Inherent benefits in microscale fractal-like devices for enhanced transport phenomena

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Abstract

Heat sinks with fractal-like branching networks and parallel channel networks are compared. Fractal-like networks are embedded in disk-shaped heat sinks, whereas parallel channel networks exist in rectangular-shaped heat sinks. The benefit-to-cost ratio, defined as the ratio of advected energy over flow power, is found to be a strong function of channel length and channel depth under the constraints imposed in the present paper. The applied heat flux, heat sink area and flow rate between heat sinks remain constant in all cases considered. The channel spacing considered in both heat sinks is constrained by fabrication techniques. For the present range of conditions, the benefit-to-cost ratio is maximized for wider, deeper, and longer channels, and is always higher for the fractal-like heat sinks. Under these conditions, the maximum wall temperature is higher for the fractal-like heat sink in comparison with a parallel channel heat sink.
Keywords: fractal, dendritic, branching, flow geometry, electronic cooling, microscale, laminar, forced convection.

1 Introduction

Societal demands have resulted in extremely compact yet powerful electronic devices, which require high watt-density cooling techniques. Heat sinks incorporating micro-scale channels are very effective in this endeavour by increasing both the convective heat transfer coefficient as well as the convective surface area per unit volume in the heat sink.

However, the improved heat transfer provided by a series of parallel micro-channels is not without drawbacks. The small diameter of the channels produces large pressure drops, and non-uniform temperature distributions along the wall of

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the channel often occur. If used to cool an electronic component, a non-uniform temperature distribution, if significant enough, could result in uneven thermal expansion of the electronic device, possibly damaging it or affecting the electrical properties.

Tuckerman and Pease [1] first introduced the idea of micro-channel heat sinks for cooling integrated circuits. Since that time, numerous investigations of single straight micro-channels and micro-channel arrays in a heat sink have been conducted. Much of the experimental data prior to 2000 were not well predicted by macro-scale correlations. However, a recent study by Garimella and Singhal [2] demonstrates the validity of using conventional macro-scale correlations for predicting laminar flow pressure drop and heat transfer through micro-scale geometries.

Bau [3] demonstrated, using a mathematical model, that a micro-channel with a variable cross-section area can be optimized to reduce temperature gradients along the channel length. Reduction in the maximum heated surface temperature was predicted by tapering a channel in the direction of flow. However, a decrease in axial channel diameter can be accompanied by an increase in velocity and, hence, an increase in pumping power.

To improve the temperature uniformity without increasing the pressure drop, Pence [4] proposed a fractal-like branching flow network, in which each new branch (daughter) has a smaller diameter than that from which it originates (mother). The fractal-like flow network was designed using fixed diameter and length scale ratios between consecutive branching levels, as were proposed by West *et al* [5]. Using an optimization approach to minimize pumping power while adhering to a minimal volume constraint, Bejan [6] identified, on average, the same branching level ratios reported by West *et al* [5].

Using macroscopic correlations, Pence [7] developed a one-dimensional model for predicting both the distribution of pressure and wall surface temperature in a fractal-like branching channel network, in which energy was applied directly to the walls of the channel network. Results were compared to an array of parallel channels having the same channel length and same convective surface area as the branching network. The maximum wall temperature along the fractal-like flow network was consistently and significantly lower than through the parallel channel array for three different cases, which include identical pressure drop, identical flow rate and identical total pumping power through both flow networks. The model of Pence [7] assumes that both hydrodynamic and thermal boundary layers redevelop at each wall following a bifurcation, but the model neglects the influence of branching angle on either pressure drop or wall temperature.

Chen and Chang [8] also investigated pressure drop and heat transfer capacity of fractal-like channel networks. Flow was assumed to be fully developed and the influence of branching angle on pressure drop was assumed negligible. Wechsatol *et al* [9] recently investigated the optimal channel length distribution in a disk-shaped geometry similar to that proposed by Pence [4,7]. Circular cross-section channels were considered assuming fully developed flow conditions. Length scale ratios that vary at each bifurcating level were deemed

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optimal in terms of minimizing flow resistance. In addition, Wechsatol *et al* [9] report that the best flow configuration depends upon which variable(s) are held fixed.

