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Optimal prestress strengthening of a masonry lighthouse for static and earthquake loading

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Abstract

The optimal prestress reinforcement of civil engineering structures is analyzed in this work. A modal analysis framework is assumed for the dynamic, aseismic design. The prestress action is modelled by means of induced temperature fields in the prestress cables. The optimal choise and the optimal placement of the tendons are treated by a ground (sub) structure technique. The proposed method is illustrated by means of numerical examples.

1 Introduction

Reinforcement of structures with prestressing elements, which modify the stress distribution and permit a more beneficial use of the materials is a widely adopted method. Especially for existing structures and for masonry buildings this method is advantageous, as it can be applied by minimal disturbance of the structure and its users, it is removable as it is required from every intervention on monuments and it is usually adopted by architects and archaeologists involved in these projects. A method suitable for the optimal design of prestressing reinforcement and which can easily be integrated in a finite element based, structural analysis model is developed and tested here.

For the static analysis the optimal prestressing problem is formulated as an optimal design problem with design variables the prestressing forces and subsidiary constraints the ones imposed by the stress and strength requirements for the existing structure, plus the technological restrictions posed by the prestressing system [1]. The dynamic analysis could, in principle, follow the same lines of modelling. Nevertheless, for civil engineering structures, the uncertaincy and the statistical nature of the loading sequence has led to the wide adoption of the modal analysis technique, which is based on appropriate earthquake design spectra as the more appropriate design method. Accordingly, the optimal prestressing problem for dynamic loads is formulated here for the modal structural analysis method. The finite element method is used for both static and dynamic analysis, since it permits a detailed modelling of all structural and material details of even complicated structures, including massive masonry structures.

Since most general purpose finite element programs do not allow for the introduction of initial prestresses, but most of them have a thermal loading option, the method proposed here incorporates a transformation that uses, instead of the prestressing forces, the element temperature as design variables. This technique has been used by various researchers for the modelling of smart, or intelligent structures [2].

The last point addressed in this paper is the one of the optimal choise, including the number and the placement of the reinforcing elements. The ground structure approach is used for this purpose. It has been applied for the solution of difficult, topology optimization problems [7] and it is also appropriate for the studied application, as it will be discussed later on. In addition, due to the nature of the problem, the consideration of a rather rough ground structure is required, so the size of the problem is not prohibitive large.

The layout of the paper is as follows: The optimal prestress problem is formulated for finite element discretized structures and for static and dynamic (modal) analysis in the next section. The induced (auxiliary) temprerature technique is also discussed there. In section four the ground structure approach of the optimal tendon choise and placement problem is studied. The applicability of the proposed approach is demonstrated in the last section by means of numerical examples. This method is also applied on a preliminary restoration study which concerns the masonry lighthouse of the venetian harbour of Chania.

2 Formulation of the Optimal Prestress Problem

The optimal prestress problem is formulated as a general nonlinear constrained optimization problem, which has the form:

$$\min_{\mathbf{A}\in\mathcal{A}_{ad}} F(\mathbf{A}) \tag{1}$$

where **A** is the vector of design variables, which characterize the prestressing elements, $F(\mathbf{A})$ is the cost function and both structural response and various constraints are included in the admissible design space $\mathcal{A}_{ad}$, i.e.

$$\mathcal{A}_{ad} = \{\mathbf{A} \in \mathbf{R}^n \quad | \quad g_j(\mathbf{A}) \leq 0, j = 1, \ldots, n_g, \\ h_k(\mathbf{A}) = 0, k = 1, \ldots, n_h, \\ A_i^L \leq A_i \leq A_i^U, i = 1, \ldots, n\} \tag{2}$$

In general the stress or displacement inequality constraints, which constitute the goal of the prestressing restoration, are included in the first set of (inequality) constraints in (2). The equality constraints in (2) usually count for the requirement that the applied tendons are arranged in groups of equal design (equal prestressing force and cross–section). The last set of block–type constraints in (2) correspond to the technologically induced constraints (here avoidance of local failure at the fastening regions is taken into account).

Note that in the above formulated problem the relation of the structural response parametrized by the prestress design variables is solved for the required quantities (stresses, displacements etc.) and the resulting expression is inputed into the inequality constraints, thus the state variables of the examined structural system are not whown in the concise writting of (2).

