

Application of a hybrid approach for the wear prediction of tools for hot forging

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Abstract

The influence of a great number of parameters on tool wear in hot die forging of steel is very complex, and thus the relations between these parameters and the wear are highly non-linear and spatially very disordered. Even small variations (though within allowed limits) of the chemical composition of the tool material, in heat treatment conditions, etc. can cause considerable scattering of data on tool wear resistance. Therefore wear can be hardly described mathematically either by phenomenological models or by common regression equations. A better solution in predicting tool wear (the arbor radius) is given by applying a hybrid model, that as presented here. A CAE neural network was used to design a model which enables the phenomenon of tool wear to be described and which is based on data from the FEM analysis of forging, the time variation of wear contours of true tools, coded expert know-how, data on tool materials, etc. Application of a hybrid model is illustrated in some cases of practical relevance. The results predicted by CAE neural network approach show good agreement with the measured ones. The efficiency of this approach is better with a bigger data base when CAE neural network in the mathematical sense only executes interpolations between the data in the problem space.

1 Introduction

The tools in hot metal forging are cyclically exposed to high mechanical, thermal, tribological and chemical loads. Wear itself in more than 71 % of all cases represents the main cause for discarding the tool (the wear exceeds the allowed tolerance), while mechanical fatigue of tool materials in 25%, thermal fatigue in 3% and plastic deformation in 1% of cases are the other causes. The most

exposed parts of the tool are the engravings with a small radius of curvature (tool arbor radius); where the highest contact pressures and sliding lengths occur. Due to the complexity of the problem, predicting tool wear even today presents a great challenge. Better wear prediction would also mean lower production costs, since unexpected tool breakdowns (failures) can increase costs by up to 30% per forging unit [1-5].

The possibility of better tool wear prediction is offered by a combination of CAE NN (Conditional Average Estimator Neural Networks) as an improvement on classical statistics and FEM analysis of the hot forging process, the accuracy of which is being continually improved [6]. Namely, most forges have extensive data bases on tool wear obtained during years of forging with tools made of various tool steels, various forging shapes, the use of different lubricants, etc. All these data cannot be used efficiently with current rigid mathematical tools, i.e. to predict the wear of other tools on the basis of known data on tool wear. In this paper a new approach is suggested in order to achieve a better use of data already existing in by means of the artificial intelligence (CAE NN) approach, data obtained by FEM analysis, expert knowledge, etc.

2 Influential parameters and models of wear prediction

The parameters most influencing on tool wear (Fig. 1) are surface hardness and toughness at elevated temperature (carbide-forming elements), workpiece deformation (contact surface traction), contact pressures, sliding lengths, relative velocity of slippage, contact time, workpiece temperature, basic tool temperature, presence of the third particles in the interface (scale), lubrication, etc. [2-5]. Their influence on wear are very complex and the relationship between wear and these parameters are is highly non-linear and spatially very disordered. When testing wear on block on ring testing equipment in the laboratory, Zhang et al [7], for example, found that wear is drastically increased at specific combinations of contact pressures and relative velocity of slippage. These kinds of changes can hardly be described by functions of the exponential type. The most often used wear model is Archard's model, which was upgraded by a number of authors by including new parameters (scale, lubricant, etc.) [8-12]. Doege et al [3-4] suggested a new equation for tool wear prediction on the arbor radius (at the point of maximum wear) on the basis of extensive data on tool wear. The equation put forward considered eight sets of parameters (their exponents were obtained by a statistical method).

Attempts were made to solve the problem of the complexity of tool wear by using expert systems on the one hand [13], and numerical simulations and neural networks on the other [14]. In recent years mathematical micromechanical models can be found in the literature [15-16] for simulating friction and wear. These models, however, are still of a more theoretical character, friction (metal forming) still being the subject of various research projects [17-18]. Behrens et al [19] described the friction conditions by means of an adaptive friction coefficient which was predicted by BP NN (Back Propagation Neural Networks) on the basis of data obtained by the FEM analysis of the compression process. Some other authors [12] also stress the importance of changeable influential parameters

(contact pressure, sliding lengths, etc.) along the sliding deformed material on the tool curvature (arbor radius). Consequently, non-uniform wear and deposition of materials can occur, especially on small tool radii [4-5]. In such cases, primarily, the above-mentioned models are not always reliable in predicting wear on the entire arbor radius. Neural networks have been efficiently applied for wear prediction on cutting tools [20-21] and on samples in laboratory wear testing [22-23], and there have also been some attempts to apply neural networks to forming tools [24-25].

