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A theoretical graphic approach to the stability of the Modelhss equations

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Abstract

A methodology [8,9] allows a construction of mainly non-linear equations with which several dynamic processes can be modelled which are defined within an ecosystem by means of a relation like: "$y(t) = F(x_1(t), x_2(t), \ldots, x_n(t))$" (1). These relations are generally proposed in a hypothetical way and mathematical expressions are determined from the experimental data of the variables. This methodology is based on the construction of a regular language [8] and this has been proved successfully within modelling [2,3]. The construction of equations is carried out through the program *Modelhss* [4] and in every execution it is possible to obtain groups of equations that express to a greater or lesser extent the relation (1) and this provides the modeller with more information.

In obtaining groups of equations, we want to carry out studies that help select particular equations as opposed to others. One of the criteria that the majority of modellers want to obtain is the selection of those equations, which are stable as opposed to the disruptions produced in variables; this is a concept we will formalise. On the one hand in this sense we start with a theoretical study of stability and control of an equation of the type (1). On the other hand, a graphic environment complements this theoretical study. This environment allows the visual analysis of the concept of stability starting from simulations of the disruptions.

The studies carried out in this article assume the continuation of the methodology *Modelhss* because they theoretically and graphically deal with the stability of an equation that models an ecological process.

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1 Introduction

Initially it is supposed that in modelling of an ecologic system, the study of a certain phenomenon is described by the values of a variable y(t) of which its evolution in an interval of time I=[T_1, T_2] is kwon [6]. That is to say, in I experimental data of the variable y(t) are kwon.

We consider as well a set of variables, $\{x_i(t)\}_{i=1}^{m}$ that have on influence an the evolution of said phenomenon, that is why it is assumed hypothetically that "$y(t) = F(x_1(t), x_2(t),...., x_n(t))$" (1). For the experimental data obtained from the interval of time, we will consider modelling of said equation starting with the methodology published in [4,8]. Within this methodology, we consider first a regular language generated from vocabulary of different orders which form a lexicon $L=\left\{\bigcup_{i=1}^{m} V^i\right\}$ [9], which is applied to the variables, $L(x_j)=\left\{\bigcup_{i=1}^{m} V^i(x_j)\right\}$.

Starting with this mathematical language equations are built that model y(t) from the experimental data obtained of said variable and of those that have an influence on its development using the program *Modelhss* which was developed within the quoted methodology [4]. Generally a modelling of the equation (1) will be written as

$$y=A_1 F_1(x_1)+..........+A_k F_k(x_k), \text{ where } A_i\in R, \text{ y } F_j\in V^{j_i}, j_i\in\{1,2,..m\}. \quad (2)$$

In this article the authors want to initiate a theory of stability of the models obtained through the equations of type (2). With this theory it will be possible as well to visualise graphically the resulting variations.

Former theoretical studies have been carried out for systems of risk in [7], but they studied theoretically the aspects of risk of equations of the type (1) as basis of the obtained adjustment (correlation coefficient) and of the variations that involve a risk.

For the study of stability of the equations of type (2) the authors develop a theory within which they will consider a stable equation where as the small variation in one (several) variable result in small variations in the obtained model [1]. In this sense properties of an equation are studied as compared to the resulting variations within the experimental data which are oriented to formalise this kind of concept.

2 Stability of an equation with respect to its variables

The following equation is considered:

$$y(t)=A_1 F_1(x_1(t))+......................+A_k F_k(x_k(t)) = M_y(x_1,x_2,...,x_n)\ t\in I, \quad (3)$$

model of the ecological process defined by the variable y(t). The value of the variable x_i and y denotes $x_i^{t_j}$, y^{t_j} respectively in the time t_j.

2.1 Definition: ε-rate simulated variation of the data

The ε-rate simulated variation of a variable x in a time t_j is defined by the variation which comes out of the sum of it with the value of a normal distribution with an average zero and deviation $\varepsilon \cdot x$. These values will be represented as:

$$\bar{x}_{i,\varepsilon}^{t_j} = x_i^{t_j} + N(0, \varepsilon.\, x_i^{t_j})$$

An ε-rate variation of the data is a simulated ε-rate variation of the variables that take part in the modelling in an interval of time considered in the construction of the model.

