Fire control algorithms and software for the modular naval artillery concept (MONARC) of the German navy

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Abstract

The German navy is about to implement a modular naval artillery concept. This means that weapon systems used by, e.g., the army will be introduced on navy vessels. One main project is the installation of the turret of the 155mmL52 Panzerhaubitze 2000 (PzH 2000, armored self propelled howitzer 2000) on frigates of type 124 and 125. It is an important goal to achieve with this weapon system high fire power, long ranges and high accuracy. High accuracy is especially important, since smart weapons (like e.g. SMArt155) shall be used. Additionally, the weapon system should be able to fire while the weapon carrier is moving, and also, while the target is moving. Looking at the desired firing ranges of up to 50 miles, this is a principal challenge for fire control algorithms and software. Not enough, the algorithms should be able to work in real time, i.e. compute fire orders in less than a second. In this paper the basic solution to this problem will be given by using a specially developed method in combination with the NATO armaments ballistic kernel (NABK). The problem of circular error probability (CEP) increase because of moving weapon/target will also be discussed.
Keywords: modular naval artillery concept, howitzer, frigate, indirect firing, moving weapon, moving target, fire control, circular error probability.

1 Introduction

A consortium consisting of Howaldtswerke-Deutsche Werft, Krauss-Maffei Wegmann and Rheinmetall W & M recently proposed a new artillery concept for navy vessels. It is named MONARC: modular naval artillery concept for naval gun fire.

Goals of this concept are:

- to increase the range and effectiveness of ship-mounted artillery and ammunition in naval warfare,
- to deliver a variety of ammunition types to distant targets from seaborne platforms onshore,
- to use well introduced German heavy-calibre artillery systems for ground forces,
- to complete the project without expensive R & D efforts in a short time.

To show the feasibility of the concept, a complete PzH 2000 turret has been temporarily mounted to the foredeck of the F124-class frigate Hamburg, see fig. 1. It was shown that the necessary modifications to existing and future warships are relatively inexpensive and that the weapon system still remained fully operational.

Figure 1: Frigate Hamburg with mounted turret of PzH 2000.

However, some problems still have to be solved:

- structural problems: mount a big gun onto a small vessel (space, mass, recoil),
- material problems: corrosion of the gun and the turret,
- ammunition: storage and handling have to be modified,

- fire control: the gun has to be stabilized, new fire control algorithms have to be developed, and the increase of CEP has to be investigated.

This paper deals with the development of fire control algorithms in case of moving weapon and/or target and the estimation of CEP increase due to movements of the ship: pitch, roll, yaw and the 3D surface velocity vector (including heaving). Since a new gun stabilization system will be used, the increase of CEP due to these factors should not be too dramatic.

For fire control the NATO armaments ballistic kernel according to STANAG 4355 (modified point mass model [1]) will be employed. It computes from a multitude of input data (e.g. weapon position, target position, ammunition data, weather data) azimuth α and elevation ϵ of the gun. In its version 6.0 (20 February 2004) it is not able to compute fire orders for moving weapons and/or targets. As it can be easily shown by some simulations, the deviations from the aimpoint are rather large if weapon and/or target are moving. Given the following data: (i) the weapon is in the point of origin of the coordinate system, (ii) the target is 10 000 m north of the weapon, i.e. $\alpha = 6389.0$ mils, $\epsilon = 417.3$ mils, then the following deflections occur:

- if the weapon is moving with 15 m/s (appr. 30 knots) east, then the resulting deflection will be appr. +330 m,
- if the weapon is moving with 15 m/s west, then the resulting deflection will be appr. -340 m.

The differences between east movement and west movement are mainly due to *Coriolis* forces. This simple example clearly shows that an appropriate fire control algorithm has to compensate for the movement of weapon and/or target to deliver ammunition precisely.

2 Fire control for moving weapon and/or target: idea

The main idea of the following approach is to find a fire solution for indirect or direct firing with a *minimal* number of solutions of the equations of motion of exterior ballistics when there is an arbitrary relative movement between weapon and target.

