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Integral equation solution of heat extraction from a fracture in hot dry rock

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Abstract

The problem of heat extraction by circulating water in a fracture embedded in hot dry rock is formulated as an integral equation solution. The mathematical model involves solving a heat transport equation in fracture and a heat diffusion equation in the infinite domain geothermal reservoir. The need to model the infinite domain is eliminated by the use of Green's function. Both two- and three-dimensional problems are formulated in this paper. Numerical implementation is given in two-dimension problems only.

Introduction

The hot dry rock (HDR) inside the earth can be an economical source of energy, owing to its large heat capacity. One of the methods for heat extraction in HDR is to drill wells to intersect pre-existing or man-made fractures in geothermal reservoirs. Cold water is injected in one well, and hot water is recovered from the other.

A mathematical model for the heat extraction involves solving a heat transport equation with known flow velocity in a two-dimensional fracture plane. This equation is then coupled with a three-dimensional heat diffusion equation in the geothermal reservoir.

The modeling of three-dimensional heat diffusion in an infinite reservoir is a cumbersome task for the domain based numerical methods, such as the finite element and finite difference methods. To avoid the difficulty, typically, the heat flow in the reservoir is simplified to one-dimensional and perpendicular to the fracture [1].

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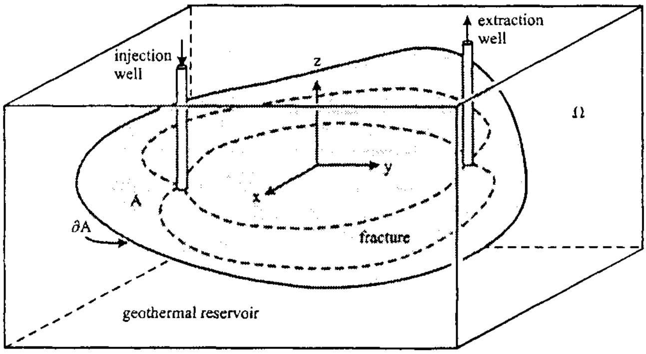

Figure 1: Heat extraction from a plan fracture in infinite geothermal reservoir.

In this paper we shall formulate the HDR heat extraction problem with fully three-dimensional heat diffusion in the reservoir. We shall demonstrate that by utilizing Green's function, the three-dimensional problem becomes a two-dimensional one defined on the fracture plane only. For the special case of a two-dimensional HDR reservoir, the integral equation formulation convert the problem into a one-dimensional Fredholm integral equation of the second kind. The integral equation is regularized and integrated using Simpson's rule. Numerical examples are presented for two-dimensional problems.

Mathematical formulation

Figure 1 gives a schematic view of heat extraction from a hot dry rock system. Consider a flat fracture with small aperture width lying on the x-y plane. Defining $\mathbf{q} = b\,\mathbf{v}$, where b is the aperture width and $\mathbf{v}$ is the average velocity over the width, the continuity condition gives

$$\nabla_2 \cdot \mathbf{q}(x,y) = Q\left[\delta(x - x_e,\, y - y_e) - \delta(x - x_i,\, y - y_i)\right]; \quad x, y \in A \tag{1}$$

In the above $\nabla_2\cdot$ stand for the divergence operator in two-dimension, Q is the injection and extraction flow rate (must be equal), δ is the Dirac delta function, and $(x_i,\, y_i)$ and $(x_e,\, y_e)$ are respectively the location of injection and extraction well. Coupled with a flow equation based on lubrication theory and a proper set of boundary conditions, the specific discharge $\mathbf{q}$ can be solved. This part, however, is not considered in the present paper. Here we assume that $\mathbf{q}$ has been solved and is given. The heat transport equation in the fracture can be written as

$$\rho_w c_w \nabla_2 \cdot [\mathbf{q}(x,y)T(x,y,0,t)] = 2K_r \left.\frac{\partial T(x,y,z,t)}{\partial z}\right|_{z=0^+} +$$
$$\rho_w c_w Q\left[T(x_e,y_e,0,t)\,\delta(x-x_e,\, y-y_e) - T_{wo}\,\delta(x-x_i,\, y-y_i)\right]; \quad x,y \in A \tag{2}$$

