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Damage mechanics applied to the fatigue behaviour of a composite material

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Abstract

Stress controlled fatigue tests were carried out on glass-polyester pultruded rods and damage evolution was monitored continuously using stiffness decay and replication techniques. Two distinct stages of damage development were identified. During Stage 1, an initially high but gradually decreasing rate of damage development took place, due to the exhaustion of new damage sites and slow growth of existing ones. The new sites were exhausted by about 10% life and the corresponding damage level was stress dependent. Stage 2 exhibited a steady increase or accelerating damage rate. During this stage, crack coalescence, longitudinal splitting and fibre fracture occurred.

A continuum damage mechanics model was developed which described damage evolution during the two distinct stages for all stress levels. Accurate fatigue lives were predicted. This model was capable of expressing the cyclic damage behaviour during a two stress level block test.

1 Introduction

In polymer matrix composites the main damage mechanisms are matrix cracking, interfacial debonding, delamination and fibre breakage. These occur in two dominant stages [1]. The first consists of homogeneous non-interactive cracking, restricted to individual plies that develops at a decreasing rate due to exhaustion of new damage sites and the slow growth of existing ones. The second stage is characterized by the localization of damage in zones of increasing crack interaction. In uni-directional composites this is seen as crack coalescence and in laminates as delamination. This leads to a steady increase or an acceleration in damage evolution until fracture takes place. The proportion and amount of

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damage occurring during each stage depends upon the configuration of the composite and the imposed stress level.

2 Stage 1 - decelerating damage evolution

Investigations such as those of Jessen and Plumtree[2] on pultruded rods and Broutman and Sahu[3] on cross-ply laminates have shown that the first failure event is matrix cracking in regions or plies where the fibre/load angle is the greatest. This general behaviour for matrix cracking may be illustrated by considering the fatigue results for $[0,90]_s$ glass-epoxy with a stress ratio of $R = 0.1$ [4]. During the first 10% of life (Stage 1) the crack density in the 90-degree plies increased very quickly, thereafter very slowly over the remaining 90% of life (Stage 2).

There is a direct relationship between this change in crack density and an accompanying decrease in stiffness which may be described by a damage variable, D (see equation 2). The relationship between crack density and stiffness (or longitudinal elastic modulus) may be used to measure damage and has been analyzed by several authors using different approaches, such as the shearlag model [4], self-consistent model [5] and the variation approach [6].

Applying continuum damage mechanics [7,8], an equation expressing the longitudinal damage tensor, ($D^{(1)}$), in terms of the number of cycles, N, takes the form

$$D^{(1)} = D_c\,[1-\exp\{-(N/\alpha)^{\beta}\}] \tag{1}$$

where D_c is the damage at the end of Stage 1 and α, β are material constants.

3 Stage 2 - steady increase or accelerating damage evolution

This stage involves the coalescence of microcracks and development of macro-cracks. For a nominal stress, σ, the damage parameter, D, is considered to be zero for the material containing no cracks and unity when rupture takes place. The term $\sigma/(1-D)$ is an "effective" stress, taking into account the weakness of the material due to the presence of voids or micro-cracks. From a thermodynamic point of view, D is an internal variable of an irreversible micro-rupture process. It appears linearly in the elastic thermodynamic potential, ψ, which is a quadratic function of elastic strain and temperature. The variable, $\gamma(=-\rho\ (\delta\psi/\delta D))$ is the energy rate of the decohesive forces [9]. This approach directly relates the damage parameter to the variation of the elastic modulus of the material or to the stiffness of the specimen. If E^* is the Young's modulus of the virgin material and E the elastic modulus of the damaged material, then

$$D=1-E/E^* \tag{2}$$

The differential equation [8] for damage evolution is:

$$dD/dN = \{\sigma_{max}/B(1-D)\}^{v}\,(1-D)^{w} \tag{3}$$

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where B, v and w, are constants dependent upon lay-up. Equation (3) takes the same differential form for the evolution of transgranular cracks during fatigue when the mean stress is 0, (R = -1), as proposed by Lemaitre and Plumtree [10]. This equation can be written in the integral form

$$D = 1 - (1-N/N_f)^{\gamma} \quad (4)$$

where N_f is the number of cycles to failure and γ is a parameter which Lemaitre and Plumtree regarded as a constant.

Equation (4) may be modified by introducing the coefficient D_a to account for failure occurring when D<1 since the critical value of D at fracture has been found to vary from 0.2 to 0.8 [11]. A critical value of 0.3 is generally accepted for long fibre composites [2]. Hence the second stage of damage may be written:

$$D^{(2)} = D_a\,[1-(1-N/N_f)^{\gamma}] \quad (5)$$

For unidirectional composites with constant fibre strength and perfect alignment such behaviour is the only one which would be expected, as illustrated by Dharan [12] for a glass-epoxy composite. Fibre fracture becomes significant only in the later part of life, $N > 0.5\ N_f$.

4 Combination of Stages 1 and 2

Combining equations (1) and (5), the general form for the total damage evolution in a composite may then be written

$$D=D_c[1-\exp\{-(N/\alpha)^{\beta}\}]+D_{\alpha}\,[1-(1-N/N_f)^{\gamma}\,] \quad (6)$$

and the total damage at failure ($N=N_f$) is the summation of D_c and D_a.

