuited input (Γ_1) and output (Γ_2) apertures. In the case of an isolated horn (object R is not present), the exterior problem can be solved using the Combined Field Integral Equation (CFIE) technique [6,8,13]. To this end, the equivalence theorem is applied to replace the perfectly conducting horn with a distribution of surface equivalent electric currents **J**. This leads to a new configuration in which the known magnetic current $\mathbf{M}_j$ and unknown electric current density sources **J** radiate in free space.

The unknown current **J** can be evaluated formulating the CFIE [13] over the exterior surface of the horn:

$$\alpha \mathbf{i}_{\mathrm{n}} \times L(\mathbf{J}) + \zeta(1-\alpha)\left(\frac{1}{2}\mathbf{J} + \mathbf{i}_{\mathrm{n}} \times K(\mathbf{J})\right) = \tag{6}$$

$$= \alpha\, \mathbf{i}_{\mathrm{n}} \times K(\mathbf{M}_j) - \frac{1-\alpha}{\zeta}\, \mathbf{i}_{\mathrm{n}} \times L(\mathbf{M}_j).$$

In equation (6), $\mathbf{i}_{\mathrm{n}}$ is the outward normal unit vector and the parameter α (in this case set to 0.5) is the coupling factor between the Electric Field Integral Equation (EFIE) and the Magnetic Field Integral Equation (MFIE). ζ is the free-space characteristic impedance and the integrodifferential operators $L(\cdot)$ and $K(\cdot)$ are [13]:

$$L(\mathbf{X}) = j\omega\mu \iint_{S'} \left[\mathbf{X}(\mathbf{r}') + \frac{1}{\omega^2\mu\epsilon}\nabla\nabla\cdot\mathbf{X}(\mathbf{r}')\right] \underline{\underline{G}}(\mathbf{r}, \mathbf{r}')\, \mathrm{ds}', \tag{7a}$$

$$K(\mathbf{X}) = \iint_{S'} \mathbf{X}(\mathbf{r}') \times \nabla \underline{\underline{G}}(\mathbf{r}, \mathbf{r}')\, \mathrm{ds}', \tag{7b}$$

where $\underline{\underline{G}}$ is the free-space dyadic Green's function, S' is teh exterior surface of the corrugated horn, and **X** is either **J** or $\mathbf{M}_j$.

The integrodifferential equation (6) can be solved with the weighted residual procedure to compute the equivalent electric current density **J** produced on the horn surface by each basis function $\mathbf{M}_j$. Once **J** is known, the total exterior magnetic field $\mathbf{H}^{ext}(\mathbf{M}_j)$ produced by $\mathbf{M}_j$ in presence of the metallic horn, and hence the elements of the matrix $[\mathbf{Y}^{ext}]$, can be computed evaluating the radiation of both the basis function $\mathbf{M}_j$ and the relative electric current **J**:

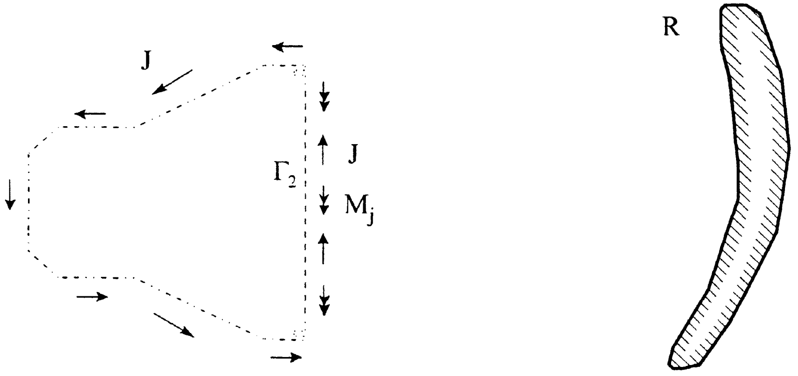

Fig. 3 – Equivalent currents introduced for the CFIE solution of the external problem.

