Transactions on the Built Environment vol 9, © 1995 WIT Press, www.witpress.com, ISSN 1743-3509

Numerical modelling of tides around Iceland

G.G. Tomasson, J. Eliasson
Department of Civil Engineering, University of Iceland,
Hjardarhagi 2-6, IS-107 Reykjavík, Iceland

Abstract

A model has been set up to solve for the tides and tidal currents in a region of the North Atlantic Ocean around Iceland. The model is based on the depth integrated, nonlinear shallow water equations, which are solved numerically using a staggered Galerkin finite element scheme on triangular elements. At the open boundaries the model is forced using results from Schwiderski's [4] global model.

So far the model has been used to simulate the semi-diurnal M2 component of the tides. The results have been compared with tidal gauge measurements at a number of stations around Iceland and elsewhere, showing in general good agreement.

Future plans for the model are to compute the complete astronomical tide, and be able to simulate actual tidal motions during particular periods of time anywhere within the model area. The model may also be used to simulate and predict storm surges at the coast of Iceland, together with various other applications of importance to Iceland, a country surrounded by the North Atlantic Ocean and totally dependent on its resources.

1. Introduction

Numerical models of the tides in the North Atlantic Ocean have been set up by many authors, see for example Flather [2] and Gjevik & Straume [3]. The resolution of these models in the coastal areas and on the coastal shelf around Iceland is however rather poor, giving little information regarding tidal variations and tidal currents on a local scale. The motivation of this work comes from the desire to study these effects is as much detail as desired. A model of the tides and tidal currents around Iceland may be used in various applications of importance to the country, for example prediction of pollutant transport in coastal areas, as well as possibly benefiting the Icelandic fish industry, on which the economy of the country is mostly based.

2. Description of the model

The modelling is done with the model AQUASEA, a commercially available program package developed by Vatnaskil Consulting Engineers in Iceland, but further developed and customised for this study. The model is based on the shallow water equations, i.e. the continuity equation

$$\frac{\partial \eta}{\partial t}+\frac{\partial}{\partial x}(uH)+\frac{\partial}{\partial y}(vH)=0 \tag{1}$$

expressing conservation of mass, and the momentum equations

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-g\frac{\partial \eta}{\partial x}+fv-\frac{g}{HC^2}(u^2+v^2)^{1/2}u+\frac{k}{H}W_x|W| \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-g\frac{\partial \eta}{\partial y}-fu-\frac{g}{HC^2}(u^2+v^2)^{1/2}v+\frac{k}{H}W_y|W| \tag{3}$$

expressing conservation of momentum. Here g is the gravitational acceleration, C is the Chezy coefficient, expressing frictional forces at the bottom, f is the Coriolis parameter, k is a wind coefficient and W_X and W_Y are wind speeds in the x- and y-directions, respectively, where x and y are Cartesian coordinates. η is the surface variation from the mean water level, $H=h+\eta$ is the total water depth, where h is the mean water depth, and u and v are depth averaged velocities in the x- and y-directions, respectively.

The model equations are approximated numerically using a staggered Galerkin finite element scheme on triangular elements. A continuous approximation is used for the water level elevation (η and H), linear within the elements, but piecewise constant approximations are used for the velocities u and v. The momentum equations are discretized in time using a one-step fully implicit approximation and substituted into the continuity equation to form an "integrated" wave equation. This equation is solved for the water level variation η and subsequently the discretized momentum equations are solved for the velocities u and v. For more details on the numerical scheme the reader is referred to Sigurdsson et. al [5] and Sigurdsson [6], [7].