A quadratic cost function of the prestress design variables is used in (1). This way the minimum cost prestress restoration is sought. Moreover, for numerical efficiency, the first set of inequality constraints are removed from the design space description (2) and are taken into account by a penalty function technique. Usually stress inequality constraints are considered. In this case the number of subsidiary constraints is very large, compared with the design space dimension. Accordingly a feasible point of (2), which is required as a starting point for the most numerical optimization schemes, either can not be determined easily or it does not exist at all. Finally the considered objective function in (1) reads:

$$F(\mathbf{A}) = \frac{1}{2}\mathbf{A}^T\mathbf{W}\mathbf{A} + \frac{z}{R}\ln\left[\sum_{j=1}^{n_g^*} \exp(Rg_j(\mathbf{A}))\right] \tag{3}$$

Here **W** is an appropriately choosen, symmetric and positive definite weighting matrix and the second term in the r.h.s. is the penalty term for the stress inequality constraints. In particular, following [3] z is the participation coefficient of the penalty function, R is a coefficient which is related to the number of violated constraints, n_g^* is the number of violated inequality constraints and $g_j, j = 1, \ldots, n_g$ are the functions which describe the constraints (cf. (1)).

For further reference we note that function (3) is differentiable and, if $g_j(\mathbf{A})$, $j = 1, \ldots, n_g$ are convex functions, the cost function (3) is a convex function too. Thus classical mathematical optimization techniques can be used for the solution of problem (1), with $\mathcal{A}_{ad}$ defined according to the previous discussion by means of the last block-type constraints of (2).

In the rest of this section the structural response modelling for static and dynamic (modal) analysis are outlined and the fictitious temperature modelling of the prestressing is discussed.

2.1 Static Analysis

A linearly elastic structure discretized by the direct stiffness or displacement finite element method is considered. In this framework the discrete equilibrium equations read:

$$\mathbf{Ku} = \mathbf{p} + \mathbf{GK}_0\mathbf{e}_0 - \mathbf{Gs}_0. \tag{4}$$

In (4) $\mathbf{u}$ denotes the vector of nodal displacements (degrees of freedom), $\mathbf{K}$ is the effective stiffness matrix, $\mathbf{K}_0$ is the natural stiffness matrix, $\mathbf{G}$ is the equilibrium matrix, $\mathbf{p}$ is the vector of externally applied loading and $\mathbf{e}_0$, $\mathbf{s}_0$ are the vectors of initial element strains and stresses. By assumming the existence of sufficient support restrictions the effective stiffness matrix $\mathbf{K}$ is symmetric and possitive definite. Moreover initial stresses and strains of one-dimensional rod elements will be used in the sequel for the modelling of the mechanical behaviour of the prestressing tendons.

Recall that by solving (4) with respect to the nodal displacements $\mathbf{u}$, they are found to be linear relations of the initial strain or the initial stress vectors. Accordingly strains and stresses which are generated in the post-processing step of the finite element method are also linear functions of the same quantities.

2.2 Dynamic, Modal Analysis

For the treatment of dynamic load cases we adopt here the modal, spectral analysis technique. This method is widely adopted for civil engineering structures and is proposed by the majority of modern design specifications. In this way the statistical nature of the earthquake induced loading is taken into account.

On the assumption of a linearly elastic system the eigenvalues and eigenmodes of the discretized system are first calculated. For large scale problems this task is accomplished by the Lanczos method which is the most effective one, since it permits the exploitation of the sparsity of the stiffness and mass matrices of the system [4].

From each eigenmodal vibration (i) the maximum stress field is obtained if the structure is subjected to the external loading

$$F_{i.max} = \psi_i S_{ai} \mathbf{M} \phi_i, \tag{5}$$

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where ψ_i is the participation factor of the i–th eigenmode, S_{ai} is the value given by the acceleration spectrum, $\mathbf{M}$ is the mass matrix of the structure and ϕ_i is the vector of the i–th eigenmode. We recall here that the eigenmodal participation factor depends on both the earthquake and the structural characteristics.

The superposition of the eigenvibration quantities follows one of the accepted statistically based methods. The complete quadratic method is used here [5], [6], which reads:

$$F_{max} = \sqrt{\sum_{i=1}^{n} F^2_{i.max}}, \quad i = 1, \ldots, n. \tag{6}$$

Here n is the number of significant eigenmodes which are considered in the structural analysis problem.

Alternative, more elaborated methods of superposition for the eigenmodal quantities have not been considered here. Their implementation within the proposed scheme and their effect on the results is a subject of further investigation.