It is important to emphasize that the number of solutions of the equations of motion has to be small. If this is not the case, computation time will be large, and the solution would be impractical. One could imagine to use two nested loops for α and ϵ with very small increments like 0.1 mils and iterate until a fire solution has been found. This was tested and the results were impractical: some 10 000 solutions of the equations of motion, or computing time in the range of some 10 minutes.

The task of finding a fire control solution with a minimal number of integrations of the equations of motion is solved as follows:

a) In specific points of the weapon and the target a coordinate system will be fixed: CS_{weapon}, CS_{target}. It makes sense to fix CS_{weapon} at the muzzle, since computations will be simplified in this case.
b) When the projectile leaves the barrel, the time t will be set to zero.

c) When the projectile leaves the barrel, the position vector of the projectile $\vec{r}$ will be set to the zero vector $\vec{0}$, cf. a).
d) When the projectile leaves the barrel, CS_{weapon} becomes the inertial system $I^\star$.
e) The resulting initial velocity $\vec{v}_0^\star$ is the vector sum of the initial velocity $\vec{v}_0$ and the velocity of the muzzle $\vec{v}_{muzzle}$ with respect to $I^\star$: $\vec{v}_0^\star = \vec{v}_0 + \vec{v}_{muzzle}$.
f) The movement of the target, represented by CS_{target} will be determined relatively to $I^\star$. Therefore, two time depending vectors can be calculated: (i) the position vector $\vec{r}_{rel}$ of the relative movement between weapon and target, and, (ii) the vector $\vec{v}_{rel}$ of the relative velocity between weapon and target.
g) The vector of the absolute wind velocity with respect to $I^\star$, $\vec{v}_{wind}$, will be corrected by using the known vector $\vec{v}_{rel}$, thus giving a corrected wind vector $\vec{v}_{wind}^{corr}$.
h) A function $J(\alpha, \epsilon)$ with the following property will be constructed: $J(\alpha, \epsilon) = 0$, if after a certain time of flight t_{flight} the position vector of the projectile $\vec{r}$ is sufficiently close to the relative position vector $\vec{r}_{rel}$.
i) The zero of $J(\alpha, \epsilon)$ can be sought with different methods for zeroing functions with two or more variables (e.g. *Newton-Raphson* method[2]). The resulting values $(\alpha^\star, \epsilon^\star)$ are the fire control solution.

3 Fire control for moving weapon and/or target: algorithm

We consider $I^\star$ to be a cartesian coordinate system with axes (x, y, z). For $t = t_{flight}$ the vectors $\vec{r}(t_{flight})$ and $\vec{r}_{rel}(t_{flight})$ should be equal in $I^\star$, thus giving:

$$x(t_{flight}) = x_{rel}(t_{flight}),$$
$$y(t_{flight}) = y_{rel}(t_{flight}),$$
$$z(t_{flight}) = z_{rel}(t_{flight}).$$

Since we have only two variables, namely α and ϵ, we need a third variable to satisfy the above equations. This will be the time of flight t_{flight}. The integration of the equations of motion will be continued, until $z(t_{flight}) = z_{rel}(t_{flight})$, or at least:

$$\|(z(t_{flight}) - z_{rel}(t_{flight}))\| \leq \tilde{\epsilon}.$$

Therefore t_{flight} is not longer unknown and we have a well defined problem: A system of 2 nonlinear equations with the two unknowns (α, ϵ). Now we can define:

$$\tilde{x}(\alpha, \epsilon) = x(t_{flight}) - x_{rel}(t_{flight}),$$
$$\tilde{y}(\alpha, \epsilon) = y(t_{flight}) - y_{rel}(t_{flight}),$$

and

$$J\begin{pmatrix} \alpha \\ \epsilon \end{pmatrix} = \begin{pmatrix} \tilde{x}(\alpha, \epsilon) \\ \tilde{y}(\alpha, \epsilon) \end{pmatrix} \Longrightarrow 0.$$

Having found the values $(\alpha^\star, \epsilon^\star)$ which provide a zero of J, the fire control solution is found.