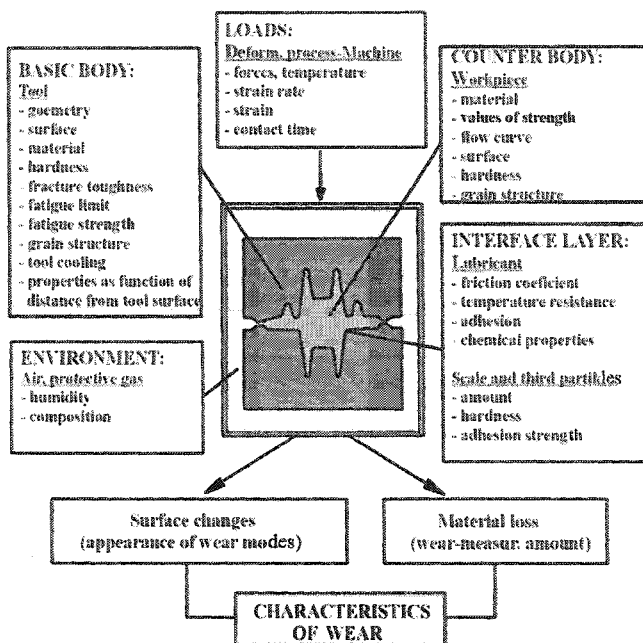


Figure 1: Influential parameters of tool wear, [3].

3 Basic characteristics of the new approach to wear prediction

Having great amounts of data on wear and influential parameters at our disposal, it is important to apply such a method that enables: us to take into account (1) the majority of essential parameters and (2) their interdependence. The CAE NN approach is one of the possible methods for doing this. The basis of the new approach (CAE and FEM) for wear prediction has already been presented [25]. A detailed description of CAE NN can be found in [26-29]; hence in this paper only the basic principles are given. Note, however, that CAE NN is not a typical neural network.

In general approach of CAE NN, each of the output variables corresponding to the vector under consideration $\hat{\mathbf{x}}$ (i.e. a vector with known input variables p_i and output variables \hat{r}_k to be predicted)

$$\hat{\mathbf{x}} = (p_1, \dots, p_i, \dots, p_L, \hat{r}_1, \dots, \hat{r}_k, \dots)^T \quad (1)$$

can be estimated by the formula

$$\hat{r}_k = \sum_{n=1}^N C_n \cdot r_{nk} \quad (2)$$

where

$$C_n = \frac{c_n}{\sum_{j=1}^N c_j} \quad (3)$$

and

$$c_n = \exp \left[\frac{-\sum_{i=1}^L (p_i - p_{ni})^2}{2w^2} \right] \quad (4)$$

Here \hat{r}_k is the k -th output variable to be predicted (corresponding to the vector $\hat{\mathbf{x}}$; in our case tool wear), r_{nk} is the same output variable corresponding to the n -th vector in the data base, p_{ni} is the i -th input variable of \mathbf{x}_n (parameters that influence tool wear), p_i is the i -th input variable of $\hat{\mathbf{x}}$, N is the number of model vectors in the data base, w is smoothness parameter, and L is the number of input variables.

