Transactions on Modelling and Simulation vol 19, © 1998 WIT Press, www.witpress.com, ISSN 1743-355X

Applications of a 3D BEM model for predicting stray capacitance in high voltage measuring devices

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Abstract

The paper presents an application of a 3D Boundary Element computational procedure to the estimation of the stray capacitance of a HV divider. The numerical model uses as unknowns the scalar electric potential and the normal component of its gradient; it allows the computation of the electric field distribution in a domain including volume charges having a known potential.

1 Introduction

Electric power equipments need dielectric tests to ensure the capability of insulating materials to withstand possible overvoltages. The most important test is undoubtedly the lightning impulse one: a transient impulse voltage with a 1.2 μs rise time, a 50 μs time to half value and a peak value up to some megavolt is applied between the insulated parts and the ground. In order to measure the transient voltage waveform and verify the values of the impulse parameters, resistive or capacitive dividers are generally used. The calibration of these devices is made by comparison with reference dividers certified by national metrological institutes. IEC standards [1] prescribe limits for the uncertainty of the reference divider which are difficult to be achieved. In particular, the dimensions of voltage dividers (up to 4 m) require complex measurements to verify the absence of stray parameters which can influence the global uncertainty.

Taking into account the measurement problems, some authors have proposed the use of numerical models to evaluate the electrical parameters of high voltage dividers. Between them, it is worth mentioning the approach proposed in [2,3] where the charge simulation method is used to evaluate the electric field generated by the device.

The aim of this work is to present a method for the computer simulation of the electric field generated by a reference divider, using the Boundary Element Method (BEM), in order to evaluate the stray capacitances. The knowledge of these parameters will successively allow to work out an accurate equivalent circuit of the divider able to simulate its dynamic behaviour. The BEM is particularly apt for this kind of application because it does not require the introduction of fictitious boundary conditions and moreover the discretization phase is limited to the surfaces of the electrodes or dielectrics [4,5].

First, the numerical method have been verified, analyzing a model structure constituted of two electrodes and a dielectric slab. The comparison with measurements has been performed on the capacitance of the system obtaining a satisfactory accuracy. The model has been then applied to the analysis of a HV divider in order to predict the electrostatic field in the surroundings and its stray capacitances. The results of this simulation, together with the comparison with the experimental results, are presented and discussed.

2 Numerical model

The simulation of high voltage dividers by means of numerical models requires the solution of the electrostatic field generated by the electrodes. The field analysis is performed introducing the electric scalar potential φ ($\boldsymbol{E} = -\nabla\varphi$) and applying the Green's theorem [6]. The value of the potential in a generic point P inside homogeneous volume V is so expressed as a function of surface integrals over the boundary Ω and a volume integral which takes into account possible volume charges ρ:

$$\zeta\varphi(P) = \int_{\Omega}(\nabla\varphi \cdot \boldsymbol{n})\psi ds - \int_{\Omega}\varphi(\nabla\psi \cdot \boldsymbol{n})ds + \int_{V}\frac{\rho}{\varepsilon}\cdot\psi dv \tag{1}$$

In Eqn. (1), $\psi = 1/4\pi r$ is the Green's function, being r the euclidean distance between computational and source point, ε is dielectric constant of the medium and $\boldsymbol{n}$ is the unit vector normal to Ω; $\zeta = 0.5$ when P is on Ω and $\zeta = 1$ when P is inside V.

Introducing the BEM approximation, the surfaces which separate media having different electric constants are divided into triangles and the values of the potential φ and its gradient are assumed to be constant on each element. Eqn. (1) is so rewritten in the discretized form as:

$$\zeta\varphi(P) = \sum_{j=1}^{N}(\nabla\varphi_j \cdot \boldsymbol{n}_j)\int_{\Omega j}\psi ds - \sum_{j=1}^{N}\varphi_j \cdot \int_{\Omega j}(\nabla\psi \cdot \boldsymbol{n}_j)ds + \int_{V}\frac{\rho}{\varepsilon}\cdot\psi dv \tag{2}$$

being N the number of triangular elements Ω_j into which Ω is subdivided.