2.3 Induced Temperature Modelling

The prestressing action is modelled by an induced, fictitious temperature field. This way is accepted by the most general purpose structural analysis software, including the MSC/NASTRAN system which has been used for the solution of the numerical examples presented in thispaper. More details of this technique is given in [1], [8].

3 Topology and Geometric Prestress Design Aspects

Besides the prestress force optimal design problem, which has been considered in the previous sections, the optimal layout of the prestressed tendons and their optimal placement are also of importance for applications. A modification of the ground structure (or structural universe) approach, which has been prooven to be effective for the solution of topology optimization problems [7], is introduced here for the treatment of the problem [8].

Following recent advances in optimal structural design, we may consider the following classes of problems [9]:

- *topology optimization*, which is related to the determination of the optimal layout of the prestressing system (such as the position and the number of tendons),

- *prestress forces optimization*, where the optimal cross-section and the prestressing forces for a given tendon system are considered, and
- *geometric optimization*, where the optimal placement (coordinates) of the fastening points for all tendons of a given system (i.e. layout and prestressing forces are given) is considered.

The method proposed in this paper treats simultaneously the first two classes of problems. A space of *potential prestressing systems* is introduced, which is defined as the prestressing systems which can be constructed by choosing tendons among all elements of a sufficiently dense grid of potential tendons. The method is analogous to the *ground structure* or *structural universe* approach which is used for the topology (layout) optimization of new structures, with the difference that in our case only the additional prestressing elements are subjected to changes in the course of optimization. The optimal prestressing algorithm of the previous section automatically chooses the tendons which are needed for the construction of the optimal prestress system.

Note here that technological limitations concerning the constructional feasibility of the prestress reinforcement and the uncertaincy of the loading and the material parameters of the structural system permit us use relatively coarse grids in the ground prestressing system. This is especially true for the problem of prestress restoration of historical buildings or monuments, which has been the main task of this work. Moreover, in this framework the geometric optimization problem is solved within the accuracy of the choosen ground prestressing system [8].

4 Numerical Examples

4.1 Cantilever reinforcement. Ground structure approach.

Let us consider a cantilever, subjected to static loading and modelled by four-node, plane stress finite elements, as shown in Fig. 1. For the elastic material of the wall we consider an elasticity modulus $E = 1.4\ 10^8 kg/m^2$ and a Poisson's ration $\nu = 0.15$. For the prestressing cables we consider the constants $E_{st} = 2.1\ 10^{10} kg/m^2$, $\nu = 0.15$ and a thermal expansion coefficient $\alpha_t = 8.3\ 10^{-6}/k^0$. Note that the thermal expansion coefficient may have an arbitrary value without affecting the results, since thermal loading is used as a fictitious way for the introduction of the prestressing.

Three ground prestress systems, with 14, 36 and 55 potential tendons have been considered, as it is shown on Figs. 2 α, β and γ. The goal of the restoration was the reduction of the tensile stresses of the cantilever. After solution, the optimal prestressing systems have been calculated, as it is schematically shown in Figs. α', β' and γ' Theoretically more elements can be added into the ground prestress system and a finer solution of the problem can be calculated. Nevertheless, as it has been discussed previously, one should consider a reinforcement system which is compatible with all uncertainties of the problem and which is technologically feasible.

4.2 Preliminary Strengthening Design of a Masonry Lighthouse

A preliminary investigation of the effect of prestressed tendons on the structural response of a masonry lighthouse at the venetian harbour of Chania is performed. The finite element analysis model and the dynamic, response spectrum analysis have been presented in [10]. Four tendons, as shown on Fig.3 are considered. The first, second, fourth and seventh significant eigenmodes are included in the analysis (see [10], p.99). The influence of prestressed tendons with prestressing forces equal to $1,9\ 10^{12}$, $5,72\ 10^{12}$, $2,1\ 10^{12}$ and $9,53\ 10^{12}$ (according to the numbering of Fig.3) on the circumferential stress component (σ_x) and at the places of tendons 1,2 and 4 are given in Figs 4,5 and 6. Since the component of the considered eigenmodes are not significant along the prestressing lines, the examined prestressing is not very effective and other reinforcing measures should be investigated.