2 Original one-dimensional model

Based on the suggestion made by West *et al* [5] for two-dimensional flow networks, Pence [4] proposed a preliminary branching channel network in a disk-shaped heat sink using the following fixed hydraulic diameter and length scale ratios across branching levels:

$$\frac{d_{k+1}}{d_k} = n^{-1/3} \tag{1}$$

$$\frac{L_{k+1}}{L_k} = n^{-1/2} \tag{2}$$

where d, L and n represent the hydraulic diameter of a channel segment, length of a channel segment length and the number of branches into which each channel splits, respectively. The value of n is equal to two. Subscripts k and $k+1$ represent the lower order and higher order branching level, respectively, at a bifurcation. Branching levels are noted in fig. 1a, with the k=0 branch emanating from the inlet flow plenum.

In the one-dimensional model developed by Pence [7], pressure and wall temperature distributions along rectangular cross-section ducts in a fractal-like branching flow network were predicted using an empirical correlation and data for pressure drop and Nusselt number, respectively. The pressure drop correlation in [10] includes increased pressure drop due to developing flow conditions. A correlation was fit to Nusselt number data from [11] for simultaneously developing thermal and hydrodynamic boundary layers. Water was used as the working fluid. In an analysis of a fractal-like flow network, a constant heat flux was applied first to the infinitely thin walls of the ducts composing the flow network. The one-dimensional model was validated by Alharbi *et al* [12] using a three-dimensional computational fluid dynamic analysis and experiments.

A heat sink with a fractal-like flow network was also compared in [7] to a heat sink with a parallel array of channels. In the heat sink analysis, the solid material composing the heat sink was assumed to have an infinite thermal conductivity. The total length of the flow channels, the exit channel hydraulic diameter, the surface area of the heat sink, and the flow power were held constant between the two heat sinks. The channels in the parallel channel heat sink were square in cross-section. In the comparison presented in [7], the channel density in the parallel channel heat sink was decreased in order to vary the convective area ratio between the fractal-like and parallel heat sinks. Results indicate that a heat sink with the fractal-like network presented in [7] was comparable to that with a series of parallel channels if the fractal-like flow heat sink had approximately 50% of the convective surface area as that of the parallel channel heat sink.

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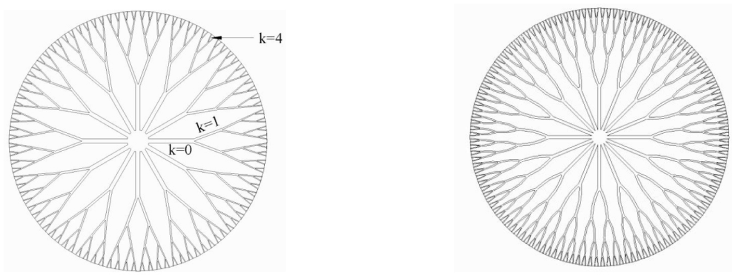
Comparable was defined by the same maximum channel wall temperature and total pressure drop.



(a) (b)

Figure 1: Four-level branching flow network (a) asymmetric branching with fixed hydraulic diameter ratio and (b) symmetric branching with fixed width ratio.

3 One-dimensional model for present analysis

For a constant channel depth, as required by in-house fabrication facilities, a fixed hydraulic diameter ratio results in an infinite channel width for the lower order branching levels. For this reason, a width ratio of

$$\frac{w_{k+1}}{w_k} = n^{-1/2} \quad , \tag{3}$$

determined by performing an optimization similar to that in [6] under the constraints of minimizing both the pressure drop and the volume, is employed in the present analysis. The fractal-like flow network for the heat sink shown in fig. 1b was developed using a fixed width ratio.