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$$\mathbf{H}^{ext}(\mathbf{M}_j) = -\frac{L(\mathbf{M}_j)}{\zeta^2} - K(\mathbf{J}). \tag{8}$$

In the general case of horn radiating in presence of conducting objects, the above procedure can be repeated. The equivalence theorem is again applied on the perfectly conducting surface of the horn which is once again replaced by the surface equivalent current **J** (Fig. 3). The CFIE can be again formulated on the extereior surface S' of the horn, however the operators $L(\cdot)$ and $K(\cdot)$ need to be modified because the equivalent currents now radiates in presence of the conducting object R (Fig. 3). In the practical case of interest in which the conducting bodies nearby the horn are the reflectors of an antenna system, $L(\cdot)$ and $K(\cdot)$ can be expressed as

$$L = L^{free} + L^{diff} \qquad\qquad K = K^{free} + K^{diff} \tag{9}$$

In the above equations, L^{free} and K^{free} account for free-space radiation and are given in equations (7), while L^{diff} and K^{diff} take care of contributions coming from field diffracted by the edges of the reflectors and can be evaluated by means of the Uniform Theory of Diffraction (UTD) [10]. Geometric Optics contributions L^{GO} and K^{GO} usually present in UTD formulations have been neglected in this case because in practical reflector antenna systems the reflectors shape and position are designed so as not to reflect signal back towards the feeder.

Although feasible, the approach outlined above is very demanding from a computational point of view. It is then more convenient resorting to a perturbative approach that can greatly reduce computational efforts maintaining acceptable accuracy.

The perturbative approach can be used because the presence of the reflectors usually only slightly modifies the free-space radiation characteristic of the feeder. Denoting with $\mathbf{J}^0$ the CFIE free-space solution, of equation (6), hereafter referred to as non perturbed solution, equation (6) can be recast in the shorter form

$$\mathcal{L}^0(\mathbf{J}^0) = g^0, \tag{10}$$

where $\mathcal{L}^0$ is the operator of the non perturbed problem (isolated horn). When the horn radiates in presence of conducting objects (perturbed problem) the CFIE is rewritten as:

$$\mathcal{L}(\mathbf{J}) = g^0 + g^1, \tag{11}$$

where $\mathcal{L}$ is the exact operator, sum of the non perturbed ($\mathcal{L}^0$) and difference ($\mathcal{L}^1$) operators. Assuming that the solution $\mathbf{J}$ of the perturbed problem can be expressed by adding a perturbative term $\mathbf{J}^1$ to $\mathbf{J}^0$, with $\mathbf{J}^1 \ll \mathbf{J}^0$, the CFIE (6) in presence of reflectors becomes:

$$\mathcal{L}^0(\mathbf{J}^0 + \mathbf{J}^1) + \mathcal{L}^1(\mathbf{J}^0 + \mathbf{J}^1) = \mathrm{g}^0 + \mathrm{g}^1 \qquad (12)$$

Exploiting the linearity of the operators and disregarding the term $\mathcal{L}^1(\mathbf{J}^1)$, equation (12) can be simplified as:

$$\mathcal{L}^0(\mathbf{J}^1) = \mathrm{g}^1 - \mathcal{L}^1(\mathbf{J}^0), \qquad (13)$$

where:

$$\mathcal{L}^0(\mathbf{J}) = \alpha \underline{\mathbf{i}}_{\mathrm{n}} \times L^{free}(\mathbf{J}) + \zeta(1-\alpha)\left\{\frac{1}{2}\mathbf{J} + \underline{\mathbf{i}}_{\mathrm{n}} \times K^{free}(\mathbf{J})\right\}, \qquad (14a)$$

$$\mathcal{L}^1(\mathbf{J}) = \alpha \underline{\mathbf{i}}_{\mathrm{n}} \times L^{diffr}(\mathbf{J}) + \zeta(1-\alpha)\left\{\underline{\mathbf{i}}_{\mathrm{n}} \times K^{diffr}(\mathbf{J})\right\}, \qquad (14b)$$

$$\mathrm{g}^1 = \alpha \underline{\mathbf{i}}_{\mathrm{n}} \times K^{diffr}(\mathbf{M}_j) - \frac{(1-\alpha)}{\zeta}\underline{\mathbf{i}}_{\mathrm{n}} \times L^{diffr}(\mathbf{M}_j). \qquad (14c)$$

The solution of eqaution (13) gives the perturbation $\mathbf{J}^1$ caused by the reflectors that can be used to compute the field $\mathbf{H}^{ext}(\mathbf{M}_j)$:

$$\begin{aligned}\mathbf{H}^{ext}(\mathbf{M}_j) = & -\frac{L^{free}(\mathbf{M}_j)}{\zeta^2} - K^{free}(\mathbf{J}^0) \\ & -\frac{L^{diffr}(\mathbf{M}_j)}{\zeta^2} - K^{free}(\mathbf{J}^1) - K^{diffr}(\mathbf{J}^0)\end{aligned} \qquad (15)$$

and subsequently the entries of the matrix $[\mathbf{Y}^{ext}]$.

3 Numerical Results

The software package X_HORN [14] implements the technique outlined in the previous section and has been used to compute the numerical results presented in this section relative to a Dual Gregorian Rotatable Coverage satellite antenna at a frequency of 13 GHz.