In addition to Eqn. (2), interface conditions are introduced to link the values of the potential and its gradient between volumes having different electric characteristics. For the generic triangle i dividing volume (a) and volume (b), the following relationships holds:

$$\varphi_i^{(a)} = \varphi_i^{(b)} \tag{3}$$

$$\varepsilon^{(a)}\left[\nabla\varphi_i^{(a)} \cdot \boldsymbol{n}_i^{(a)}\right] = -\varepsilon^{(b)}\left[\nabla\varphi_i^{(b)} \cdot \boldsymbol{n}_i^{(b)}\right]$$

The second equation of (3) imposes the continuity of normal component of electric field, taking into account that $\boldsymbol{n}_i^{(a)} = -\boldsymbol{n}_i^{(b)}$. The unknowns of the problems are so reduced to the potential $\varphi_i = \varphi_i^{(a)} = \varphi_i^{(b)}$ and the normal component of the gradient considered with respect to volume (a) $\left[\nabla\varphi_i^{(a)} \cdot \boldsymbol{n}_i^{(a)}\right]$.

Having assumed constant values of the potential and it gradient over each triangle, the computation of the integral terms in equation (2) is reduced to the evaluation of $\int_{\Omega j}\psi ds$, $\int_{\Omega j}\nabla\psi ds$ together with the volume integral $\int_{V}\psi dv$. The computation of these integrals is numerically performed using ad adaptive quadrature rule based on Kronrod scheme [7]. Following this procedure, the number of integration points over the triangle is increased until convergence is reached, so that a good accuracy can be obtained without increasing too much the computational time. In fact, the number of points is lower for source points far from integration triangle, but it is strongly increased when distance r in the Green function decreases.

Once the terms of system matrix are computed, the linear system is solved using a direct method based on Gaussian elimination. The knowledge of the potential and the normal component of its gradient over each triangle allows the successive computation of the potential and the electric field in each point of the domain. In particular, the electric field is estimated evaluating the gradient of the scalar potential; as an example, the E_x component is given by:

$$E_x = -\frac{\partial\varphi}{\partial x} = \sum_{j=1}^{N} \varphi_j \left[\int_{\Omega_j} \frac{\partial^2\psi}{\partial x^2} ds\, n_x + \int_{\Omega_i} \frac{\partial^2\psi}{\partial x \partial y} ds\, n_y + \int_{\Omega_i} \frac{\partial^2\psi}{\partial x \partial z} ds\, n_z \right] - \sum_{j=1}^{N} \left(\nabla\varphi_j \cdot \boldsymbol{n}_j\right) \int_{\Omega_j} \left(\nabla\psi \cdot \boldsymbol{i}\right) ds - \int_{V} \frac{\rho}{\varepsilon} \left(\nabla\psi \cdot \boldsymbol{i}\right) \mathrm{dv} \tag{4}$$

where n_x, n_y, n_z are the component of vector $\boldsymbol{n}$ along the cartesian axis and $\boldsymbol{i}$ is the normal unit vector along x axis. Similar relationships can be derived for $\partial\varphi/\partial y$ and $\partial\varphi/\partial z$.

The second derivative of Green function are directly expressed from the coordinates of integration point (x, y, z) and source point (x_o, y_o, z_o) by

$$\left.\begin{aligned} \frac{\partial^2\psi}{\partial a^2} &= \frac{3\cdot(a-a_o)^2}{r^5} - \frac{1}{r^3} \\ \frac{\partial^2\psi}{\partial a\, \partial b} &= \frac{3\cdot(a-a_o)\cdot(b-b_o)}{r^5} \quad \text{with } a \neq b \end{aligned}\right\} \tag{5}$$

where $a = x$ or $a = y$ or $a = z$, $b = x$ or $b = y$ or $b = z$, and $r = \sqrt{(x-x_o)^2 + (y-y_o)^2 + (z-z_o)^2}$.