where T is the water temperature in fracture, T_{wo} is the temperature of injected water, ρ_w is the water density, c_w is the specific heat of water, and K_r is the rock thermal conductivity. Next we consider heat conduction in the rock, which is governed by the diffusion equation in three-dimension:

$$K_r \nabla_3^2 T(x,y,z,t) = \rho_r c_r \frac{\partial T(x,y,z,t)}{\partial t} \qquad x,y,z \in \Omega \tag{3}$$

where ρ_r is the rock density, c_r is the specific heat of rock, and ∇_3^2 is the Laplacian operator in three-dimension.

The governing equations (1)–(3) are subject to initial and boundary conditions. The initial temperature is assumed to be constant everywhere:

$$T(x,y,z,0) = T_{ro} \tag{4}$$

At the water injection point $(x_i, y_i, 0)$, the temperature is fixed to the injection water temperature

$$T(x_i,y_i,0,t) = T_{wo} \tag{5}$$

It is in fact easier to write the above equations in terms of the dimensionless temperature deficit:

$$T_d = \frac{T_{ro} - T}{T_{ro} - T_{wo}} \tag{6}$$

which has the limits $0 \le T_d \le 1$. Equations (2) and (3) becomes

$$\rho_w c_w \nabla_2 \cdot [\mathbf{q}(x,y)T_d(x,y,0,t)] = 2K_r \left.\frac{\partial T_d(x,y,z,t)}{\partial z}\right|_{z=0^+} +$$
$$\rho_w c_w Q\left[T_d(x_e,y_e,0,t)\,\delta(x-x_e,\, y-y_e) - \delta(x-x_i,\, y-y_i)\right] \tag{7}$$

and

$$K_r \nabla_3^2 T_d(x,y,z,t) = \rho_r c_r \frac{\partial T_d(x,y,z,t)}{\partial t} \tag{8}$$

The initial condition is

$$T_d(x,y,z,0) = 0 \tag{9}$$

and boundary condition

$$T_d(x_i,y_i,0,t) = 1 \tag{10}$$

Next we apply Laplace transform to the above equations

$$\rho_w c_w \nabla_2 \cdot \left[\mathbf{q}(x,y)\widetilde{T}_d(x,y,0,s)\right] = 2K_r \left.\frac{\partial \widetilde{T}_d(x,y,z,s)}{\partial z}\right|_{z=0^+} + \rho_w c_w Q\left[\widetilde{T}_d(x_e,y_e,0,s)\,\delta(x-x_e,\,y-y_e) - \frac{1}{s}\,\delta(x-x_i,\,y-y_i)\right] \quad (11)$$

$$K_r \nabla_3^2 \widetilde{T}_d(x,y,z,s) = s\rho_r c_r \widetilde{T}_d(x,y,z,s) \quad (12)$$

where s is the Laplace transform parameter.

Integral equation

The temperature in reservoir due to a continuous point heat source with strength $\widetilde{\mu}$ is given by the Green's function

$$\widetilde{G} = \frac{\widetilde{\mu}}{4\pi K_r}\frac{1}{R}\exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}\,R\right) \quad (13)$$

where $R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$, and $\mathbf{x}'$ is the source location. The temperature deficit in the reservoir resulting from a distribution of sinks on the fracture surface A is given by

$$\widetilde{T}_d(x,y,z,s) = \frac{1}{4\pi K_r}\int_A \widetilde{\mu}\,(x',y',s)\,\frac{1}{R_1}\exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}\,R_1\right)\,dx'dy' \quad (14)$$

where $R_1 = \sqrt{(x-x')^2 + (y-y')^2 + z^2}$. The heat source strength can be represented as

$$\widetilde{\mu}\,(x,y,s) = -2K_r \left.\frac{\partial \widetilde{T}_d(x,y,z,s)}{\partial z}\right|_{z=0^+} \quad (15)$$