In general the relevant processes regarding wear can be investigated at several levels, i.e. the nano, micro and macro level, as well as at the external (visible) level. The higher the level used to obtain data about the relevant processes the more difficult it seems to be to follow them quantitatively during the wear process. These problems still occur at the micro level, whereas FEM analysis, the precision of which has advanced greatly in the recent years, can bring more reliable data at the macro level. Here, the temperature calculated on the surface layer of the tool is still an exception, which is the reason why only the measured temperature of the workpiece is taken into consideration. The data basis is thus formed by following the essential parameters (mechanical and thermal loads) at the macro level (the FEM analysis), which indirectly affect the processes at other levels. For each point determined on the observed tool curvature (arbor radius, Fig. 2) the temporal course of the influential parameters must be calculated. This includes normal and tangential pressures, sliding velocity, sliding length, the temperature on the tool surface, their relative values along the contour in the direction of sliding, etc. The temporal course of sliding lengths on the mentioned arbor radius computed in this way (FEM) is shown on Fig. 3. It can be clearly seen that sliding lengths increase in the first part of the arbor radius and rapidly decrease in the second part, which means that there is a low value of relative sliding between the tool and the deformed material. The FEM analysis of contact pressures also indicates that in the first part relatively high pressures occur, whereas they decrease in the second part. Both cases explain why, on the one

hand, the removal (wear) of material occurs in the first part of the arbor radius and, on the other hand, why deposition of material can be noticed on the second part (Fig. 4). The data base is furthermore formed by the physical properties of the tool materials (chemical composition, hardness, tensile strength, etc.), the expert knowledge, as well as wears data on the entire arbor radius at a various number of strokes. In the Table 1 all the applied influential parameters are given except for parameters describing the chemical composition, which are given in the Table 2.

The scattering of tool wear data can be caused by variation of chemical composition (above all of the carbide-forming elements) of tools, though still within the allowed limits, by heat treatment of tool steels (a different temperature of austenitising and of tempering, etc.), by varying composition of the lubricant, and so on. Small variations in the chemical composition cannot simply be expressed in terms of other hardness data (tensile strength, etc.), but additionally by forming eight new vectors (parameters) for chemical composition. The number of parameters in such models can vary from 15 to 62, depending on the type of model.

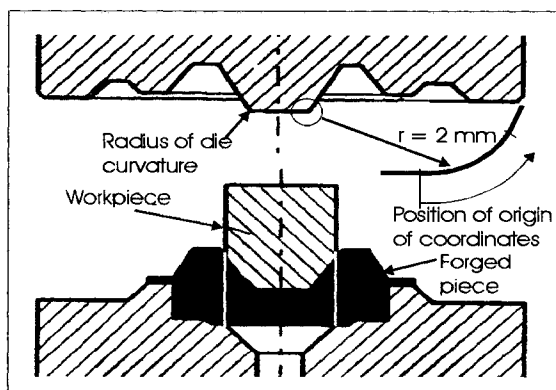


Figure 2: Applied tool with arbor radius and origin of coord. system defined, [5].

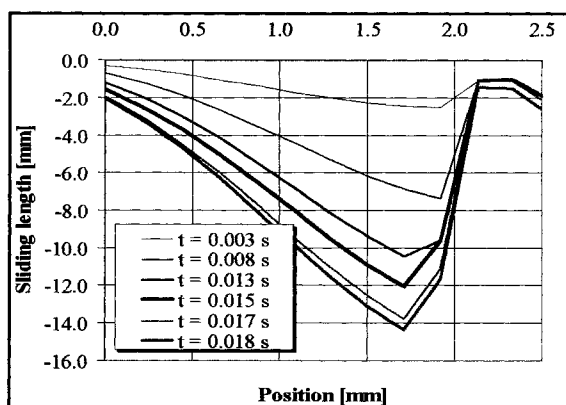


Figure 3: Temporal course of sliding lengths on the arbor radius.

Table 1: Variables - components of model vectors for description of wear.

$\sigma_{N(t,i)}$	Normal pressure at time t and on point i
$l_{(t,i)}$	Sliding length at time t and on point i
$v_{(t,i)}$	Relative velocity of slip at time t and on point i
N_s	Number of strokes
$W_{(N_s,i)}$	Wear at N_s on point i
$\sigma_{Nmax}/\sigma_{N(t)}$	Ratio of max. pressure on the whole die curvature to max. pressure on point i
$l_{max}/l_{max(i)}$	Ratio of max. sliding length on the whole die curvature to max. sliding length on point i
$v_{max}/v_{max(i)}$	Ratio of max. sliding velocity on the whole die curvature to max. sliding velocity on point i
$\sum \sigma_{N(t,i)} \Delta t$	Sum of products of normal pressure and time
v	Temperature of forgings
i	Position of point on the curvature
t	Time
σ_s	Tensile strength
v_{au}	Austenitising temperature
v_t	Tempering temperature

Table 2: Allowed deviations (%) of chemical composition of tool steels used.

