Prediction of kinetic energy correction factor for flow past butterfly valve

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Abstract

The paper presents numerical analysis of kinetic energy correction factor at the outlet of elbow penstock without and with butterfly valve for two different geometries.

The butterfly valve inside the penstock is vertical. That is why 3-D calculation of turbulent flow is taking into account. The basic aim of this analysis is comparison of kinetic energy correction factor for different geometries and its influence on the flow inside spiral casing.

For numerical analysis two equation $k - \epsilon$ turbulent model of FIDAP computer code has been used. Calculation was done on computer CONVEX 220.

Introduction

The main task of the flow analysis is to find out the influence of velocity profile on the turbine efficiency for two cases: with butterfly valve and without butterfly valve. The problem arose during an acceptance test of a model of Francis turbine for hydro power plant Diablo. It has been a refurbishement where the existing runner would be replaced by a new runner developed according to up to date technology. The existing flow tract has bend and a butterfly valve of large dimensions that can obstruct the flow at turbine inlet. Using CFD methods the influence of the kinetic energy correction factor on efficiency in different cases has been predicted.

The Turbulence Model

To solve the flow conditions a suitable mathematical model should be found approximated to real conditions as far as possible. In cases when local Reynolds number is high enough the turbulence occurs. In analysis of flow through hydraulic turbomachines the turbulence should be considered. The most prevalent model for the computation of turbulent flow is two-equation $k - \epsilon$ model.

In statistical approach to the mathematical treatment of turbulence every variable is considered as the sum of its average value and fluctuation. These values being introduced in the Navier-Stokes equations and averaged, some new unknowns are obtained which should be additionally modeled.

The basis of the computation of turbulent flow by two-equation model are Navier-Stokes equations to which two transport equations for turbulence kinetic energy and dissipation, are added. By means of two new quantities we can calculate the eddy viscosity being a new unknown in the average Navier-Stokes equations, if the Boussinesq supposition is considered. The system of equations to be solved increased considerably, thus the application of the segregated method can save a lot of computer memory on account of the number of iterations which increases considerably. The segregated method is used particularly with the 3-D models.

Viscosity is modeled as the sum of laminar and eddy viscosity

$$\mu = \mu_o + \mu_t, \tag{1}$$

where μ_t depends on k and ϵ

$$\mu_t = c_\mu \varrho k^2 / \epsilon. \tag{2}$$

Turbulent flow is described by means of the following system of equations

$$u_{j,j} = 0 \tag{3}$$

$$\varrho(\frac{\partial u_i}{\partial t} + u_j u_{i,j}) = -p_{,i} + \varrho f_i + [\mu(u_{i,j} + u_{j,i})]_{,j} \tag{4}$$

$$\varrho(\frac{\partial k}{\partial t} + u_j k_{,j}) = (\frac{\mu_t}{\sigma_k} k_{,j}) + \mu_t \Phi - \varrho\epsilon \tag{5}$$

$$\varrho(\frac{\partial \epsilon}{\partial t} + u_j \epsilon_{,j}) = (\frac{\mu_t}{\sigma_\epsilon} \epsilon_{,j}) + c_1 \frac{\epsilon}{k} \mu_t \Phi - \varrho c_2 \frac{\epsilon^2}{k}. \tag{6}$$

The constants used by $k - \epsilon$ model are:
$c_\mu = 0.09$
$\sigma_k = 1.00$
$\sigma_\epsilon = 1.30$
$c_1 = 1.44$

$c_2 = 1.92$.

Empirically determined constants appearing in this turbulence model were tested and optimized over a wide range of different practical cases and found appropriate for the analysis of flow around the profile.

Two-equation $k - \epsilon$ model is applicable for flow computation at high Reynolds number. In the flow having high Re high gradients of all searched quantities are to be found near the walls. For detailed description of such conditions the near-wall region should be divided to very large number of elements which would require high consumption of CPU time and more computer memory. In these cases wall functions can be used for solving the flow near the walls. In special elements near the walls the basic equations of motion and transport equations of turbulence kinetic energy and dissipation are not calculated, the solutions of flow are obtained by the application of functions considering Van-Driest's mixing length approach. Special elements thus represent the region between fully turbulent flow and wall.