The number of branching levels (5 to 11), number of branches emanating from the inlet plenum (2 to 172), channel depth (100 to 500 μm), width of the terminal branch segment (30 to 150 μm), and total channel length of the fractal-like flow network (10 to 35 mm) in the heat sink are varied in the present analysis and compared to a heat sink with a parallel array of channels. Fractal-like flow configurations from the above combinations are only considered if the center-to-center spacing of the terminal branches is between 1.5 and 2 times the width of the terminal branch and the ratio of the inlet plenum diameter over the disk diameter is less than 0.1. The channels in the parallel channel heat sink are assigned lengths identical to the total flow length, from inlet to exit, in the fractal-like heat sink. In addition, the heat sink surface area and the applied heat flux are made identical between the two heat sinks. For most analyses, the height and width of the parallel channels are equal to those of the terminal branch in the fractal-like heat sink, with a center-to-center channel spacing equal to twice the channel width. Further details of the one-dimensional model are available in [7, 12].

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4 Results

4.1 Wall temperature and pressure distributions

Stream-wise channel wall temperature, *T(F)*, and static pressure, *P(F)*, distributions are presented in fig. 2 for the heat sink in fig. 1b, supplied with a heat flux of 100 W/cm^2 and a water flow rate of 10 ml/s. The heat sink has a total channel length of 20 mm, terminal channel width of 100 μm, and a channel depth of 250 μm.

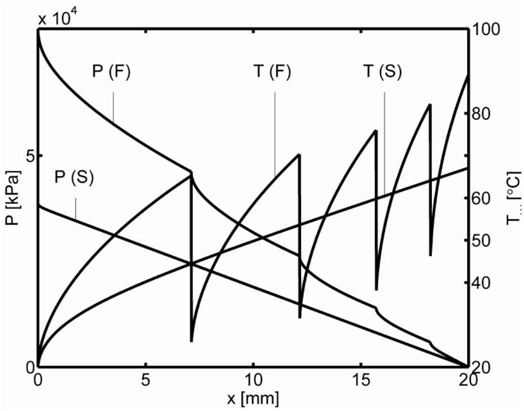

Figure 2: Channel wall temperatures and static pressure distribution through fractal-like and parallel channel networks.

The wall temperature, *T(S)*, and pressure distributions, *P(S)*, for a heat sink with a parallel array of straight channels having the same heat sink surface area, total channel length, total flow rate and heat flux as the fractal-like heat sink are also shown in fig. 2. The channel depth and width are also identical to those of the terminal branch in the fractal-like heat sink and are spaced at a center-to-center distance of twice the channel width. For the comparison presented in fig. 2, the ratio of the fractal-like convective area over the parallel channel convective area, designated by *AR*, is 0.23. The total pressure drop and maximum wall temperature in the fractal-like heat sink are approximately 2.1 times higher and 27ºC higher than in the parallel channel heat sink. A comparable analysis, in which the parallel channels are square in cross-section with the same hydraulic diameter as the terminal branch in the fractal-like flow heat sink, yields a 35% higher pressure drop and 9.1ºC higher wall temperature in the fractal-like heat sink. In this latter comparison, *AR* is 0.39. Recall from Pence [7] that given identical convective surface areas, the fractal-like flow network significantly outperforms the parallel channel array. It is evident from these results that an increase in *AR* is desirable for the fractal-like heat sink.

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4.2 Convective area ratio variations

The maximum wall surface temperature, T_w, pressure drop, ΔP, and benefit-to-cost ratio, *BCR*, are plotted as a function of the convective area ratio, *AR*, in figs. 3, 4, and 5, respectively. The benefit-to-cost ratio, *BCR*, is defined as the ratio of energy advected from the system over the flow power, defined as the product of pressure drop and flow rate. The convective area ratio is increased by increasing the number of branching levels, *k*, and/or the number of branches emanating from the inlet plenum, N_b. The heat flux applied to both fractal-like and parallel channel heat sinks is 100 W/cm^2. The fractal-like channel depth, terminal channel width and total channel length are 250 μm, 100 μm and 20 mm, respectively. The influence of channel geometry in the parallel channel heat sink is considered by comparing the fractal-like performance first to a parallel channel heat sink having channels with the same depth and width as the terminal branch in the fractal-like heat sink, referred to as the "rectangular" comparison and denoted by *R*, and also to a parallel channel heat sink having channels with a square cross-section and the same hydraulic diameter as the terminal branch in the fractal-like heat sink. This latter comparison is referred to as the "square" comparison and denoted by *S*. The influence of flow rate is considered by changing the flow rate from 10 ml/s to 20 ml/s.