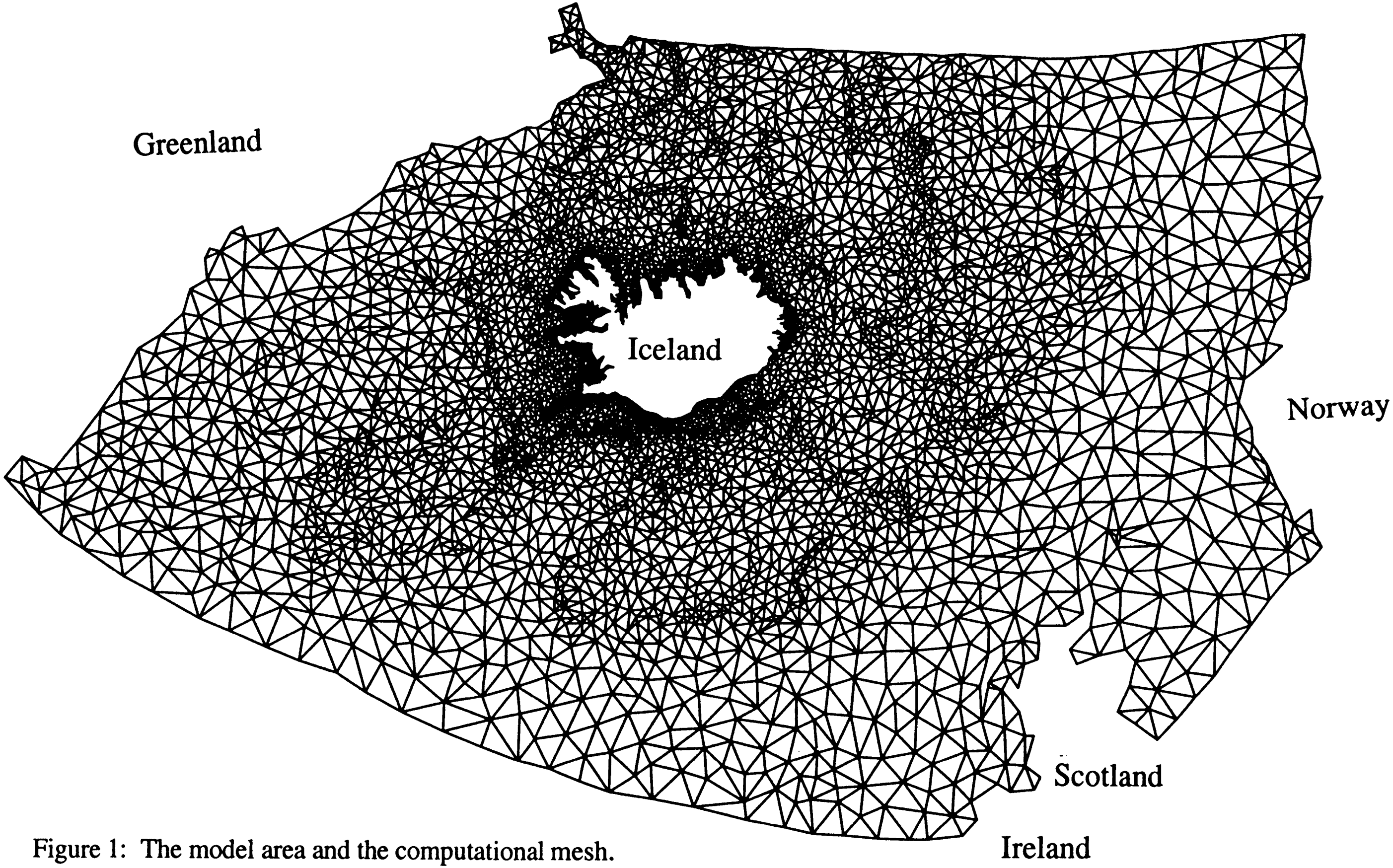

Figure 1: The model area and the computational mesh.

The model area is shown in figure 1. It extends from a line between the southern tip of Greenland and the northern part of Ireland in the south to around 70°N in the north, and from the coast of Greenland in the west to the coast of Norway and Scotland in the east, an area covering around 3.3 million km^2 of the ocean. The computational mesh consists of around 5000 nodes and 9500 elements, with the greatest resolution on the shelf and in the coastal areas around Iceland.

At the open boundaries of the model the water level variation η is forced using results from Schwiderski's [4] global model. However, to include radiation boundary conditions, i.e. to allow outgoing disturbances to propagate out of the model area without reflection at the open boundaries, a modified form of the Sommerfeld radiation boundary condition is applied at the nodes on the open boundaries

$$\frac{\partial \eta}{\partial t} + c\frac{\partial \eta}{\partial n} = -\frac{\eta - \eta_F}{T_f} \tag{4}$$

Here η_F is the desired water level variation, c is the linear wave speed and n defines the direction normal to the boundary. T_f is a timescale for the radiation boundary conditions, such that for $T_f = 0$ the equation expresses pure clamped boundary conditions, but for $T_f = \infty$ a pure radiation boundary condition is applied. This simple way of including the radiation conditions has been used by a number of authors, including Blumberg & Kantha [1], and shown to give reasonably good results.