Utilizing (11) and (15), we can write (14) as

$$\widetilde{T}_d(x,y,z,s) = \frac{-\rho_w c_w}{4\pi K_r}\int_A \nabla_2 \cdot \left[\mathbf{q}(x',y')\widetilde{T}_d(x',y',0,s)\right] \frac{1}{R_1}\exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}\,R_1\right)\,dx'dy' + \frac{\rho_w c_w Q}{4\pi K_r}\left[\widetilde{T}_d(x_e,y_e,0,s)\frac{1}{R_e}\exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}\,R_e\right) - \frac{1}{s}\frac{1}{R_i}\exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}\,R_i\right)\right] \quad (16)$$

where $R_e = \sqrt{(x-x_e)^2 + (y-y_e)^2 + z^2}$ and $R_i = \sqrt{(x-x_i)^2 + (y-y_i)^2 + z^2}$. Applying the divergence theorem to the

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above, and utilizing the no flux condition $q_n = 0$ on the fracture boundary ∂A, we obtain the following integral equation

$$\widetilde{T}_d(x, y, z, s) = \frac{\rho_w c_w Q}{4\pi K_r}\left[\widetilde{T}_d(x_e, y_e, 0, s)\frac{1}{R_e}\exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}\,R_e\right) - \frac{1}{s}\frac{1}{R_i}\exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}\,R_i\right)\right]$$
$$-\frac{\rho_w c_w}{4\pi K_r}\int_A \widetilde{T}_d(x', y', 0, s)\frac{1}{R_1^2}\left(1+\sqrt{\frac{\rho_r c_r s}{K_r}}R_1\right)\exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}\,R_1\right)\left[q_x(x', y')\frac{\partial R_1}{\partial x'} + q_y(x', y')\frac{\partial R_1}{\partial y'}\right]\,dx'dy' \quad (17)$$

If we apply the above equation on the fracture surface, $(x, y) \in A$ and $z = 0$, we then obtain

$$\widetilde{T}_d(x, y, 0, s) = \frac{\rho_w c_w Q}{4\pi K_r}\left[\widetilde{T}_d(x_e, y_e, 0, s)\frac{1}{r_e}\exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}\,r_e\right) - \frac{1}{s}\frac{1}{r_i}\exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}\,r_i\right)\right]$$
$$-\frac{\rho_w c_w}{4\pi K_r}\int_A \widetilde{T}_d(x', y', 0, s)\frac{1}{r^2}\left(1+\sqrt{\frac{\rho_r c_r s}{K_r}}r\right)\exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}\,r\right)\left[q_x(x', y')\frac{\partial r}{\partial x'} + q_y(x', y')\frac{\partial r}{\partial y'}\right]\,dx'dy' \quad (18)$$

where $r = \sqrt{(x-x')^2 + (y-y')^2}$, $r_e = \sqrt{(x-x_e)^2 + (y-y_e)^2}$, and $r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2}$. We note that (18) is entirely defined on the fracture surface, $x, y \in A$. It can be utilized for a boundary element solution. There is no need to discretize the domain Ω. After the solution of temperature in the fracture, $\widetilde{T}_d(x, y, 0, s)$, using (18), we can use (17) to find temperature everywhere in the reservoir. As the last step of the solution, we apply the approximate Laplace inversion to invert the solution back to the time domain.

Two-dimensional problem

As a special case, we also investigate the two-dimensional fracture problem as shown in Figure 2. The geothermal reservoir is bound at top and bottom by impermeable and non-conducting layers, but is of infinite extent in the horizontal directions. The governing equations expressed in temperature deficit and under Laplace transform become

$$\frac{d\widetilde{T}_d(x, 0, s)}{dx} = \frac{2K_r}{q\rho_w c_w}\left.\frac{\partial \widetilde{T}_d(x, y, s)}{\partial y}\right|_{y=0} \quad (19)$$

$$\frac{\partial^2 \widetilde{T}_d(x, y, s)}{\partial x^2} + \frac{\partial^2 \widetilde{T}_d(x, y, s)}{\partial y^2} = \frac{\rho_r\, c_r\, s}{K_r}\,\widetilde{T}_d(x, y, s) \quad (20)$$