With the *Jacobian* $\bar{J}$ of the problem:

$$\bar{J} = \begin{pmatrix} \frac{\partial \tilde{x}}{\partial \alpha} & \frac{\partial \tilde{x}}{\partial \epsilon} \\ \frac{\partial \tilde{y}}{\partial \alpha} & \frac{\partial \tilde{y}}{\partial \epsilon} \end{pmatrix}$$

a *Newton-Raphson* iteration scheme with index $i = 1, 2, \ldots$ can be written as follows:

$$\begin{pmatrix} \alpha \\ \epsilon \end{pmatrix}^{i+1} = \begin{pmatrix} \alpha \\ \epsilon \end{pmatrix}^{i} - \bar{J}_i^{-1} \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}^{i},$$

with

$$\bar{J}^{-1} = \frac{1}{\left(\frac{\partial \tilde{x}}{\partial \alpha}\frac{\partial \tilde{y}}{\partial \epsilon} - \frac{\partial \tilde{x}}{\partial \epsilon}\frac{\partial \tilde{y}}{\partial \alpha}\right)} \begin{pmatrix} \frac{\partial \tilde{y}}{\partial \epsilon} & -\frac{\partial \tilde{x}}{\partial \epsilon} \\ -\frac{\partial \tilde{y}}{\partial \alpha} & \frac{\partial \tilde{x}}{\partial \alpha} \end{pmatrix}.$$

To start the computation of a fire order an initial approximation (α_0, ϵ_0) is needed. It turned out that the actual azimuth of the target is a good approximation for α_0. The initial guess for the elevation ϵ_0 can be provided by a built in NABK function or a simple ballistical table, since the distance to the target is known. The following loop will give a precise result after a maximum of 4 iterations with three integrations per step:

1. Integrate the equations of motion with the initial guess (α_0, ϵ_0).
2. Integrate the equations of motion with the actual $\alpha + \delta\alpha$ and save the results.
3. Integrate the equations of motion with the actual $\epsilon + \delta\epsilon$ and save the results.
4. Compute from steps 1,2,3 (or 6., respectively) the partial derivatives $\frac{\partial \tilde{x}}{\partial \alpha}$, $\frac{\partial \tilde{y}}{\partial \alpha}$, $\frac{\partial \tilde{x}}{\partial \epsilon}$, $\frac{\partial \tilde{y}}{\partial \epsilon}$.
5. Perform a *Newton-Raphson* step.
6. Integrate the equations of motion with the new values. If the fire solution is precise enough: stop, else goto 2.

4 Fire control for moving weapon and/or target: example

Combining the developed algorithm with NABK provides a software which is able to compute fire orders in less than a second on common Personal Computers. Therefore a high flexibility of warfare and also an engagement with several targets is possible.

Figure 2 shows an example for the application of the fire control software. The moving weapon W is in the center of the picture. The numbers indicate, how many iterations were necessary to find a fire order. The fixed data of the example are:

- weapon and target altitude 0 m,
- low angle of fire,
- target grid spacing 1000 m,
- weapon azimuth is 1600 mils, i.e. weapon is moving east (to the right in the picture) with 15 m/s,

- target azimuth is 4800 mils, i.e. the target is moving west (to the left in the picture) with 15 m/s,
- weapon is PzH 2000.

It can be seen that in this example (i) after a maximum of 3 iterations a fire control solution was found, if one existed for the used ammunition and target distances, and (ii) the larger the distance of the moving target, the larger is the number of iterations necessary to find a fire order.

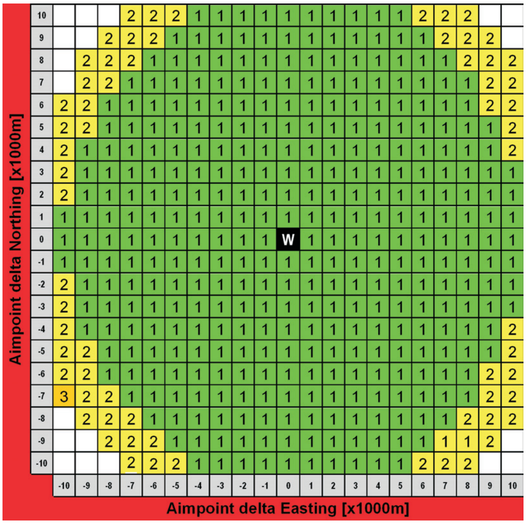

Figure 2: M107, DM74, M4/6W, maximum shooting distance 10 000 m.