The disadvantage of model is observed in cases where flow separation and recirculation regions occur. In describing the turbulent flow it is very important to prescribe the correct boundary conditions. For viscous fluid on all solid walls zero velocity components are prescribed. At the inlet the velocity distribution obtained by measurements or constant velocity distribution is assumed, while the turbulence kinetic energy is assumed to be one half percent of flow inlet kinetic energy. To describe the dissipation at the inlet there is a considerable number of empiric formulae which are dependent on turbulence kinetic energy, turbulent flow mixing length and friction velocity. In our case the following equation is used (Zhang and Sousa, 1990):

$$\epsilon = c_\mu \frac{\chi y k^2}{L_m{}^2 u^*}. \tag{7}$$

Prediction of kinetic energy correction factor

For viscous flow inside the pipe the nonuniform distribution of velocity has been observed. In Bernoulli equation the average velocity instead of parabolic profile is taken into account. That is why is necessary to improve kinetic energy term with kinetic energy correction factor. Kinetic energy for a flow in a pipe is

$$W_k = \frac{\varrho}{2} \int_A v^3 dA. \tag{8}$$

If the average velocity $\bar{v}$ has been used the kinetic energy has been

$$\bar{W}_k = \frac{\varrho}{2}\bar{v}^3 A. \tag{9}$$

The ratio between two different results of kinetic energy is more than one

$$\frac{W_k}{\bar{W}_k} = \frac{\int_A v^3 dA}{\bar{v}^3 A} = \delta > 1. \tag{10}$$

The coeficient δ is called kinetic energy correction factor or Coriolis coefficient. Numerically the problem was treated by finite element method which is highly suitable for treatment of water flow in hydraulic turbomachines. Complex geometries can be well described by a small number of curvilinear elements.

The average velocity is obtained by the method where the area of each element is taking into account

$$\bar{v} = \frac{\sum v_{el} pl_{el}}{pl}. \tag{11}$$

In our case for finite element computational mesh the Coriolis coefficient is calculated using following equation

$$\delta = \frac{\sum_{el} v^3{}_{el} pl_{el}}{\bar{v}^3 pl}. \tag{12}$$

Computational domain, grid and boundary conditions

In the flow throught the bending pipe and past a butterfly valve there are presented 3-D effects. So numerically the problem has been treated three dimensionally with FIDAP package and with the two equation $k - \epsilon$ turbulent model. Mesh for an empty bending pipe has about 4500 elements and mesh for a pipe with butterfly valve has about 5000 elements. Further on the outlet geometry has been changed. In this case pipe has been shorter and the diameter of the pipe has become smaller from the valve to the end of the pipe. The diameter of a pipe is 434 mm and the width of the butterfly valve is 109 mm what is more than a quarter of the pipe diameter. Boundary conditions at the inlet have been obtained from measurement of flow rate. All calculations have been done on computer Convex 220 and from half an hour to one hour have been taken of CPU time.

Results and discussion

For empty pipe and Reynolds number about 10^6 the velocity distribution at the outlet is almost uniform. Only secondary flow can be seen, but the main flow is 25-times bigger than secondary flow. The shape of secondary flow are two vortexes. The kinetic energy correction factor for the flow in empty pipe is negligible small, less than 1.01. Quite opposite results have been obtained in the case with butterfly valve inside the pipe. The kinetic energy correction factor for cross section 350 mm behind the centre of butterfly valve has been about 1.08. Behind the pipe is a wide wake which is the consequence of a very width valve. The velocity distribution behind the valve is not uniform and Coriolis coeficient is not negligible. Results for two different geometries of the pipe behind the butterfly valve are shown on the following figures.

References

Ruprecht, A., 1990, "Finite Element Method, Part 1", *Notes of CFD '90 - Intensive Course on Computational Fluid Dynamics and Heat Transfer*, Ljubljana.

Zhang, C., A. C. M. Sousa, 1990, "Numerical Simulation of Turbulent Shear Flow in a Isothermal Heat Exchanger Model", *ASME Journal of Fluids Engineering, Vol. 112, No. 1, March 1990, pp. 48 - 55.*

FIDAP USERS MANUAL, Revision 6.0.