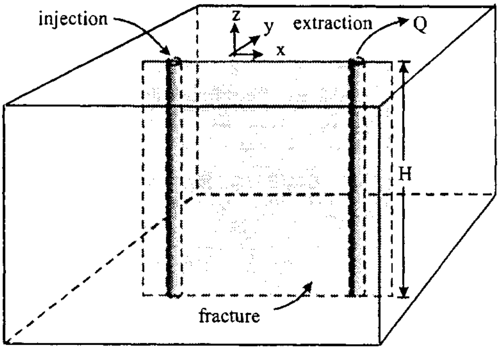

Figure 2: Heat extraction from a fracture in a bounded geothermal layer.

The heat source in two-dimension is given by

$$G(x,y,x',y',s) = \frac{\widetilde{\mu}}{2\pi K_r\, s}\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\, r\right) \tag{21}$$

where K_0 is the modified Bessel function of the second kind of order zero. The temperature in the reservoir due to the distribution of source along the fracture trajectory $0 \leq x \leq L$ is given by

$$\widetilde{T}_d(x,y,s) = \frac{1}{2\pi K_r}\int_0^L \widetilde{\mu}\,(x',s)\ \mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\, r\right)\ dx' \tag{22}$$

where

$$\widetilde{\mu}(x,s) = -2K_r\ \frac{\partial \widetilde{T}_d(x,y,s)}{\partial y}\bigg|_{y=0^+} \tag{23}$$

Following (19) and (23), we can express (22) as

$$\widetilde{T}_d(x,y,s) = -\frac{q\rho_w c_w}{2\pi K_r}\int_0^L \frac{d\widetilde{T}_d(x',0,s)}{dx'}\,\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\, r\right)\ dx' \tag{24}$$

Performing integration by parts, the above equation is transformed into

$$\begin{aligned}\frac{2\pi K_r}{q\rho_w c_w}\widetilde{T}_d(x,y,s) &= \frac{1}{s}\,\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}\,(x^2+y^2)}\right)\\ &-\widetilde{T}_d(L,0,s)\,\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}\left[(L-x)^2+y^2\right]}\right)\\ &+\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\int_0^L \widetilde{T}_d(x',0,s)\,\frac{x-x'}{\sqrt{(x-x')^2+y^2}}\mathrm{K}_1\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\, r\right)\ dx'\end{aligned} \tag{25}$$

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If we apply the above equation on the fracture trajectory, $y = 0$ and $0 \leq x \leq L$, we obtain

$$\frac{2\pi K_r}{q\rho_w c_w}\widetilde{T}_d(x,0,s) = \frac{1}{s}\,\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,x\right) - \widetilde{T}_d(L,0,s)\,\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,(L-x)\right) + \sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\int_0^L \widetilde{T}_d(x',0,s)\,\frac{x-x'}{|x-x'|}\mathrm{K}_1\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,|x-x'|\right)\,dx' \tag{26}$$

Equation (26) is defined in one-dimension in the interval $0 \leq x \leq L$. It can be used to solve for the water temperature in the fracture. Equation (25) is then used to evaluate the temperature everywhere.

Numerical implementation

The numerical implementation presented below is limited to two-dimensional problems. Equation (26) is identified as a Fredholm integral equation of the second kind. It is Cauchy singular due to the $1/r$ singularity contained in the Bessel function K_1, which needs to be regularized before a numerical integration can be performed. This can be accomplished by subtracting this exact integration

$$\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\int_0^L \widetilde{T}_d(x,0,s)\frac{x-x'}{|x-x'|}\mathrm{K}_1\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,|x-x'|\right)\,dx' = \widetilde{T}_d(x,0,s)\left[\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,(L-x)\right) - \mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,x\right)\right] \tag{27}$$

from (26) to obtain

$$\left[\frac{2\pi K_r}{q\rho_w c_w} - \mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,(L-x)\right) + \mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,x\right)\right]\widetilde{T}_d(x,0,s) = \frac{1}{s}\,\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,x\right) - \widetilde{T}_d(L,0,s)\,\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,(L-x)\right) + \sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\cdot \int_0^L \left[\widetilde{T}_d(x',0,s) - \widetilde{T}_d(x,0,s)\right]\,\frac{x-x'}{|x-x'|}\mathrm{K}_1\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,|x-x'|\right)\,dx' \tag{28}$$