5 Application

In the case of fatigued cross-ply and angle-ply laminates, a variety of microscopic damage mechanisms, such as matrix cracking, fibre/matrix debonding, fibre pull-out, longitudinal cracking and fibre fracture, have been observed in glass-polyester rods pultruded. Initially, decelerating damage evolution occurred due to the early fracture of weak or damaged fibres and matrix cracking in resin rich areas, particularly where the fibres were misaligned. Consequently the material displayed Stage 1 damage behaviour. With perfectly aligned fibres of uniform strength this stage would have been absent. In the case of the pultruded glass-polyester rods with some misaligned fibres, [2], it was necessary to apply the two-stage model (equation 6) to describe the total damage evolution.

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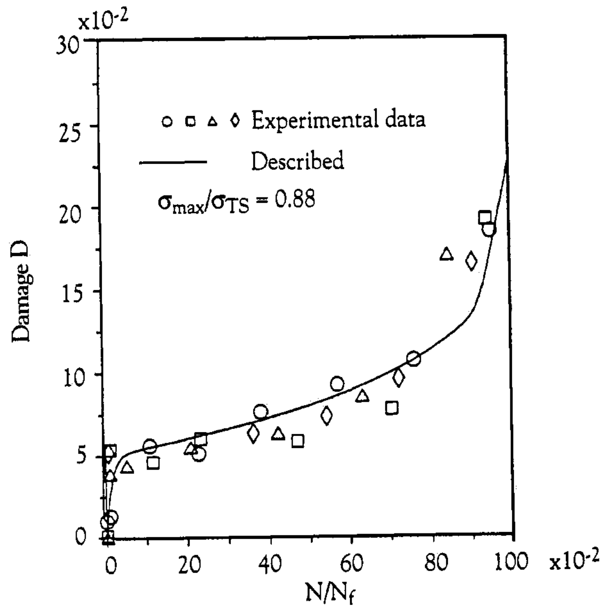

Figure 1. Damage evolution, pultruded glass-polyester [2]

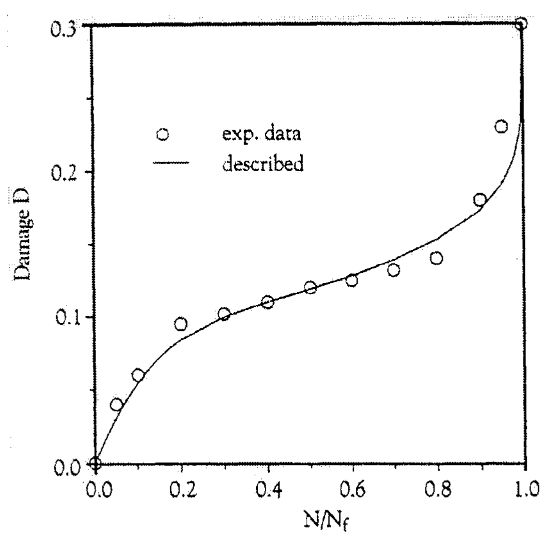

Figure 2. Damage evolution,$[0.90]_{2s}$ glass-epoxy [13]

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Figure 1 gives the experimental data and the described damage evolution curve for a stress level of σ_{max}/σ_{TS} =0.88(R=0.05)[2]. The parameters α (15.17), β(1.04) and γ(0.23) in equation (6) were obtained by least squares fits. It is seen that the damage curves described by equation (6) are in very good agreement with the experimental data. Stage 1 is complete by $0.1N_f$. The parameter α is sensitive to the applied load level [8] and may be expressed by

$$\log \alpha = 8.03 - 7.78\,(\sigma_{max}/\sigma_{TS}) \tag{7}$$

By contrast, β, is relatively constant (1.04 to 1.12). It is interesting to note that the value of 1.08 for cross-ply glass-epoxy also falls within this narrow range. Similarly, Stage 2 exponent, γ, for the pultruded rods appeared to be independent of stress level and constant as suggested by Lemaitre and Plumtree [10]. Further work is in progress to investigate the extent to which these parameters remain constant while addressing material properties, lay-up and cyclic loading conditions.

Poursatip et al [13] measured stiffness changes during cyclic tests on $[0,90]_{2S}$ glass-epoxy composites at R = 0.1 and maximum stress of 200 MPa. The corresponding experimental data are shown in Figure 2. Applying values for the parameters α(=1100), β(1=1.1) and γ (=0.22) derived from the work of Jessen and Plumtree [2] and values D_c (=0.09) and D_a (=0.21) from the experimental data, then equation (6) may be used to describe the damage evolution. This is shown in Figure 2 and clearly indicates that the two stage model may be applied satisfactorily to predict the actual damage (or stiffness) changes for the cross-ply laminate.

In order to demonstrate the versatility of the two stage method for describing the fatigue damage of long fibre composites, the life of a two stress-level test (high to low) was considered. A pultruded glass-polyester rod similar to that used by Jessen and Plumtree was first cycled at a high stress level σ_{max}/σ_{TS} = 0.88 (R=0.05) for 200 fatigue cycles (N/N_f =0.19), then the stress was decreased to a low level of σ_{max}/σ_{TS} = 0.51 (R = 0.05). Failure occurred after a further 330,590 cycles (N/N_f = 0.48). Using the two stage damage model (equation (6)), the predicted number of cycles to failure was 431,870 at the lower stress level. This is reasonable when it is realized that the expected number of cycles using the linear damage rule would have been 564,020, with a corresponding error of 70.5% compared to 30.6% when the two stage damage model was applied.

6 Conclusions

The accumulation of fatigue damage in a composite material may be described by a general two stage model which includes decelerating (Stage 1) and steadily increasing or accelerating (Stage 2) damage components. This approach accounts for the variety of damage mechanisms that develop throughout the life of the material and may be applied to predict the fatigue life of cross-ply laminates and unidirectional long fibre polymer based composites.

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