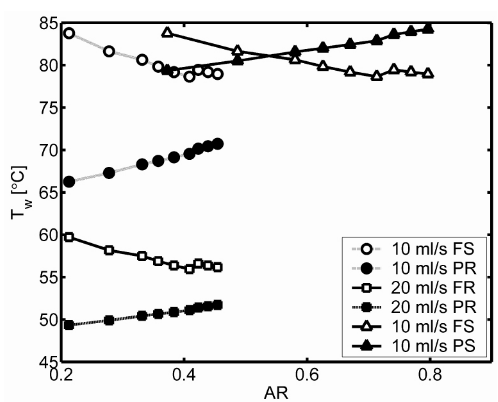

Figure 3: Maximum channel wall temperature as a function of convective area ratio.

In fig. 3, for both flow rates the wall temperature in the fractal-like heat sink (*FR*) is higher than that in the parallel channel heat sink with rectangular cross-section channels (*PR*) over the range of *AR*. Note, however, that the temperature, was well as the temperature difference between *FR* and *P*R, decrease as the flow rate is increased from 10 ml/s to 20 ml/s. Recall that the energy added by heat transfer remains constant; hence, a higher flow rate is expected to yield a lower exit bulk fluid temperature, and because the maximum wall temperature occurs at the channel exit, a lower maximum wall temperature is also anticipated.

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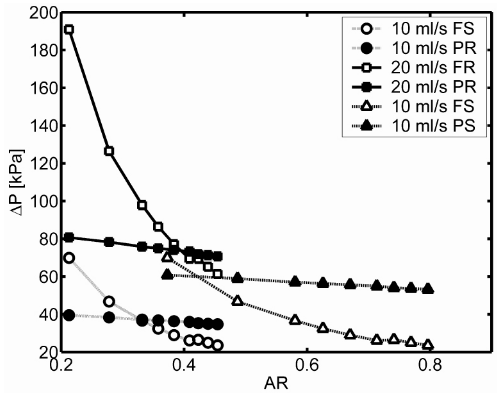

Figure 4: Pressure drop as a function of convective area ratio.

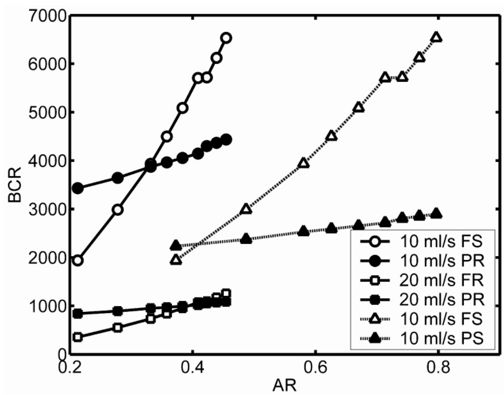

Figure 5: Benefit-to-cost ratio as a function of convective area ratio.

Note that the range of *AR* in fig. 3 is lower for the rectangular comparison, 0.2 to 0.5, than for the square comparison, 0.4 to 0.8. This is because of the decreased convective area in the parallel channel heat sink when square channels are used instead of rectangular channels. Comparing the fractal-like heat sink (*FS*) to a parallel channel heat sink with square cross-section channels (*RS*) suggests that lower wall surface temperatures in the fractal-like heat sink can be achieved for *AR* values greater than 0.5. This is consistent with results in [7].

Because the inlet plenum must increase to accommodate an increasing number or width of zero order branches, necessary for increasing *AR*, and because the channel length is fixed, the total plate area increases for a fixed channel length. Because heat sink areas are identical between the fractal-like and parallel channel heat sinks, this slight increase in plate surface area allows for more channels in the parallel channel heat sink, which in turn results in a decrease in flow through each channel for the fixed flow rate. The decrease in flow through

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each channel results in an increase in wall surface temperature and a decrease in total pressure drop. These increases in T_w and decreases in ΔP for the parallel channel heat sinks are observed in fig. 3 and fig. 4, respectively.