We notice that the integral is no longer singular; hence the equation is regularized.

To approximately solve (28), we shall apply a quadrature rule. The well known Simpson's rule is selecte. Hence (28) becomes

$$\left[\frac{2\pi K_r}{q\rho_w c_w} - \mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,(L-x_i)\right) + \mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,x_i\right)\right] T_i =$$
$$T_0\,\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,x_i\right) - T_n\,\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,(L-x_i)\right)$$
$$+\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,\Delta x \sum_{j=0}^{n} w_j\,(T_j - T_i)\,\frac{x_i - x_j}{|x_i - x_j|}\mathrm{K}_1\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,|x_i - x_j|\right) \tag{29}$$

for $i = 1,\dots,n$, where n is the number of equal intervals, $\Delta x = L/n$ is the size of the intervals, $x_i = i\,\Delta x$ are the abscissas, w_i are weights, and T_i denote the discrete values $\widetilde{T}_d(x_i, 0, s)$. The first value T_0 is given by the boundary condition $T_0 = 1/s$. Using Simpson's $\frac{3}{8}$-rule, the weights are

$$w_j = \frac{3}{8}, \frac{7}{6}, \frac{23}{24}, 1, 1, \dots, 1, 1, \frac{23}{24}, \frac{7}{6}, \frac{3}{8}; \qquad \text{for } j = 0,\dots,n \tag{30}$$

Some caution is needed in evaluating the quadrature in (29) at the node $x_i = x_j$. Although the singularity has been removed by the term $T_j - T_i$, the program comes to a $0/0$ division. A proper limit needs to be taken:

$$\lim_{x'\to x}\left[\widetilde{T}_d(x', 0, s) - \widetilde{T}_d(x, 0, s)\right]\frac{x - x'}{|x - x'|}\mathrm{K}_1\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,|x - x'|\right)$$
$$= -\sqrt{\frac{K_r}{\rho_r\, c_r\, s}}\,\frac{\partial \widetilde{T}_d(x', 0, s)}{\partial x'}\bigg|_{x'=x} \approx \sqrt{\frac{K_r}{\rho_r\, c_r\, s}}\,\frac{T_{i-1} - T_{i+1}}{2\,\Delta x} \tag{31}$$

Equation (29) is then ready for evaluation.

We notice that (29) contains $n+1$ discrete values of T_i. Only n of them are unknown because T_0 is given by the boundary condition. Hence there exist n unknowns and n equations for $i = 1,\dots,n$. This forms a linear system

$$[\mathbf{A}]\{\mathbf{T}\} = \{\mathbf{b}\} \tag{32}$$

which can be solved for T_i.

If the temperature in the geothermal reservoir is needed, the quadrature rule can be applied to (25) such that the temperature at any location (x, y) can be found:

$$\widetilde{T}_d(x, y, s) = \frac{q\rho_w c_w}{2\pi K_r}\left[T_0\,\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}\,(x^2 + y^2)}\right)\right.$$
$$-T_n\,\mathrm{K}_0\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}\left[(L-x)^2 + y^2\right]}\right) + \sqrt{\frac{\rho_r\, c_r\, s}{K_r}}\,\Delta x \sum_{j=0}^{n} w_j T_j\,\cdot$$
$$\left.\frac{x - x_j}{\sqrt{(x - x_j)^2 + y^2}}\mathrm{K}_1\left(\sqrt{\frac{\rho_r\, c_r\, s}{K_r}\left[(x - x_j)^2 + (y - y_j)^2\right]}\right)\right] \tag{33}$$