3. Results for the M2-tide

So far the model has only been used to solve for the semi-diurnal M2 component, which is the dominating component of the tides at all stations around Iceland. The model simulations are started from rest, i.e. for the initial conditions $\eta = u = v = 0$. To ensure that the solution has reached a steady state before determining the M2 harmonic constants, the model is first run through seven tidal cycles (86.8 hours). The harmonic constants are generated from the solution through the next four cycles (49.6 hours) by decomposing the computed sea level variation at all nodes in the model into the M2 component and its first few multiples, which are generated in the model by nonlinear interactions through the nonlinear terms in the governing equations. In these simulations, a Chezy coefficient of 50 $m^{1/2}/s$ has been found to give best agreement with observations.

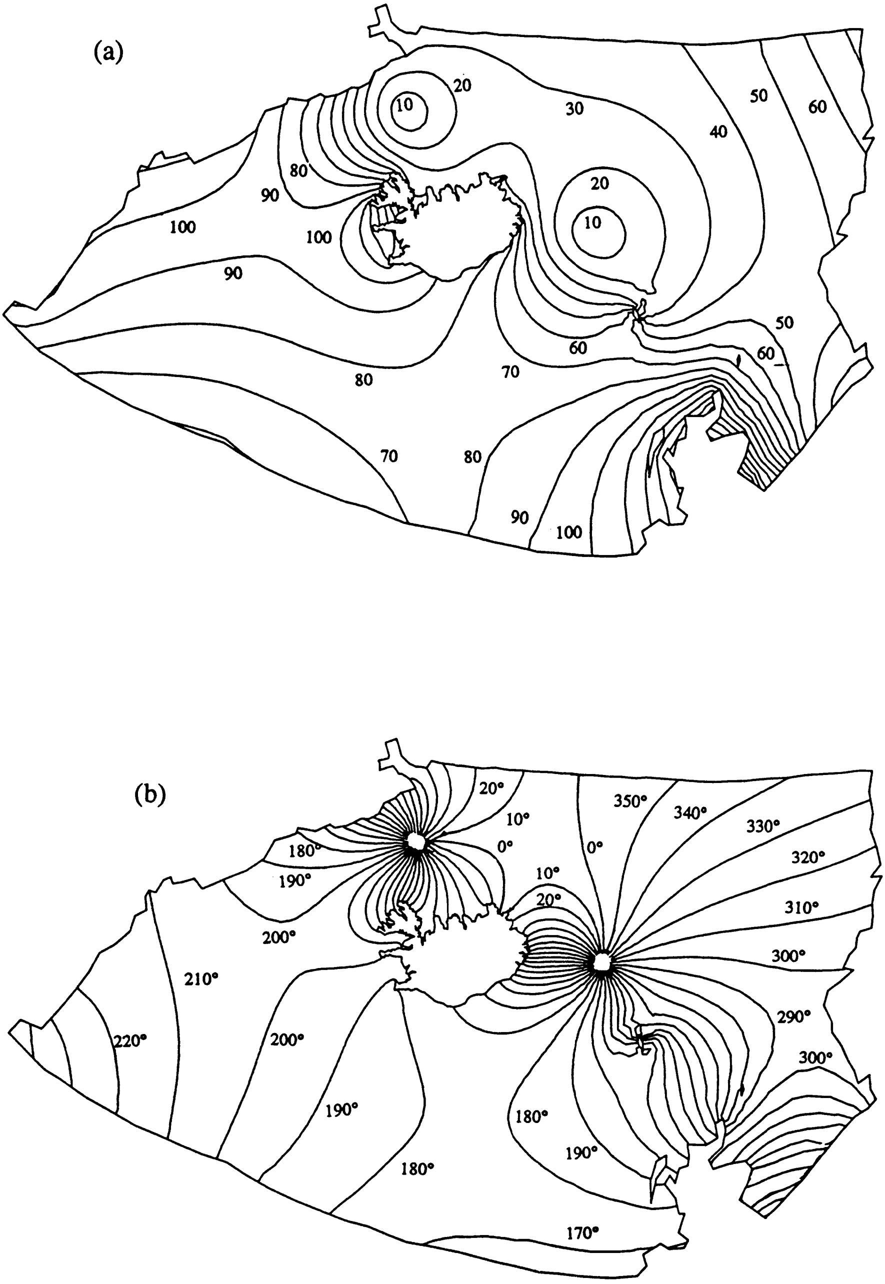

Figure 2: A map of the results for the M2 tide.
(a) The amplitude (cm). (b) The phase lag (°).