Figure 4 shows the pressure drop for the same comparisons noted in fig. 3. Increases in flow rate from 10 ml/s to 20 ml/s result in an increase in pressure drop, as is expected. The pressure drop in the parallel heat sink with channel dimensions identical to the terminal branch in the fractal-like heat sink is lower than that in the square cross-section parallel channel heat sink for the same flow rate. Given the restrictions on channel spacing, more rectangular channels can be packed into a fixed area heat sink than can square channels. The result is a lower flow rate through each of the rectangular channels, hence a lower pressure drop. This is accompanied by a higher wall temperature in fig. 3.

In all cases presented in fig. 4, there exists an *AR* above which the fractal-like heat sink pressure drop falls below that of the parallel heat sink for the present geometric and flow conditions. The transition occurs at a 15% lower *AR* value for 10 ml/s compared with 20 ml/s. Likewise, for the same flow rate, the transition occurs at a 25% lower AR value for rectangular channels than for square channels.

The *BCR* is shown in relation to *AR* in fig. 5. Increases in flow rate, from 10 ml/s to 20 ml/s, result in a 6-fold decrease in the *BCR* for the fractal-like heat sink. This is due to a 6-fold increase in flow power, as the advected energy remains constant. The major advantage of increasing the flow rate is the decrease in the maximum wall temperature, as was noted in fig. 3.

4.3 Channel length, depth and width variations

The influence of channel depth, terminal channel width and the total channel length on the performance of a fractal-like heat sink, specifically the benefit-to-cost ratio, are presented in figs. 6 and 7. The *BCR* as a function of channel depth, h, and terminal channel width, w_N, are plotted in fig. 6. The *BCR* is plotted as a function of the total channel length, L_{tot}, in fig. 7. The total flow rate is 10 ml/s and the applied heat flux is 100 W/cm^2. With the exception of the variable under investigation, all other variables of the fractal-like heat sink remained fixed and equal to those in the base case. For the base case, the terminal width is 100 μm, the channel depth is 250 μm and the total length is 20 mm. Also shown in the figures are the *BCR* for parallel channel *(P)* heat sinks with both rectangular (*R*) and square (*S*) cross-section channels. The surface areas of the fractal-like and parallel channel heat sinks are identical.

Figures 6 and 7 were generated by collecting the maximum *BCR* for both the parallel and fractal-like heat sinks for each of the specified variables. In the legend w and h represent width and height, respectively, whereas *F*, *PR* and *PS* represent the fractal-like heat sink, parallel heat sink with rectangular channels and parallel heat sink with square channels, respectively. Evident from fig. 6 is that the channel height, h, has a greater influence on *BCR* than does the terminal channel width, w_N. Increases in h result in an increase in *BCR* for both parallel and fractal-like channel heat sinks; however, the increase is steeper for the fractal-like heat sink. Because eqns (2) and (3) are independent of channel depth, changes in *h* do not influence the plate area of the fractal-like heat sink. Therefore, maximum wall temperature variations with *h* are a consequence of the Nusselt number dependence on channel aspect ratio and an increase in convective surface area. For all data corresponding to the range of channel depths for the fractal-like heat sink, shown as hollow squares and denoted as *h F* in fig. 6, the number of branching levels is 5 and the number of *k*=0 branches is 24. Note that all *BCR* converge at *h*=100 μm, which yields square channels.

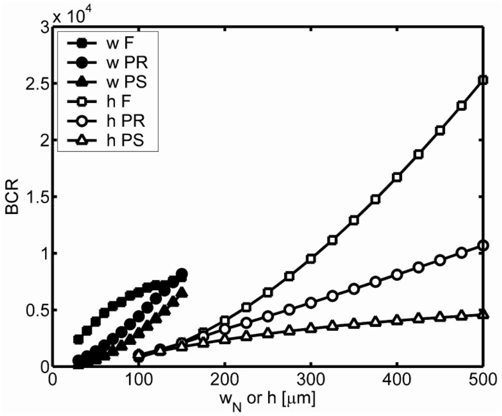

Figure 6: Benefit-to-cost ratio as a function of channel depth and terminal channel width.