On the basis of the formulation here described, a numerical code has been worked out. An essential feature required by this package is the capability of describing and discretizing the complex structures of high-voltage (HV) devices. For such a purpose, a specific pre-processor has been developed. The mesh of 3D surfaces can be generated by translating and/or rotating a 2D grid previously prepared; the volumes which contain charge volume densities are described, in general, by 8 or 20 vertex hexaedra. An example of the discretization of a resistive HV divider is shown in Fig. 1.

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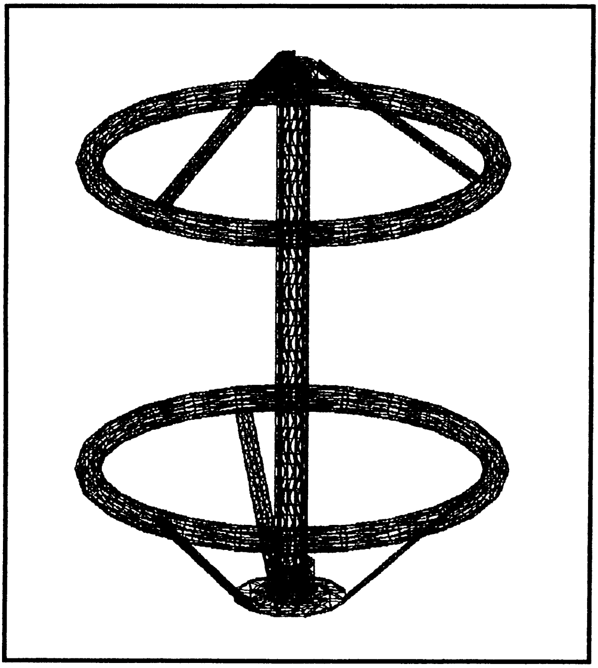

Figure 1 - 3D mesh of a HV divider

3 Analysis of model problem

The numerical model based on BEM formulation has been first tested on a model problem having a simple 3D geometry. One of the electrodes is the torus shown in Fig. 2, whose outer diameter is 172 mm; the other one is a square sheet of side equal to 0.5 m. The distance between the electrodes has been fixed at 2 mm and a dielectric slab with 6.6 relative dielectric constant has been inserted between them. In this structure, the influence of the surrounding structures on the capacitance measurement is much lower than in the dividers where the distance between the electrodes is higher. The measured value of capacitance is 95 pF, to be compared with the computed value of 81 pF. Taking into account the uncertainties of the parameters (dimensions and dielectric constant) and of the measurement of capacitance, the result of the simulation can be considered very satisfactory.

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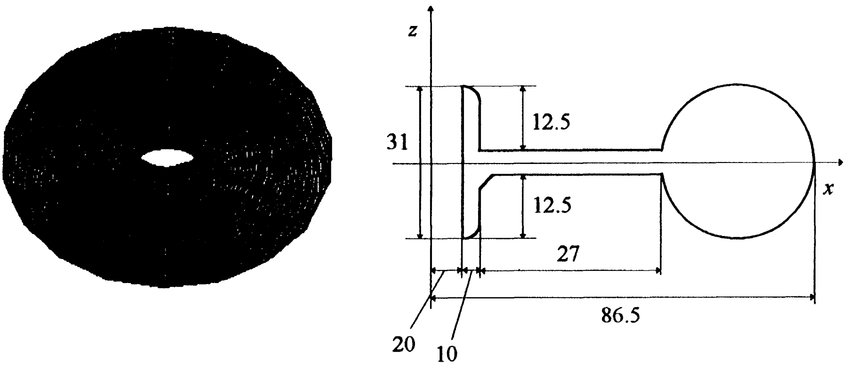

(a) (b)

Figure 2 - Electrode of the model problem:
(a) three dimensional view; (b) section with dimensions in millimiters

4 Experimental setup for field measurements

The measurement of the electric field between the electrodes of the HV divider has been performed using a miniature probe constituted of two parallel plates separated by a dielectric. The voltage induced between the plates of the probe is proportional to the component of the electric field normal to them. The small dimensions of the probe allow spot measurements which are required by the high gradient of the electric field. The calibration of the probe is preliminarily performed by measuring a known uniform electric field; the linearity of the response has also been verified in the range of values of the electric field to be measured.