Acknowledgements

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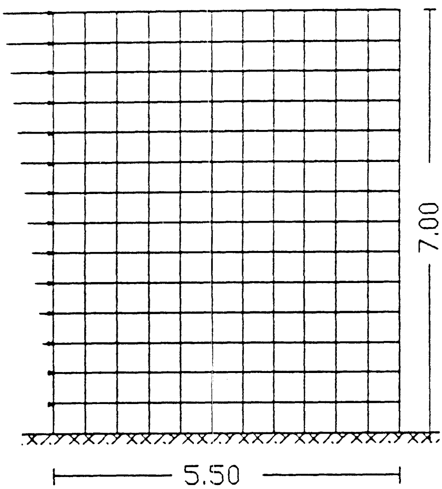

Figure 1: Finite element model of a cantilever.

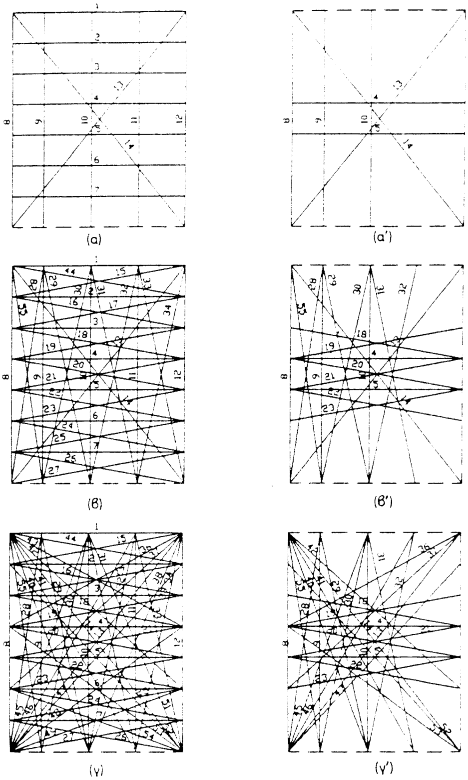

Figure 2: Ground prestress systems and optimal prestress systems for the cantilever of Fig. 1.

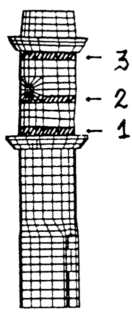

Figure 3: Four peripheral tendons and finite element discretization of the Lighthouse.

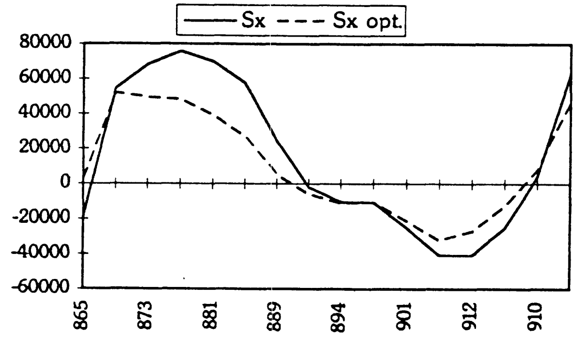

Figure 4: Stresses σ_x before and after prestressing. Cross–section 1.

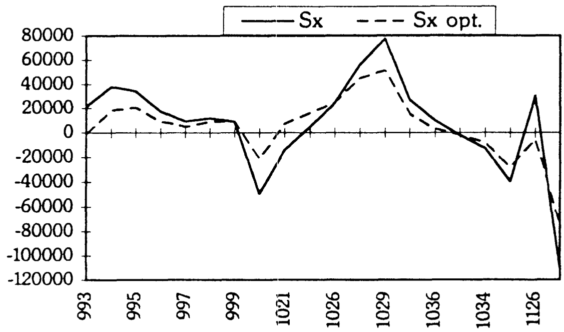

Figure 5: Stresses σ_x before and after prestressing. Cross–section 2.

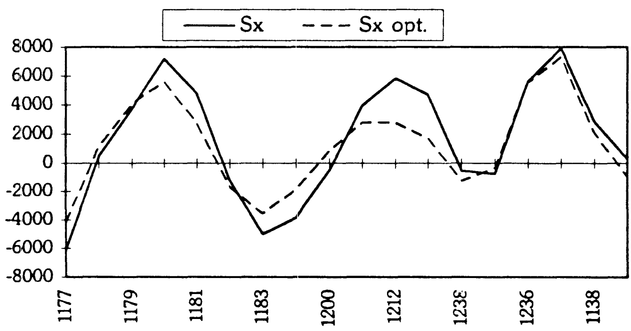

Figure 6: Stresses σ_x before and after prestressing. Cross–section 3.