5 Circular error probability (CEP)

5.1 Definition of CEP

CEP is the radius of a circle which includes x % of the values of a sample [3]. The percentage x is often written as a suffix, e.g. CEP_{20} for 20%. If no suffix is provided, 50% is the usual default value for military purposes.

The general definition of CEP is:

$$P_\rho = \frac{1}{2\pi\sqrt{|\Phi|}} \int\int_{x^2+y^2<\rho^2} e^{-\frac{1}{2}(x,y)\Phi^{-1}(x,y)^t} \mathrm{d}x\mathrm{d}y.$$

P_ρ is the probability that a random vector $N(0,\Phi)$ will be mapped in a circle with the radius ρ. It is possible to simplify the above double integral with the help of a

Bessel function of order 0, I_0, which results in:

$$P_\rho = \frac{b}{a} \int_0^{\frac{\rho}{b}} I_0 \left(\frac{1}{4} t^2 \left(\frac{b}{a} - 1 \right) \right) e^{-\frac{1}{4} t^2 \left(1 + \left(\frac{b}{a}\right)^2\right)} t \mathrm{d}t,$$

with $t = \frac{r}{b}$ and a, b as the eigenvalues of the variance-covariance matrix Φ:

$$\Phi := \begin{pmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{yx}^2 & \sigma_y^2 \end{pmatrix}$$

For numerical computations the following formula can be derived by using a *Frobenius* series:

$$P_\rho = 1 - \frac{2\beta}{\beta^2 + 1} \sum_{k=0}^{\infty} \frac{\left(\beta^2 - 1\right)^{2k}}{(k!)^2 \, 4^k \left(\beta^2 + 1\right)^{2k}} \Gamma \left(2k + 1, \frac{\beta^2 + 1}{4} \left(\frac{\rho}{b} \right)^2 \right),$$

with $\beta = \frac{b}{a}$, and $\Gamma(k, x)$ as the incomplete Gamma function which is defined as:

$$\Gamma(k, x) = \int_x^{\infty} \xi^{k-1} e^{-\xi} \mathrm{d}\xi.$$

However, it is still difficult to estimate CEP with the above formula. The computer algebra system *MAPLE* uses the following formula for the incomplete Gamma function:

$$\Gamma(k, x) = \Gamma(k) - \frac{x^k}{k} {}_1F_1 \left(k, 1 + k, -x \right).$$

Here $\Gamma(k)$ is the complete Gamma function and ${}_1F_1(n, d, z)$ is *Barnes's* extended hypergeometric function:

$$\Gamma(x) = \int_{t=0}^{\infty} e^{-t} t^{x-1} \mathrm{d}t,$$

$${}_1F_1 = \sum_{t=0}^{\infty} \frac{\Gamma(k + t)\Gamma(k + 1)(-x)^t}{\Gamma(k)\Gamma(k + 1 + t)t!}.$$

To compute the complete Gamma function we use the *Lanczos* approximation. It provides the Gamma function for positive real numbers with an accuracy of $|\tilde{\epsilon}| < 2 \cdot 10^{-10}$ which is completely sufficient for the estimation of CEP:

$$\Gamma(x) = \left(\frac{\sqrt{2\pi}}{x} \left(p_0 + \sum_{n=1 \ldots 6} \frac{p_n}{x + n} \right) \right) (x + 5, 5)^{x + 0,5} \, e^{-(x + 5,5)},$$

with

$$p_0 = 1.000000000190015,$$

$$p_1 = 76.18009172947146,$$
$$p_2 = -86.50532032941677,$$
$$p_3 = 24.01409824083091,$$
$$p_4 = -1.231739572450155,$$
$$p_5 = 1.208650973866179 \cdot 10^{-3},$$
$$p_6 = -5.395239384953 \cdot 10^{-6}.$$

5.2 Estimating CEP

CEP can be estimated either by using experimental data or by simulations. Using experimental data, one can straightforward estimate the elements of the variance-covariance matrix Φ from the coordinates of the impact points and compute CEP according to the equations given in the last section.