	C	Si	Mn	Cr	Mo	V	Ni	Co	Ti
W.Nr. 1.2344	0.37 0.43	0.90 1.20	0.30 0.50	4.80 5.50	1.20 1.50	0.90 1.10	- -	- -	- -
W.Nr. 1.2365	0.28 0.35	0.10 0.40	0.15 0.45	2.70 3.20	2.60 3.00	0.40 0.700	- -	- -	- -
W.Nr. 1.2714	0.50 0.60	0.10 0.40	0.65 0.95	1.00 1.20	0.45 0.55	0.07 0.12	1.50 1.80	- -	- -
W.Nr. 1.2799	0.01 0.05	- -	- -	- -	7.80 8.30	- -	11.80 12.20	7.85 8.25	0.45 0.55

4 Results of wear prediction by CAE NN

In the case of a small data base a reasonable method seems to be taking the wear data at a lower number of strokes. These data already indicate the direction of scattering of end wear data (above or below the average). This method of intermediate control of tool wear is regularly used in forges; usually, just the data on the point of maximum wear suffices. The accordance between the measured and CAE predicted wear values are estimated by the coefficient of determination (B):

$$B = 1 - \frac{\sum_{k=1}^M (\hat{r}_k - r_k)^2}{\sum_{k=1}^M (r_k - \bar{r}_k)^2} \quad (5)$$

\bar{r}_k in equation 5 represents the mean value of r_k , and M is the number of model vectors tested.

4.1 Prediction of tool wear for other chemical compositions

In order to be able to predict wear by CAE NN a minimal data base had to be constructed. It was formed by means of FEM analysis of the forging process (temporal course of parameters), as well as by data characterizing the material properties (tensile strength, chemical composition, etc.). The base also included data on the wear of arbor radii at a various number of strokes (100 and/or 200, 500 and 1000). The tools with tensile strength 1500 MPa were made of W.Nr. 1.2344, W.Nr. 1.2365 and W. Nr. 1.2714, having the same dimensions and austenitised at the same temperature (1100 °C). The workpieces (heated on 1100 °C) were made of C 45, their dimensions were $\Phi=30 \times 40$ mm, contact time 0.020 s, time of one cycle 13 s, the lubricant delta 31 (friction factor $m=0.2$), etc. [4-5]. The fourth tool (W.Nr. 1.2799) also had the same shape and dimensions (Fig. 2), and knowing the intermediate data on tool wear (e.g. at 500 strokes), enabled us to predict (extrapolate) the tool wear of this fourth tool at a higher number of strokes, e.g. at 1000 strokes. The predicted CAE wear results for the fourth tool are shown in Fig. 4 (Example 1).

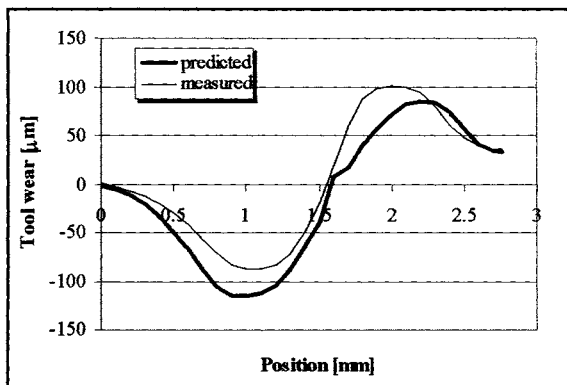


Figure 4: Comparison between measured and CAE predicted wear at 1000 strokes, W.Nr. 1.2799; tensile strength 1500 MPa, intermediate tool wear at 500 strokes is known, example 1.

It can be clearly seen that even a small data base (wear data for only three tools!) enables a relatively good estimate to be made of the temporal course of wear on the entire arbor radius ($B=0.709$). In Table 3 the values of B for CAE predicted

wear values at 1000 strokes are given (Example 2-4), also for the other three tools (procedure as in Example 1). The values for B for the given data set up (including the last measured wear data) can also be found in Table 3.