The probe has been employed to map the electric field in the volume between the electrodes of the divider. Particular care has been devoted to avoid the inaccuracies introduced by the significant capacitance of the probe versus ground. In fact, when the electric potential of the probe is different from the ground one, the small derived currents alter the validity of experimental results. Therefore, for a fixed supply voltage, the electric potential of each electrode has been varied, until a zero potential is reached in the point where the probe is located. As shown in Fig. 3, the potential of the electrodes is varied moving the ground connection (point A) in the divider used to supply the device under experiment. The correct position is found when the current derived from the probe to ground has a minimum, but the evaluation of this position is a very complex task, also taking into account the step variation of the potentials.

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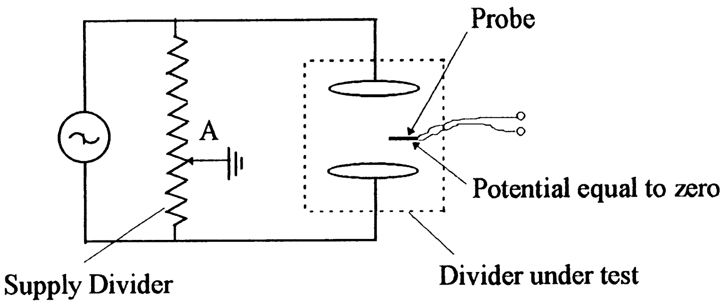

Figure 3 - Scheme of electric field measurement

It must be taken into account that several causes of uncertainty are present in this measurement procedure; between them it is worth mentioning that the position of minimum potential is not univocal and the correct position of the probe is known with an uncertainty of some millimeters.

5 Comparison between numerical and experimental results

The numerical and experimental analysis has been developed on the high voltage divider sketched in Fig. 4, where the leading dimensions are also reported. The electrodes are separated by a dielectric tube, whose relative dielectric constant is found to be equal to 5.8. A conducting sheet is placed under the divider and connected to ground during the experiments.

The device has been discretized into about 5000 surface elements, as shown in Fig. 1, also including the conducting sheet under the divider. The distribution of the electric field generated by the device is represented in Fig. 5, considering a part of the divider.

The comparison between numerical and experimental outcomes has been developed considering the values of the vertical component E_z of the electric field. First, the computed and measured values in a point of the domain are compared in Fig. 6 at the increasing of the voltage between the electrodes. As can be seen, the discrepancies are always lower than 12% and the response of the probe is practically linear.

The diagram of the normalized values of vertical component of the electric field versus x at y = 0, z = 0 is reported in Fig. 7, comparing measured and computed outcomes. As can be noted, the agreement between the results is good.

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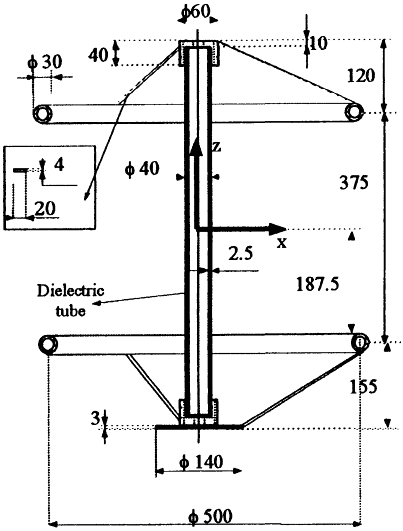

Figure 4 - Sketch of domain under considerations (dimension in millimeters)

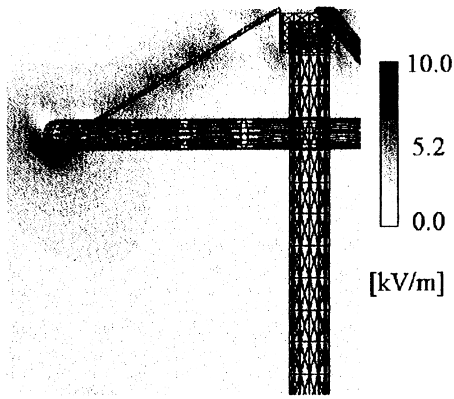

Figure 5 - Distribution of the electric field generated by the divider (voltage supply equal to 1 kV)