The feeder of this antenna system is the V5A circular corrugated horn [8-9], whose copolar and cross polar radiation patterns (when considered as isolated) are shown in Fig. 4. These results show a very good agreement with available measurements [8] and validates the approach for the case of free-space radiation (no interaction).

The radiation pattern of the isolated horn is also useful to assess the level of interactions between the horn and the two reflectors of the dual Gregorian system. From the geometry of the system (sketched in Fig. 5) and the radiation pattern of the isolated horn reported in Fig. 4, it is seen that the subreflector's edges are strongly illuminated (− 12dB with respect to the peak of radiation in

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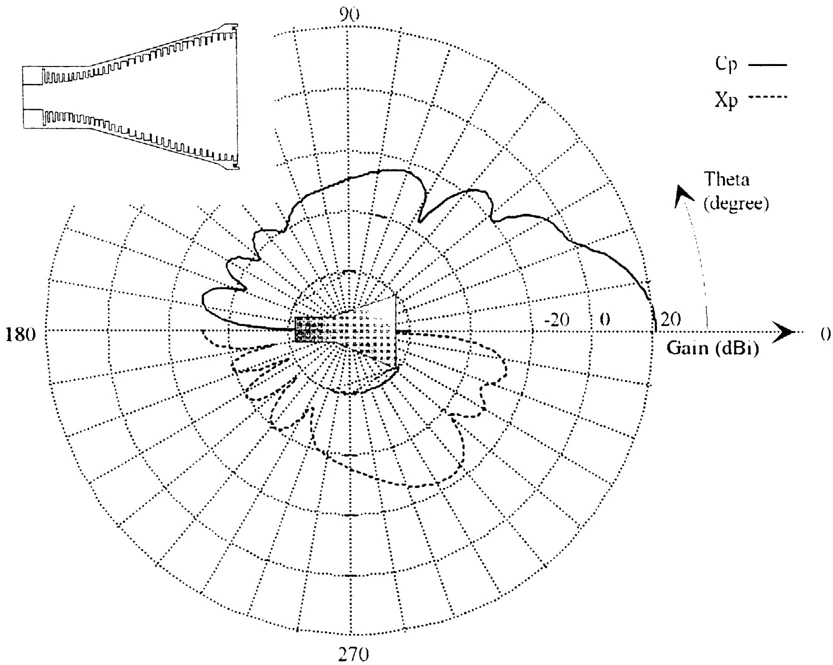

Fig.4 – Radiation pattern of the copolar (cp) and crosspolar (xp) field components at $\varphi = 45°$ for the VA5 horn.

the direction $\theta = 0°$), while the level of field radiated by the horn which directly reaches the main reflector is well below -45dB with respect to the peak of radiation. Consequently, the effect of direct interaction between the horn and the main reflector can be neglected without significantly degrading the accuracy of the technique.

Fig. 5 shows cross-polar radiation pattern of the horn when the horn is isolated and when the coupling between the V5A feeder and the subreflector of the Dual Gregorian antenna is taken into account. Only the crosspolar radiation patterns are shown because they are more significant than the copolar patterns due to their lower field levels. It is seen that when interaction between the feeder and the subreflector is taken into account, the level of the crosspolar pattern in the solid angle subtended by the main reflector rises up to 5 dB. In some satellite antennas applications this increase can be critical and its detection in the early stage of design can be very important to reduce development time.

The non-perturbed and perturbed magnetic and electric field amplitude distributions over the horn opening are shown in Fig. 6. These results serve as an *a posteriori* validation of the perturbative approach since they show that the corrective term is about 30dB lower than the non-perturbed field over most of the horn aperture.