Table 3: Values of coefficients of determination B at smoothness parameter $w=2$.

	Example 1	Example 2	Example 3	Example 4
W.Nr.	1.2799	1.2365	1.2344	1.2714
Given data	0.862	0.941	0.945	0.994
Extrapolation	0.709	0.567	0.862	0.959

4.2 Prediction of the wear of a tool with different mechanical properties

By using different temperature for tempering the tool material, a different tensile strength can be achieved, which influences its wear resistance. Figure 5 shows the measured and CAE predicted values of wear obtained by interpolation from the existing data base and for the case of known wear data at 500 strokes (extrapolation). The data base originally only contained wear data on the steel W.Nr. 1.2799, W.Nr. 1.2344, W.Nr. 1.2365 and W. Nr. 1.2714 with a tensile strength of 1500 MPa; if then data about the wear of tool steel material with tensile strengths of 1300 and 2000 MPa were added to the base, a CAE wear prediction at 1000 strokes for tool steel (1400 MPa) could be carried out (Example 5, interpolation). Here a coefficient of determination $B=0.697$ was obtained. If wear of a tool with a tensile strength of 1400 MPa at 1000 strokes was predicted by known wear data at 500 strokes, the value of B obtained was 0.649 (Example 6, extrapolation). It should be noticed that the results for Example 5 were obtained for $w=0.15$ and for Example 6 for $w=2$. The different values applied for the smoothness parameter w were the consequence of the largeness of the data bases and the description of the model. In Example 5 the data base was relatively extensive (approximately 500 model vectors), but in Example 6 only five model vectors were used.

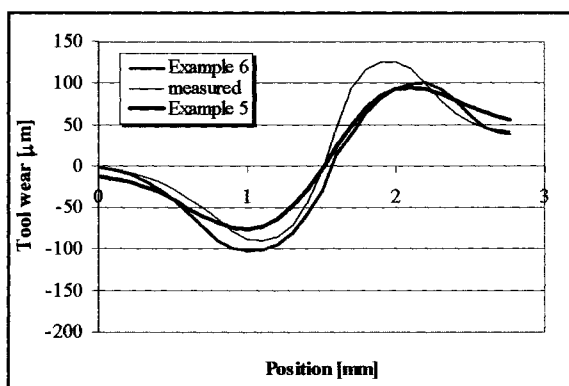


Figure 5: Comparison between measured and CAE predicted wear at 1000 strokes, W.Nr 1.2365, tensile strength 1400 MPa (Example 5, interpolation; Example 6, extrapolation, known wear at 500 strokes).

4.3 Wear prediction of a tool austenitised at a different temperature

The temperature of austenitising has great influence on the dissolution of carbides and consequently also on the wear resistance of tool steels. The Fig. 6 shows the measured and CAE predicted values at 1000 strokes of a tool austenitised at 1050 °C. Example 7 shows the CAE predicted wear obtained by interpolation (known wear data at 1100 °C and 1000 °C). Example 8 again shows the predicted value, considering the known wear data at 500 strokes (extrapolation). In Example 7 the value of $B=0.657$, while in Example 8 the value of $B=0.924$, indicating good concordance, as is also shown in Figure 6. In this case there is also a great accordance between the measured and predicted values.

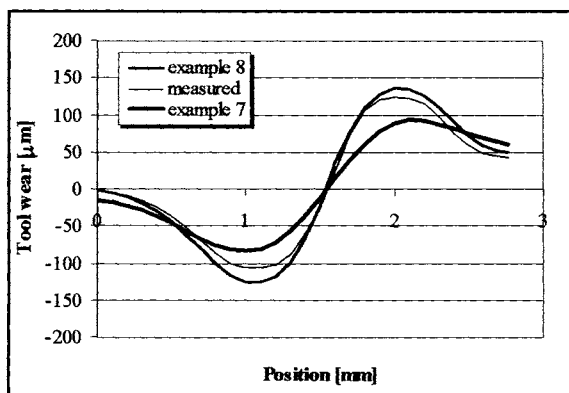


Figure 6: Comparison between measured and CAE predicted wear at 1000 strokes, W.Nr. 1.2365, temperature of austenitising (1050 °C); Example 7, interpolation; Example 8, extrapolation.