Increasing the number of the branching levels significantly reduces the pressure drop, suggesting that *k* should be maximized if the sole goal is to minimize pressure drop. However, increasing *k* increases the number of terminal branches in a manner proportional to $2^k N_b$, which subsequently results in a reduction in the terminal channel spacing for a fixed channel length. Increasing *k* also results in an increase in the *k*=0 channel width. The combined decrease in terminal branch spacing and an increase in the ratio of inlet plenum diameter over heat sink diameter consistently resulted in the elimination from consideration of heat sinks with a large number of branching levels.

Increases in w_N result in an increase in *BCR* for fractal-like heat sinks for terminal channel widths below 120 μm, beyond which the *BCR* levels off. According to eqn (3), increases in terminal branch width result in increases in lower order branch widths for a constant value of *k*. Therefore, the number of branches that can originate from the inlet plenum, under the inlet plenum diameter restriction, decreases with an increase in the terminal branch width. The number of branching levels corresponding to the range of terminal channel widths considered for the fractal-like heat sink, denoted as *w F* in fig. 6, is 5. The number of k=0 branches decreases from 49 for a 50 μm terminal width to 16 for a 150 μm terminal width. There is a 70% decrease in convective surface area in

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the fractal-like heat sink with this increase in w_N. Consequently, there is an increase in maximum wall temperature, from 66ºC to 88ºC, respectively, for terminal channel widths of 50 μm and 150 μm. Also, the fractal-like heat sink compares more favourably when compared to heat sinks with parallel channels of square cross-section than of rectangular cross-section of the same hydraulic diameter.

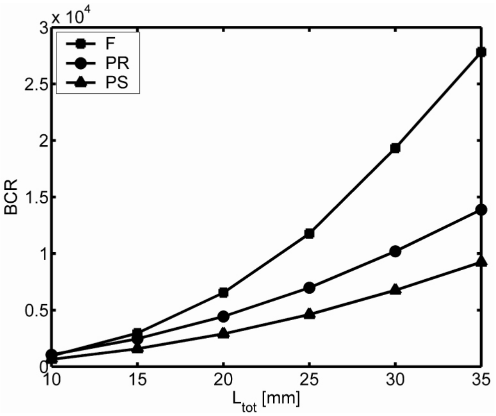

Figure 7: Benefit-to-cost ratio as a function of total channel length.

The *BCR* is plotted relative to L_{tot} in fig. 7. Increases in the channel length result in an increase in *BCR* for both fractal-like and parallel channel heat sinks; however, the increase is steepest for the fractal-like heat sink. Unfortunately, the temperature increase is also steeper for the fractal-like heat sink. Under the specified geometric and thermal and flow conditions, the maximum wall temperature increases from 43ºC for a total length of 10 mm to approximately 100ºC at 25 mm. The increase in wall temperature is a consequence of the increase in plate area in combination with a fixed heat flux and fixed flow rate. The performance of the fractal-like heat sink is again more favourable when compared with the square cross-section parallel channel heat sink than the rectangular cross-section parallel channel heat sink, on the order of 2.8 higher times and 2 times higher, respectively.

5 Conclusions

Heat sinks with fractal-like branching flow networks were compared with heat sinks having parallel channel arrays. All comparisons were made under identical flow rate and heat flux conditions. The fractal-like flow networks had fixed width ratios and length ratios. The ratio of convective surface areas between the fractal-like and parallel channel heat sinks, and the channel depth, channel length and terminal channel width were varied to determine the influence of these variables on the fractal-like heat sink performance. Heat sink performance was assessed using a cost-to-benefit ratio. Parallel heat sinks with square and rectangular cross-section flow areas, both having the same hydraulic diameter as the terminal branch in the fractal-like heat sink, were considered. Channels in the parallel heat sinks were at a fixed spacing equal to twice the channel width. Increasing the length and depth had a greater influence on the heat sink performance than did the terminal channel width, with a preference for larger values of each. In most cases considered, the maximum wall temperature in the fractal-like heat sink was higher than in either parallel heat sink. It is recommended that in future investigations maximum wall temperature be specified and the flow rate varied to assess the benefit-to-cost ratio.

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