2.2 Definition: ε -variation of the model defined by the equation M_y

ε-variation of the model is the name of the value of equation (3) for an ε-rate simulated variation of the data, that is to say

$$\bar{y}_{\varepsilon}^{t_j} = M_y(\bar{x}_{1,\varepsilon}^{t_j}, \bar{x}_{2,\varepsilon}^{t_j}, \ldots\ldots\ldots\ldots, \bar{x}_{n,\varepsilon}^{t_j})$$

2.3 Definition: control of variability of the data

The interval predetermined by the modeller in which the variation of the data is admitted is called *range* of variability of the data, D_ε. An interval will be of the form $D_\varepsilon = (0, M_\varepsilon]$.

2.4 Definition: function of stability – $E(\varepsilon)$

Let us suppose an equation of type (3), $M_y(x_1, x_2, \ldots, x_n)$ $t \in I$,as well as a sample of the size M of the variables that take part in the equation. For all the ε-variations of the model, $\varepsilon \in I\varepsilon$, the function of stability $E{:}R \rightarrow R$ is defined as:

$$E(\varepsilon) = \frac{\sum_{j=1}^{M} \left| \frac{\bar{y}_{\varepsilon}^{t_j} - y^{t_j}}{y^{t_j}} \right|}{M}$$

2.5 Stability of an equation regarding a set of variables

The concepts given in the previous paragraphs are valid when one only wishes to analyse the stability of an equation regarding a variable or a set of variables.

For a variable it will be sufficient to consider an ε-rate simulated variation of a variable x_i in the interval of time considered and to define an ε-variation of the model regarding the variable x_i, which is defined by the equation M_y.

2.5.1 Definition: ε-variation of the model regarding the variable x_i

The ε-variation of the model is defined with regard to the variable x_i in the equation (4) M_y to the values of the dependent variable.

$$\overline{y}^{t_j}_{\varepsilon,x_i} = M_y\,(x^{t_j}_{1,\varepsilon}, x^{t_j}_{2,\varepsilon}, \ldots \overline{x}^{t_j}_{i,\varepsilon}, x^{t_j}_{i+1,\varepsilon} \ldots\ldots\ldots, x^{t_j}_{n,\varepsilon})$$

2.5.2 Definition: function x_i – stability – $E_{x_i}(\varepsilon)$

This is the function of stability defined from the ε-variation of the model with regard to the variable x_i by,

$$E(\varepsilon) = \frac{\sum_{j=1}^{M} \left| \frac{\overline{y}^{t_j}_{\varepsilon,x_i} - y^{t_j}}{y^{t_j}} \right|}{M}$$

The concepts for a sub-set of variables is generalised by varying ε-rate all the variables and by introducing its variations to the variation of the model the same way as carried out for all the variables.

2.5.3 Definition: ε -variation of the model with regard to the variable $\{x_i\}_{i=1}^{h}$

The ε-variation of the model is defined with regard to the variable$\{x_i\}_{i=1}^{h}$ within the equation (3), M_y to the values of the dependent variable

$$\overline{y}^{t_j}_{\varepsilon,x_i} = M_y\,(\overline{x}^{t_j}_{1,\varepsilon}, \overline{x}^{t_j}_{2,\varepsilon}, \ldots \overline{x}^{t_j}_{k,\varepsilon}, \ldots\ldots\ldots, x^{t_j}_{n,\varepsilon})$$

2.5.4 Definition: function $\{x_i\}_{i=1}^{h}$ – stability – $E_{\{x_i\}_{i=1}^{h}}(\varepsilon)$

It is the function of stability defined from the ε-variation of the model regarding the variables $\{x_i\}_{i=1}^{h}$.

$$E(\varepsilon) = \frac{\sum_{j=1}^{M} \left| \frac{\overline{y}^{t_j}_{\varepsilon,\{x_i\}_{i=1}^{k}} - y^{t_j}}{y^{t_j}} \right|}{M}$$

2.6 Types of stability

Now the stability of the equation is defined with regard to the similarity of its variations to the ones produced in a certain function H(ε).