In case of simulations one has to know specific data of the weapon platform, i.e. the gun and the ship. It is known that in case of resting weapon and target the maximum error of the gun is $0.3\% \ldots 0.4\ \%$ of the shooting distance. So the question arises, whether in case of moving weapon and/or target there will be a substantial increase in CEP. In that case mainly the errors of the weapon stabilization system and the measurement error of the velocity vector of the ship are important input data for numerical simulations. Using that information one can generate data sets consisting of nominal values of the ship velocity vector, azimuth and elevation $\pm$ small deviations. The deviations can be generated by using a random number generator which provides a desired probability density function, e.g. a *Gaussian*. With these data one can determine the deviations from the coordinates of the nominal hitpoint caused by the random fluctuations by integration of the equations of motion. Consequently, the elements of Φ can be estimated, and finally, CEP can be computed.

Since it might be time consuming to write a computer program for the computation of CEP according to the equations of the last section, a self-explaining Maple script is given, which computes CEP from the elements of Φ.

```
> restart:with(stats):with(describe):
> # Computes CEP from the elements of Phi
> cep:=proc(sigmaX,sigmaY,covXY)
> local sigmaXk,sigmaYk,og,sigma,PGamma,beta,dummy:
> with(linalg):
> if abs(covXY) > 0.95*sigmaX^2 then return(`covariance too large`) fi:
> if abs(covXY) > 0.95*sigmaY^2 then return(`covariance too large`) fi:
> sigma:=matrix(2,2,[sigmaX^2,covXY,covXY,sigmaY^2]):
> sigmaXk:=evalf(sqrt(eigenvalues(sigma)[1]));
> sigmaYk:=evalf(sqrt(eigenvalues(sigma)[2]));
> if abs(sigmaYk-sigmaXk)<5e-9 then sigmaYk:=sigmaYk+1e-8: fi:
> if (sigmaYk>=sigmaXk) then
> dummy:=sigmaYk; sigmaYk:=sigmaXk; sigmaXk:=dummy;
> fi;
> beta:=sigmaYk/sigmaXk;
```

```
> PGamma:=(rho,beta,sigmaYk)->1-2*beta/(beta^2+1)
> *
> sum(
> (beta^2-1)^(2*k)/(k!*k!*4^k*(beta^2+1)^(2*k))
> *
> GAMMA(2*k+1,(beta^2+1)/4*(rho/sigmaYk)^2)
> ,k=0..20
> );
> # Now CEP x% (x=0.5-> CEP 50%)
> og:=5.0*sqrt(sigmaXk^2+sigmaYk^2):
> fsolve(PGamma(rho,beta,sigmaYk)=0.5,rho,0..og); # Result is CEP
> end proc:
```

5.3 CEP example

The common parameters for the CEP examples according to table 1 are: (i) shooting distance 25 000 m, (ii) projectile M549A1, (iii) high angle, (iv) vessel goes north (0 mils), (v) standard deviations are $\sigma_v \approx 0.3$ m/s, $\sigma_\alpha, \sigma_\epsilon \approx 0.3$ mils which is close to real values.

In table 1 it can be seen that CEP resulting from moving weapon and/or target is much smaller than the CEP of the gun itself. Therefore concept MONARC can be deployed from the point of view of exterior ballistics.

Table 1: CEP examples: + means that the corresponding standard deviation was present in the simulation. CEP was computed from 500 simulation runs.

aimpoint/mils	σ_v	σ_α	σ_ϵ	CEP/m
0	+	-	-	1.35
1600	+	-	-	1.49
0	-	+	+	13.01
1600	-	+	+	12.79
0	+	+	+	13.18
1600	+	+	+	12.81

6 Conclusions

In this paper a fire solution for moving weapon and/or target for direct and indirect firing was proposed. The increase of CEP due to this approach was estimated. Examples show that the approach works well and the increase of CEP due to the relative motion of weapon and target can be neglected. The next step will be the experimental verification of the theoretical fire solution.

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