4.4 Wear prediction of a tool at a different forging temperature

If the temperature of the workpiece is changed, the mechanical and thermal loads are also changed (at lower temperature the contact pressures are increased and the temperature on the tool surface decreases, and vice versa). As known from the literature [2-5], lower thermal loads on the tool surface also mean lower wear values, since the temperature is the most influential wear parameter. Mechanical loads increase at lower workpiece temperatures.

From an existent data base on the temporal course of the influential parameters and wear data on all the mentioned types of materials at a workpiece temperature of 1100 °C, the wear on W.Nr. 1.2365 at a workpiece temperature of 900 °C was predicted.

An FEM analysis was again carried out. It is worth mentioning that in this case there were no known wear data for any of the materials in the existing data base at lower temperature. Again, the method of input of a single wear data at 500 strokes was applied. B obtained in this case was 0.789 (Fig. 7, Example 9). By enlarging the wear data at lower temperature, interpolation in the problem space can be used. If the wear data are for lower temperature (900 °C) for at least one

more of the above mentioned tool materials (W.Nr. 1.2714), then the wear data of W.Nr. 1.2365 can be predicted more reliably (Example 10, interpolation). The value B in this case is even higher than in the previous example where it amounted to $B = 0.864$, ($w=0.075$).

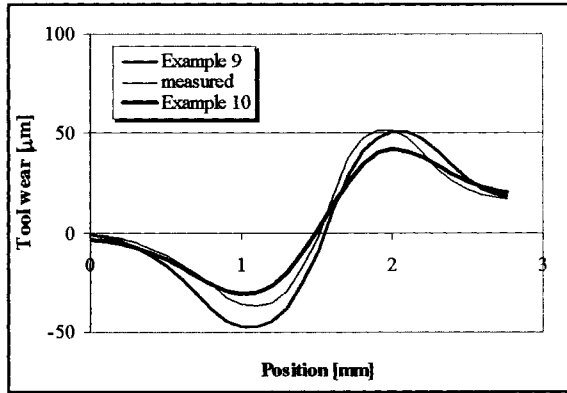


Figure 7: Comparison between measured and CAE predicted wear at 1000 strokes, W.Nr. 1.2365, temperature 900 °C, (Example 9, extrapolation; Example 10, interpolation).

A similar approach was used for predicting tool wear when using other lubricants; the results met our expectations.

The values for the coefficient of determination (B) in particular cases show that by enlarging the data base the CAE predicted wear results improve. This can be said for both models and also for both predicting modes (interpolation and extrapolation). Namely, increasing the amounts of data (model vectors) fills the vectors of problem space which, due to the complexity of the wear process, is very extensive. Having a large enough data base, the CAE NN approach will be only an interpolation in the problem space. In this case it can be expected with a high degree of certainty that the description even of such a non-linear problem as wear is going to be very precise.

5 Conclusions

The application of a hybrid model using FEM, CAE NN and coded expert knowledge has proved to be very reliable and suitable for predicting tool wear in forges for hot die forging, and offers an alternative to existing regression models. With the hybrid model most of the essential parameters on tool wear are considered, in the procedure of tool (and tool steel) making and in forging technology these parameters can vary, thus affecting tool life.

In this paper a procedure is described for systematically enlarging the data base (a time consuming process) and how to utilize this limited data for predicting wear. The examples presented in this paper are adapted to the actual conditions in forges. Though most forges possess extensive data bases, obtained in the many years of forging programs, this data could not be used to the best effect due to the

use of rigid mathematic tools, i.e. prediction of tool wear on the basis of known wear data. This paper offers a new approach for better usage of such data bases. Depending on the extensiveness of the data base, the procedures of extrapolation and interpolation by CAE NN may be applied. Extrapolation was carried out on the basis of known individual wear data at a lower number of strokes and this enables reliable predictions of wear on the entire arbor radius at a higher number of strokes. In the case of a smaller data base, the input of expert knowledge can be of great help. A sufficient amount of data also promises an even more reliable prediction of tool wear.

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