H(ε) will be a real continuous function in the domain of variability of the data D_E, $H(\varepsilon) \in C^{0}(D\varepsilon)$. Let us suppose furthermore a function of stability E(ε) defined within an equation M_y with regard to an ε-variation of the model.

2.6.1 H(ε) – range of stability

The points of the control of stability whose function of stability is minor or the same as the function H(ε) are called range of stability

$$R_{H_\varepsilon} = \{\varepsilon \in D\varepsilon \;/\; E(\varepsilon) \leq H(\varepsilon)\}$$

2.6.2 H(ε) –stable equation

An equation M_y is H(ε)-stable with regard to the variation of a set of variables if and only if the range of stability and the domain of stability coincide, $R_{H_\varepsilon} = D\varepsilon$.

Normally, we can consider as functions H(ε) the set of straight lines and the set of curves that pass through the following origin:

$$H(\varepsilon) = m\varepsilon \,,\; H(\varepsilon) = m\varepsilon^2,\; H(\varepsilon) = m\sqrt{\varepsilon}\ .$$

3 Applications to models

Now we will illustrate all the concepts that were presented previously by means of two models. Like this we will be able to obtain conclusions about its stability.

3.1 Floral model

The reproductive model of *Cistas albidus* [5]. In this model we consider the variables humidity, temperature, rainfall, number of hours of sun, number of gems which are represented by the variables (**H,T,PLU,NHS,NGEM**). **CFLOR** is the variable of the observed floral growth. The observed data and the equation of the model correspond to the following graphic representation:

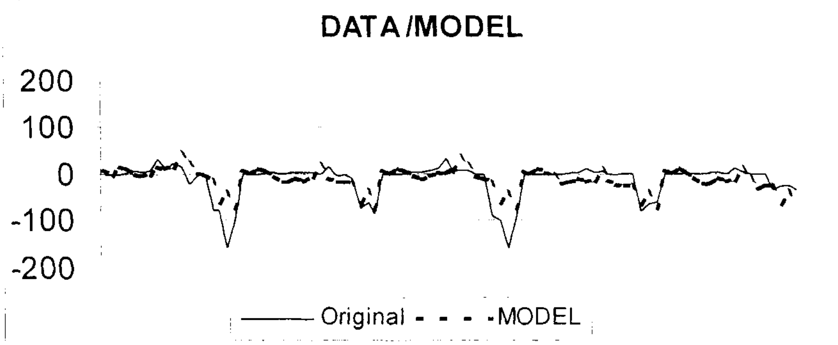

Figure 1: Observed and model data.

Now we set out to simulate several values of the variables H,T,PLU,NHS,NGEM, with the values ε∈(0, 0.3]. Like this we will obtain different graphs of which we have selected as representatives $y^{t_j} / y^{t_j}_{0.01}$,, $y^{t_j} / y^{t_j}_{0.2}$.

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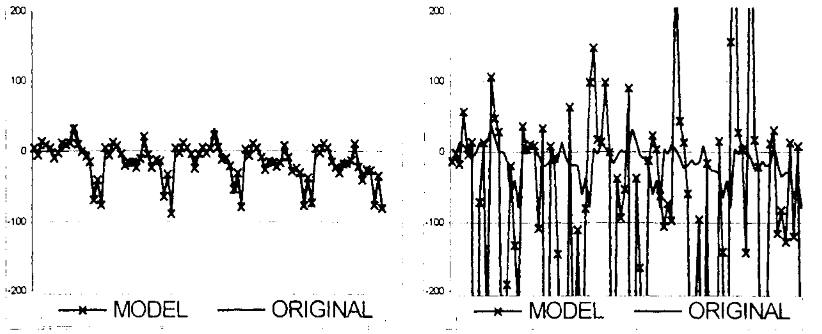

Figure 2: Relationship $y^{t_j}/y^{t_j}_{0.01}$ and $y^{t_j}/y^{t_j}_{0.2}$.