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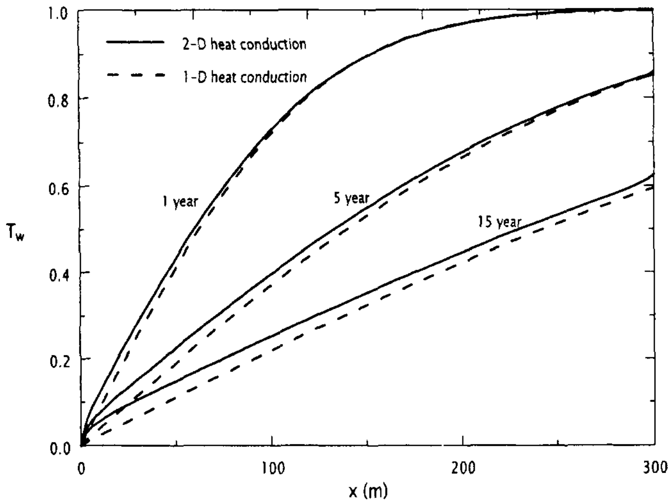

Figure 3: Normalized temperature distribution in the fracture based on 1-D and 2-D heat conduction model.

The above formula does not involve any unknown quantity; hence can be directly evaluated.

Numerical results

For the numerical example, we use the following set of parameters:

$$\rho_w = 1.0 \text{ g/cm}^3; \quad \rho_r = 2.65 \text{ g/cm}^3; \quad K_r = 6.2 \times 10^{-3} \text{ cal/cm} \cdot \text{s} \cdot {}^\circ C;$$
$$c_w = 1.0 \text{ cal/g} \cdot {}^\circ C; \quad c_r = 0.25 \text{ cal/g} \cdot {}^\circ C; \quad L = 300 \text{ m}; \quad q = 0.15 \text{ cm}^2/\text{s}$$

Figure 3 presents the temperature profile along the fracture length at three different times, 1, 5, and 15 years after production. The temperature shown is the normalized temperature $T_w = 1 - T_D = (T - T_{wo})/(T_{ro} - T_{wo})$. The two-dimensional heat conduction solution (solid lines) is computed based on (29). For comparison, we also present the analytical solution based on one-dimensional diffusion [2]. We observe that the 2-D case predicts a higher temperature for the obvious reason that more heat is supplied from the additional hot rock region. The increase in temperature is more significant near the inlet than outlet, and at large than small times.

The most useful information for the application is the extraction temperature. We plot in Figure 4 the normalized extraction temperature $T_w(L, 0, t)$ for 30 years of production time. We observe that the 2-D heat conduction

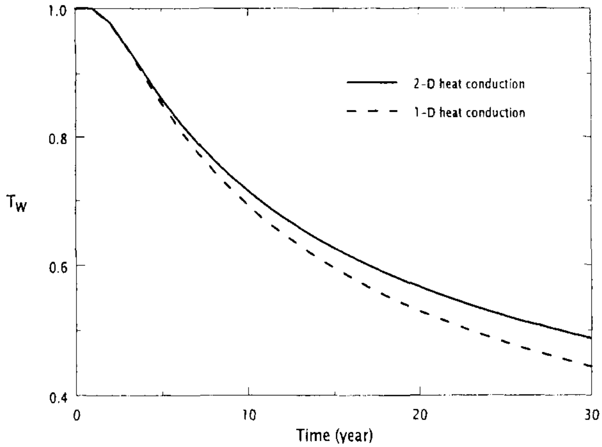

Figure 4: Normalized extraction temperature based on 1-D and 2-D heat conduction model..

case predicts a higher extraction temperature. The difference is more pronounced at large production time. At the end of 30 years, the predicted temperature T_w of the 2-D case is about 10% higher than the 1-D case. Another way to interpret the difference is that the 1-D case predicts a production life of 23 years for the heat extraction efficiency to drop below 50%, and the 2-D case predicts 29 years.

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