Figure 2 (a) shows a map of the amplitude of the M2 tide and figure 2 (b) shows its phase lag relative to Greenwich. These are in general in good agreement with earlier published results, for example Flather [2] and Gjevik & Straume [3]. In particular, the location of the amphidromes east and north of Iceland is similar.

In table 1 the model results for the amplitude and phase of the M2-tide are compared with amplitude and phase derived from tidal gauge measurements at various stations around Iceland and elsewhere.

Table 1: Computed and measured amplitude and phase.

Station	Measured Amplitude (cm)	Measured Phase (°)	Computed Amplitude (cm)	Computed Phase (°)	Difference Amplitude (cm)	Difference Phase (°)
Reykjavík	131.9	183.3	135.0	192.4	3.1	9.1
Stykkishólmur	138.8	201.9	144.0	207.0	5.2	5.1
Ísafjörður	70.0	242.1	71.5	250.0	1.5	7.9
Skagaströnd	48.2	292.6	40.6	307.4	-7.6	14.8
Ólafsfjörður	42.4	312.0	36.3	326.6	-6.1	14.6
Raufarhöfn	43.6	335.5	39.0	353.7	-4.6	18.2
Seyðisfjörður	48.6	31.7	51.4	49.9	2.8	18.2
Djúpivogur	65.1	97.3	68.8	113.5	3.7	16.2
Vestmannaeyjar	87.9	157.7	94.1	172.5	6.2	14.8
Sandgerði	119.2	178.6	125.0	186.9	5.8	8.3
Reykjanes-ridge	91.0	180.0	93.5	189.0	2.5	9.0
Bergen	43.9	284.4	41.9	295	-2.0	10.6

In general the agreement is quite satisfactory. The computed amplitude is within 10 cm of the measured one at all the stations, showing both positive and negative differences. The amplitude along the north and east coasts of Iceland is very sensitive to the precise location of the amphidromes north and east of the country. The amphidrome north of Iceland seems to be located too close to the country in the current model, resulting in an underestimation of the amplitude at stations along the north coast (Skagaströnd, Ólafsfjörður and Raufarhöfn). On the other hand, Flather [2] and Gjevik & Straume [3] seem to predict the amphidrome too far north, resulting in overestimation of the amplitude along the north coast of Iceland.

In the current model the error in the phase lag is however systematically positive, from around 5° to 20°. Again, the largest errors are at stations along the north and east coast of Iceland (Skagaströnd, Ólafsfjörður, Raufarhöfn Seyðisfjörður and Djúpivogur), where the phase lag changes very rapidly due to the proximity of the amphidromes. There may be several reasons for the systematically positive phase lag error, including a delay of the forcing at the open boundary due to the application of the radiation boundary conditions through eqn [4], and the lack of direct tide generating forces within the model domain, as will be discussed in §4.

Up to now the emphasis has been put on comparing the model results with tidal gauge measurements, but close comparison with current measurements in the ocean around Iceland is planned.

4. Discussion

A numerical model of the tides and tidal currents in the ocean around Iceland has been set up. Preliminary results for the M2 tide show promising results, indicating a good agreement with observations and earlier results. However, a closer comparison with measurements, especially current measurements, is planned, together with some refinements of the model, including a variable Coriolis coefficient (which is currently constant in the model), and addition of tide generating forces within the model domain.

Future plans for the model are to compute the complete astronomical tide, and be able to simulate actual tidal motions during particular periods of time anywhere within the model area. One benefit of basing the model on the finite element method is the ability to increase the spatial resolution of the model locally within areas of special interest. Such simulations of the local tidal currents may have applications in various fields, such as predictions concerning transport and mixing of water bodies, rescue at sea, migration of fish, etc. After some extensions of the model, including the addition of the atmospheric pressure force terms in the momentum equations, it may also be used to simulate and predict storm surges at the coast of Iceland, an issue of importance to communities on the south coast of Iceland, which regularly experience coastal flooding due to storm surges.

Acknowledgements:

This work has been supported through grants from the National Science Foundation of Iceland and the University of Iceland. The program AQUASEA is made available by Vatnaskil Consulting Engineers, Reykjavík, Iceland.

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