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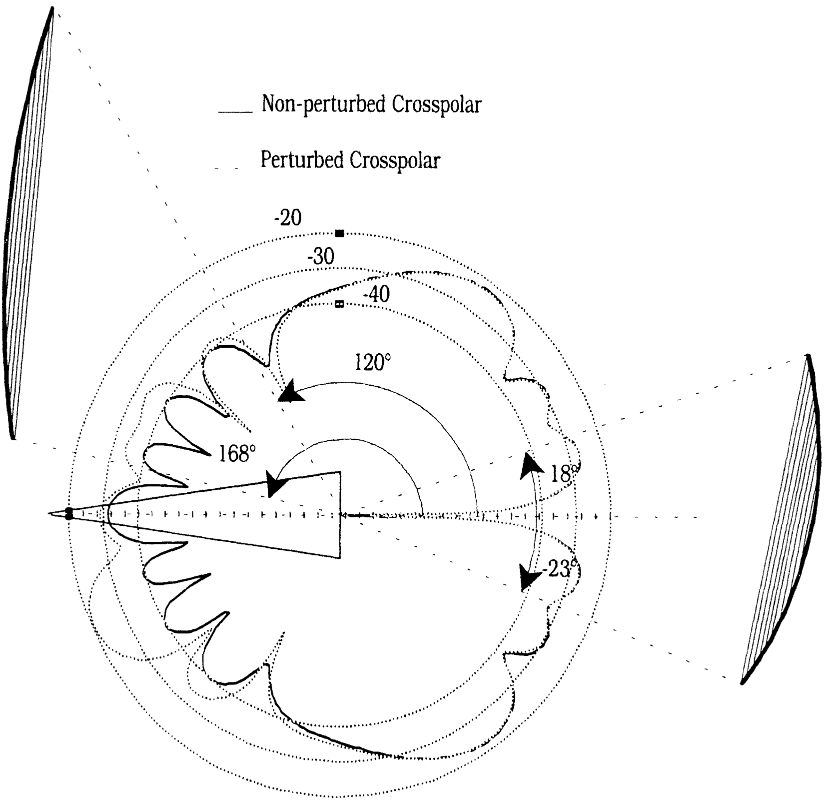

Fig. 5 – Comparison between the radiation pattern (dBi) of the cross-polar field component for the isolated horn (non-perturbed) and the cross-polar field component arising because of the interactions with the subreflector of the dual Gregorian antenna (perturbed); $\varphi = 45°$.

4 Conclusions

A software package for the analysis of circular corrugated horns radiating in presence of nearby perfectly conducting structures has been presented. The software implements a hybrid method which combines several techniques (MM, CFIE and UTD), each suitable for the analysis of a particular region of the problem space.

The computational cost of the problem is minimized by exploiting a perturbative approach to correct the solution relative to the isolated horn with contributions due to the interactions with nearby conducting structures. This approach is particularly convenient for the analysis of circular horns used as feeder in reflector antenna systems where unwanted interactions are mainly due to field diffracted by the reflector's edges and are responsible for increasing the cross-polarization level.

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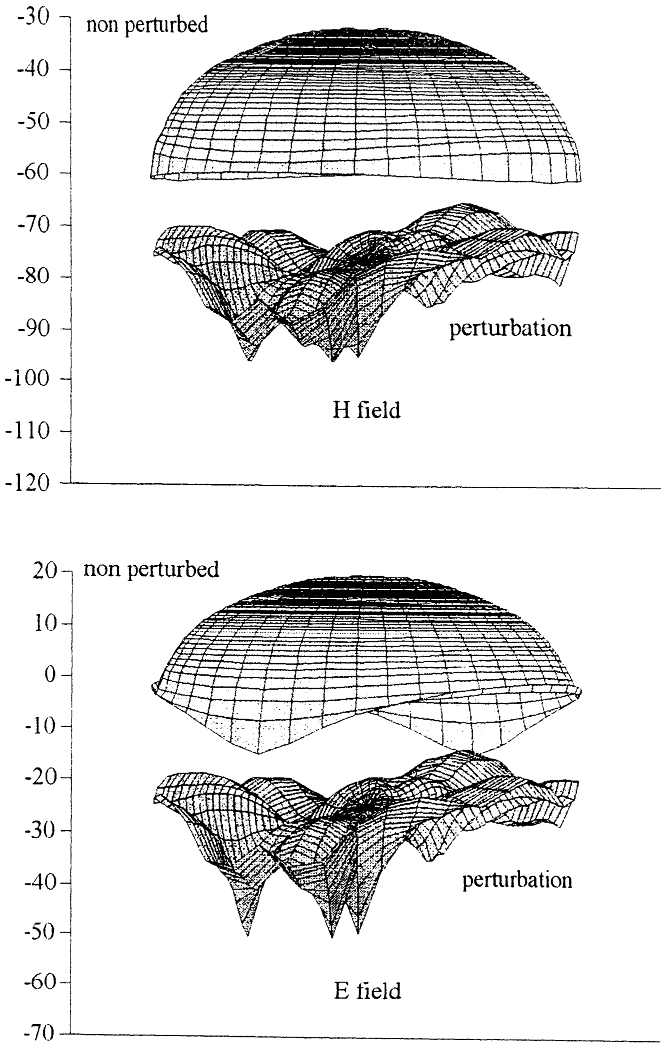

Fig. 6 – Comparison between amplitude (dB) distribution over horn aperture of non-perturbed field and field induced by diffraction (perturbation); elevation=0°, azimuth= – 60°.

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