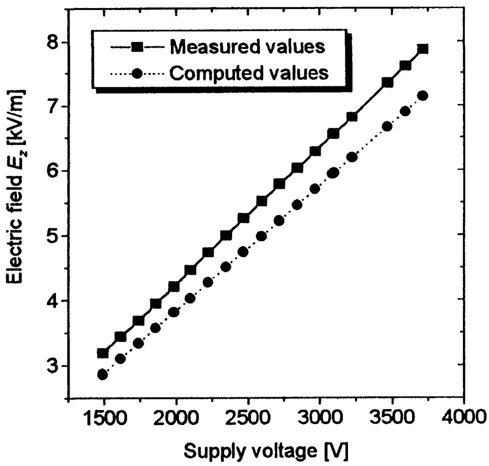

Figure 6 - Component E_z of the electric field in a point of coordinates (x = 0.25 mm, y = 0, z = -5.3 mm)

A comparison between measurements and computations has been also developed along the vertical direction, fixing the x and y coordinates. In Fig. 8, the normalized values of vertical component of the electric field are reported versus z. Even if a qualitative agreement is found, some local discrepancies are evidenced. These can be probably attributed to the

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difficulties in measurements, also considering the presence of unavoidable stray capacitances of the probe versus ground and the electrodes, which are not taken into account in the simulation. The computed values of the capacitance between the electrodes of the divider is about 15 pF.

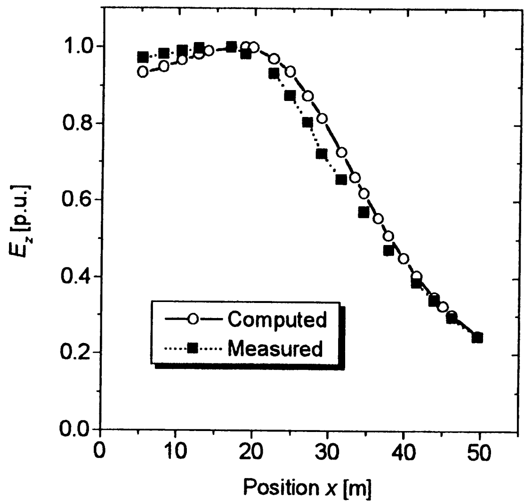

Figure 7 - Measured and computed values of component E_z of the electric field (normalized values) along a horizontal line with $y = 0$, $z = 0$.

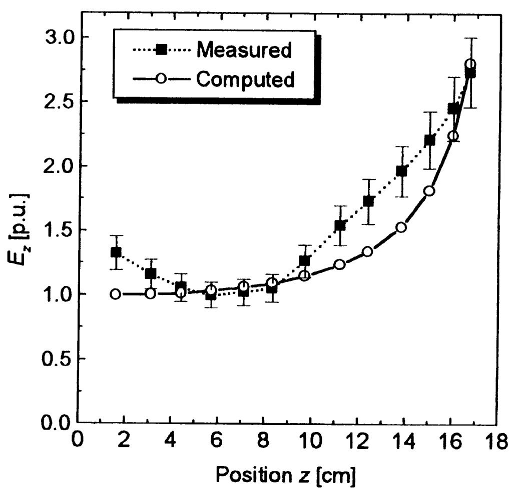

Figure 8 - Measured and computed values of component E_z (normalized values) of the electric field along a vertical line with $x = 25$ mm and $y = 0$

6. Conclusions

The paper has presented the preliminary work on the simulation by Boundary Element Method of the electric field generated by resistive HV voltage dividers. The results compared with the experiment are found to be in satisfactory agreement, even if the accuracy may be improved. In particular, the experimental set-up has been found very sensible to the environment conditions. In the next future, the measurement chain will be improved in order to reduce the noises and the influence of the probe on the field to be measured.

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