The equation obtained by the *Modelhss* is

y(t) = M_y (H,T,PLU,NHS,NGEM)=4.693359 +
0.017653*((0.211283*H-13.8874)^2)^2 -
0.075472*((0.407963*T-4.32471)^2)^2 -
0.022558*((2.54041*PLU+3.35925)^2)^2 -
0.048227*((1.55799*NHS-18.1045)^2)^2 +
85.0515*ln(abs(ln(abs(0.017724*NGEM+3.3391)))).

Within this sequence we clearly observe the *no-proportion* between the variations of the variables (ε) and the variations of the equation $\bar{y}^{t_j}_{\varepsilon}$. But in any case we focus the study of stability on the relation to the function E(ε).

With the following graphic representation we represent the relation $\bar{y}^{t_j}_{\varepsilon}/\varepsilon$. For this objective the values of E(ε) for all the values of $\varepsilon \in (0, 0.3]$ have been counted.

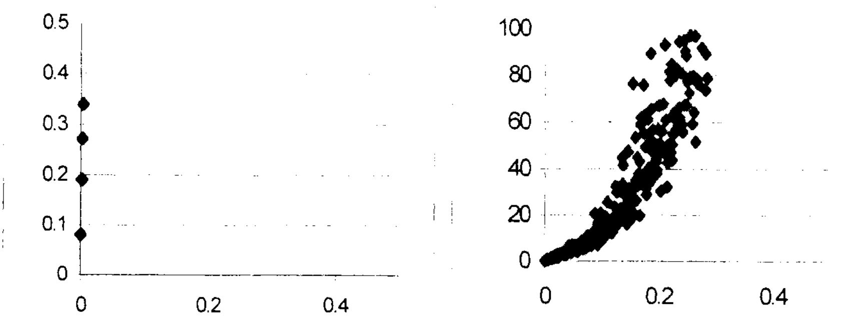

Figure 3: E(ε) with two scales.

This is an example within which we can observe the poor stability of the equation within the studied range of possible values of the parameters.

We also observe the necessity of changing the scale of the axis y to be able to appreciate the behaviour of the function E(ε).

3.2 Populations of acacias

We have taken the data corresponding to a population of Acacia Melanoxylon of a mountain of Azul (Province of Buenos Aires) [2].

In this model only two variables take part, **D** and **DH** with which we obtain the value of the height H. The data obtained and modelled are represented graphically in the next figure. The equation obtained is:

H=13.8595+30.8945/(0.28273***D**+0.006232)-45.2536/(0.020552***DH**+1.02577), where D represents the diameter and DH represents the relationship diameter/height.

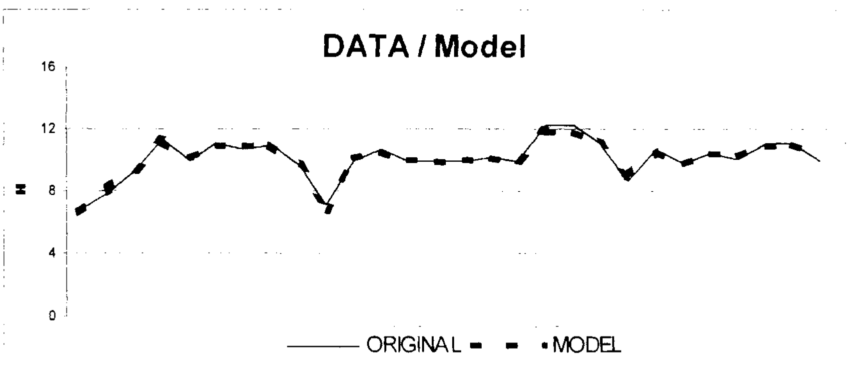

Figure 4: Observed and modelled data.