Nomenclature

|	
c_1, c_2, c_μ	= turbulent model constant
k	= turbulent kinetic energy
L_m	= the Nikuradse mixing length
M_l	= hydraulic torque for profile
p	= pressure
pl	= area
pl_{el}	= area of element
Re	= Reynolds number, $Re = \frac{\varrho l U_o}{\mu}$
u	= velocity
u^*	= friction velocity
U	= streamwise velocity
U_o	= throughflow velocity
v_{el}	= velocity on each element
$\bar{v}$	= average velocity
W_k	= kinetic energy
x, y	= coordinates
ϵ	= turbulent dissipation rate
μ	= effective dynamic viscosity
μ_o	= laminar dynamic viscosity
μ_t	= turbulent dynamic viscosity
ϱ	= density
σ_e, σ_k	= turbulent Prandtl numbers for ϵ and k, respectively
χ	= the von Karman constant
Φ	= volumetric rate of generation of turbulent dissipation rate
δ	= kinetic energy correction factor or Coriolis coeficient
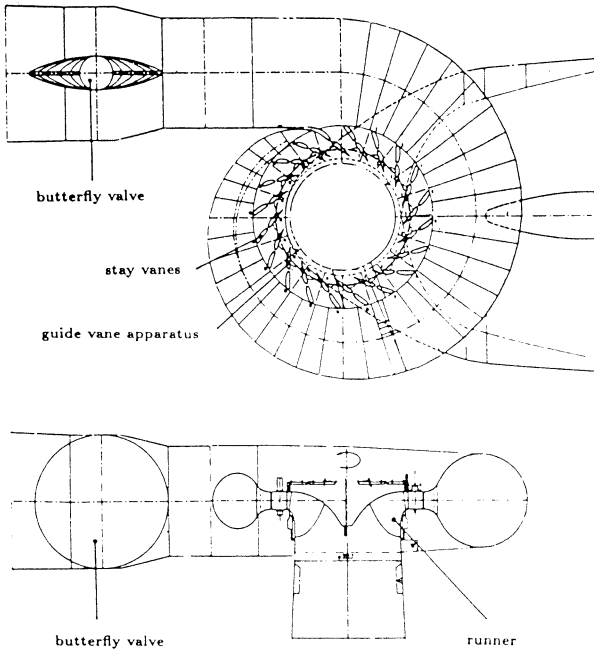
Figure 1. Turbine model
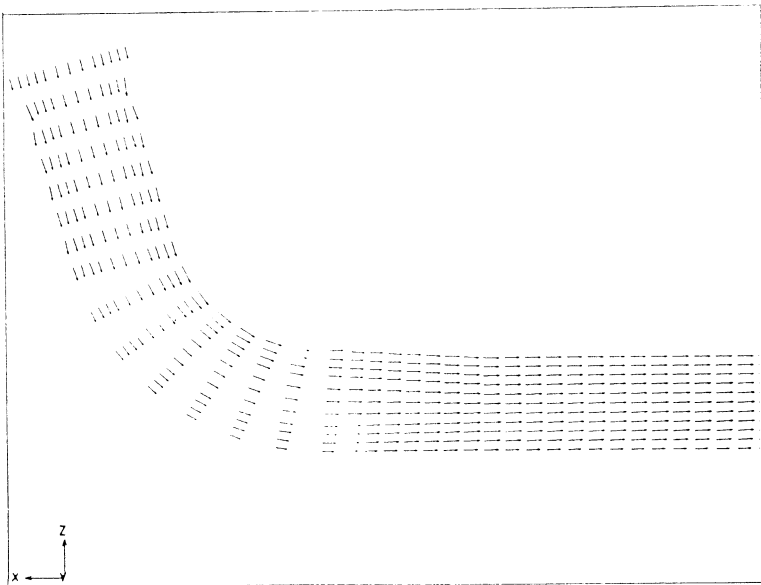
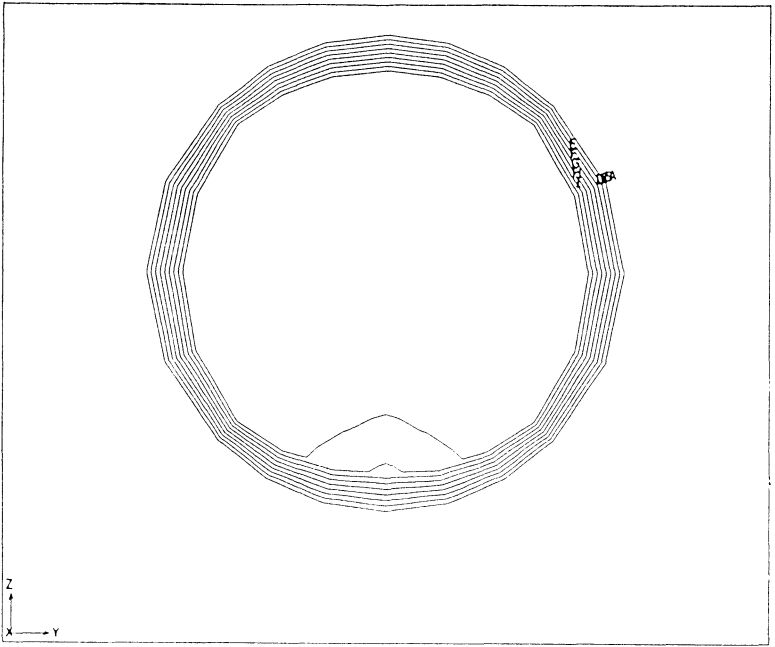
Figure 2. Velocity distribution in empty pipe