Here we can observe that the approximation possesses a coefficient of superior determination. We will repeat the same steps as in the previous example with the graphs corresponding to $y^{ij} / y^{ij}_{0.01}$, $y^{ij} / y^{ij}_{0.2}$:

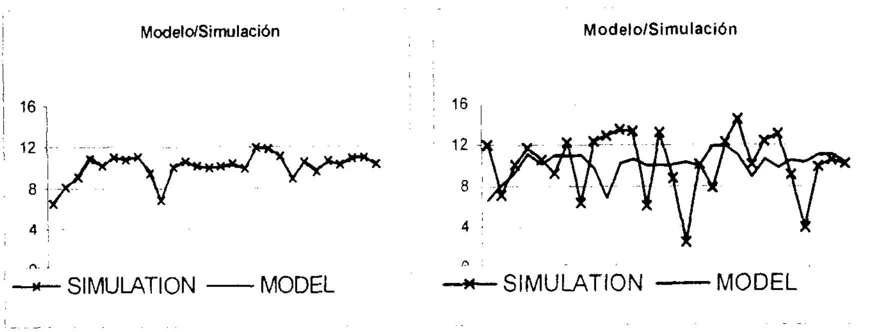

Figure 5: $y^{t_j}/y^{t_j}_{0.01}$, $y^{t_j}/y^{t_j}_{0.2}$.

In the following graph we represent the relation $\overline{y}^{t_j}_{\varepsilon}/\varepsilon$. For this we have counted the values of E(ε) for all the values of $\varepsilon \in (0,0.3]$

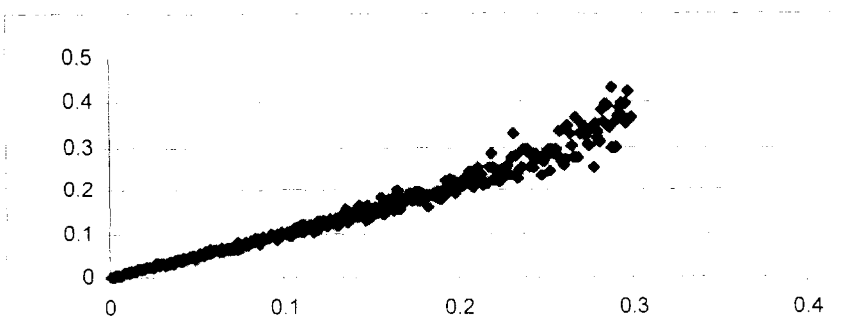

Figure 6: E(ε) in the example of *acacias*.

As can be observed from the two examples of degree at first sight of stability it is much better in this second example than in the first.

Now we represent within the same scale the function E(ε) of both examples.

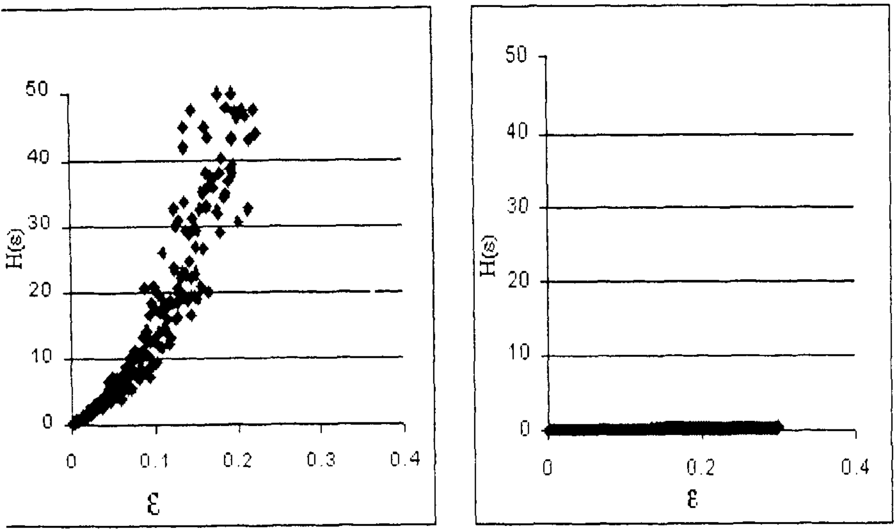

Figure 7: E(ε) in floral growth and acacias.

4 Conclusion

The methodology presented allows on one hand to consider a level of variation of the independent variables and its effect on the resulting variation in the model. On the other hand, this study lays the foundations and permits the specification of a graphic environment that makes it easier for the modeller to chose one equation or another.

This environment will allow to choose which variables are sensitive to change, it allows to choose their ranges of variation and it will also allow to choose the function H(ε) that best adjusts itself to the case which is the subject of study.

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