Figure 3. Speed contour plot at the outlet cross section for empty pipe
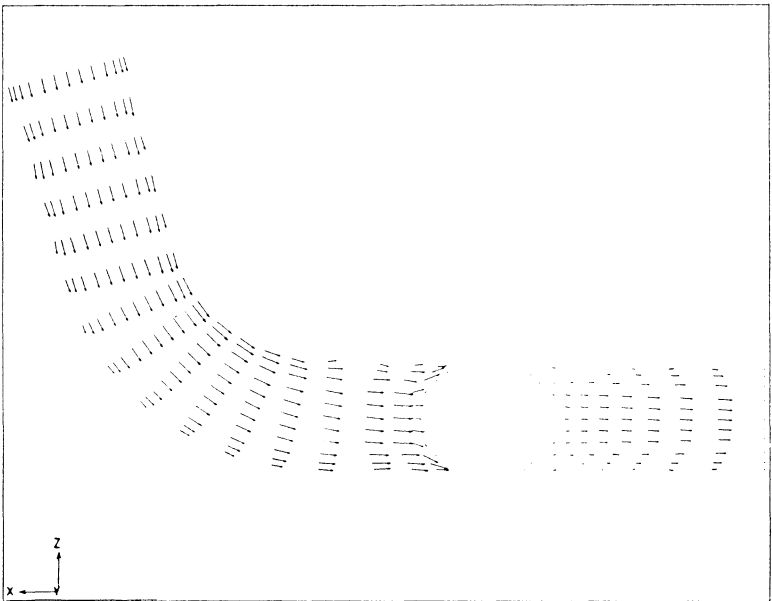

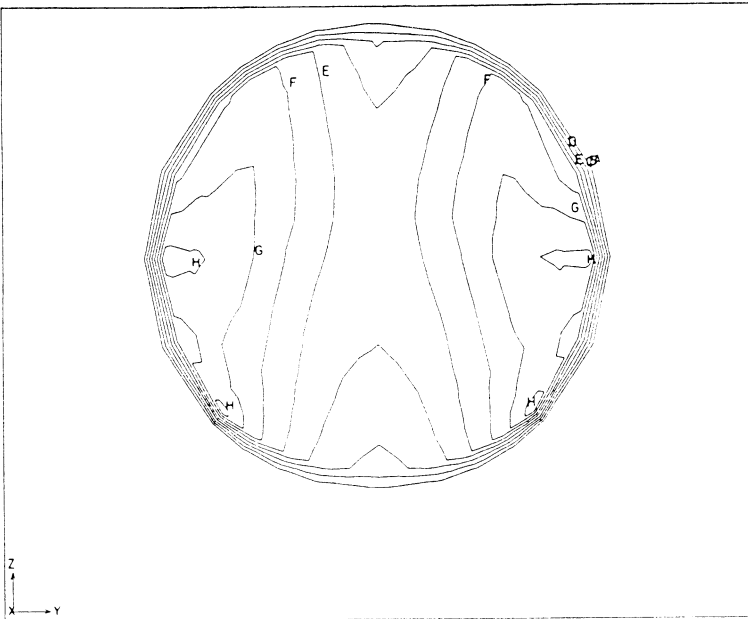Figure 4. Velocity distribution in the pipe with butterfly valve

Figure 5. Speed contour plot at the outlet cross section for the pipe with butterfly valve
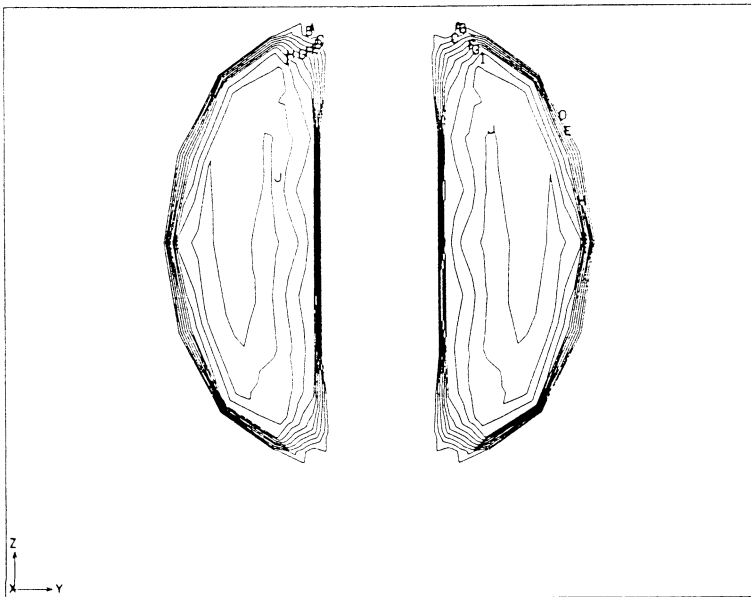

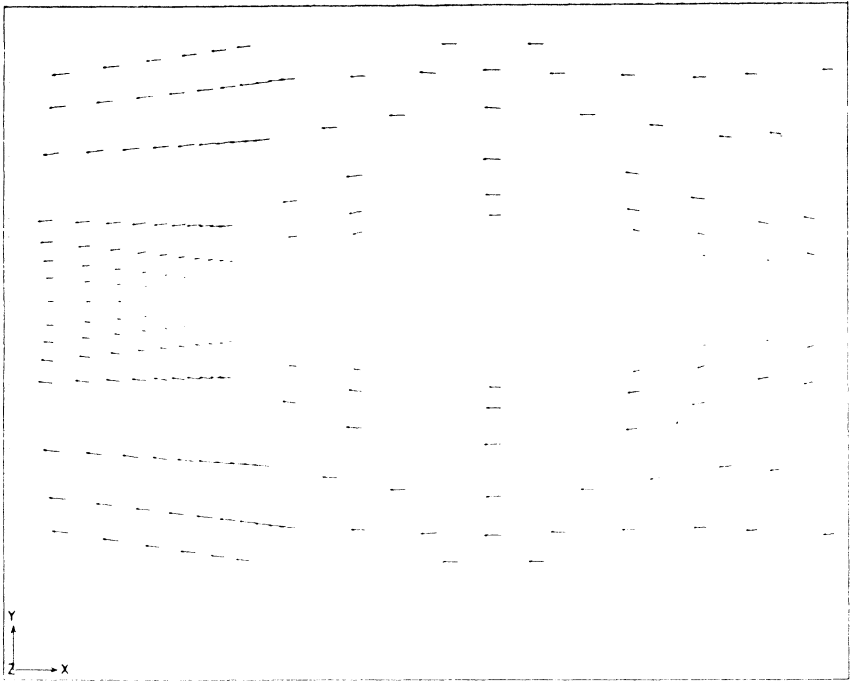Figure 6. Speed contour plot at the centre of butterfly valve

Figure 7. Velocity distribution for conic pipe behind the butterfly valve