Transactions on Modelling and Simulation vol 20, © 1998 WIT Press, www.witpress.com, ISSN 1743-355X

Extension of the dual reciprocity boundary element method to the problems of phase-change and thermal wave propagation

Wen-Qiang Lu[a], Yuntao Zeng[a] & Jing Liu[b]
[a]*Department of Physics, Division of Thermal Science, The Graduate School at Beijing, University of Science and Technology of China, Academia Sinica, PO Box 3908, Beijing 100039, P.R. China*
[b]*Thermal Engineering Department, Tsinghua University, Beijing 100084, P.R. China*
Email: luwq@sun.ihep.ac.cn

Abstract

The numerical schemes of the Dual Reciprocity Boundary Element Method are developed and used to simulate the problems of phase change and thermal wave propagation. The two-dimensional thawing characters of frozen working medium in microgravity heat pipe are presented in this paper. The superposition, resolution, reflection and phase jumping phenomenon of the thermal wave propagation in biological tissues are also shown in the paper.

Introduction

Recently, the Dual Reciprocity Boundary Element Method (DRBEM) is developed as the effective numerical method of pure boundary integral without domain integral [1]. The DRBEM has been successfully applied to solve the problems as the steady and unsteady heat conduction, the sonic propagation in subsonic non-homogeneous flow, the flow and heat transfer with dissipation effect and inner heat source. However the reports to solve the problems about the heat conduction with phase-change and the thermal wave propagation by the DRBEM have been not yet seen. In this paper the DRBEM is extended to simulate the thawing process of the working medium in the microgravity heat pipe of high capacity parallel channel and the thermal wave propagation in biological tissues.

The DRBEM for simulating these problems

The basic idea of the DRBEM is to transform the control equation into boundary integral equation by a fundamental solution corresponding to the Laplace equation. The non-homogeneous terms (which can not be easily transformed to boundary integration through traditional boundary element method) in the original equation are further transformed into the boundary integral through a procedure which involves a series expansion using global approximating functions and the application of reciprocity principles [1].

The control equation to describe these problems can be written as

$$\nabla^2\theta = a\frac{\partial^2\theta}{\partial t^2} + b\frac{\partial\theta}{\partial t} + c\theta + d = h(x,y;t) \tag{1}$$

Employing the dual reciprocity theorem, equation (1) can be transformed into pure boundary integral equation, and its discretizing form is as the follow:

$$C_i\theta_i + \sum_{k=1}^{N}\int_{\Gamma_k} q^*\theta d\Gamma - \sum_{k=1}^{N}\int_{\Gamma_k}\theta^* q d\Gamma =$$
$$\sum_{j=1}^{N+L}\alpha_j\left(C_i\hat{\theta}_{ij} + \sum_{k=1}^{N}\int_{\Gamma_k} q^*\hat{\theta}_j d\Gamma - \sum_{k=1}^{N}\int_{\Gamma_k}\theta^*\hat{q}_j d\Gamma\right) \tag{2}$$

Where $C_i = \gamma / 2\pi$ (γ is internal angle at point i in radius). $q = \partial\theta / \partial n$, $\theta^* = \ln(1/r)$, $q^* = \partial\theta^* / \partial n$, n is the unit outward normal to boundary Γ. $\alpha = F^{-1}h = F^{-1}\left(a\ddot{\theta} + b\dot{\theta} + c\theta + d\right)$, where each column of F consists of a vector f_j containing the values of the function $f = 1 + r$ at the (N+L) DRBEM collocation points. N are the numbers of boundary nodes. L are the numbers of internal nodes. $\ddot{\theta} = \partial^2\theta / \partial t^2$,

$\dot{\theta}=\partial\theta/\partial t$, $\hat{q}_j=\left(r_x\frac{\partial x}{\partial n}+r_y\frac{\partial y}{\partial n}\right)\left(\frac{1}{2}+\frac{r}{3}\right)$, $\hat{\theta}_j(x,y)=\frac{r^2}{4}+\frac{r^3}{9}$. r is the distance from the point i of the application of the concentrated source to any other point j under consideration.

Then the matrix form of equation (2) becomes

$$-\left(H\hat{\Theta}-G\hat{Q}\right)F^{-1}\left(a\ddot{\theta}+b\dot{\theta}+c\theta+d\right)+H\theta=Gq \tag{3}$$

The time discretizing schemes are: $\ddot{\theta}=\frac{1}{\Delta t^2}\left(\theta^{m+1}+\theta^{m-1}-2\theta^m\right)$,

$\dot{\theta}=\frac{1}{\Delta t}\left(\theta^{m+1}-\theta^m\right)$, $\theta=\left(1-\theta_u\right)\theta^m+\theta_u\,\theta^{m+1}$, $q=\left(1-\theta_q\right)q^m+\theta_q\,q^{m+1}$,

Where θ_u and θ_q are parameters which position the values of θ and q between time levels m and $m+1$, respectively. Letting $S=-\left(H\hat{\Theta}-G\hat{Q}\right)F^{-1}$, after substituting the time discretizing schemes into equation (3), the equation of numerical calculation is obtained as the follow:

$$\left(\frac{Sa}{\Delta t^2}+\frac{Sb}{\Delta t}+Sc\theta_u+\theta_u H\right)\theta^{m+1}-\theta_q Gq^{m+1}+Sd$$
$$=\left(\frac{2Sa}{\Delta t^2}+\frac{Sb}{\Delta t}-Sc\left(1-\theta_u\right)-H\left(1-\theta_u\right)\right)\theta^m-\frac{Sa}{\Delta t^2}\theta^{m-1}+\left(1-\theta_q\right)Gq^m \tag{4}$$

The thawing process of the frozen working medium of modeling liquid channel in parallel channels microgravity heat pipe

Recently, some kinds of high capacity parallel channels heat pipe are being researched and applied to astrovehicle, space station, space shuttle and so on. For example, the Grumman monogroove heat pipe and the Lockheed graded-groove heat pipe are two of them. They are comprised of two parallel circular channels: vapor flow and liquid flow channels. When the parallel channels heat pipe is started up in space, total working medium exists as solid phase since the temperature is too low in space. Besides, the working environment of these heat pipes in space is very complex. The working liquids of the heat pipes may be frozen in extreme case. Therefore, for start-up or restart-up heat pipe, it is necessary to thaw frozen working medium.

The numerical simulation for the two-dimensional thawing process of the radial cross section in the evaporator of the heat pipe had been made by Green's function boundary element method[2]. In this paper, the DRBEM extended to calculate the thawing process of the frozen working medium of the axial cross section in modeling liquid channel of the heat pipe (see Fig. 1).

During the working medium in the pipe is thawed, the flow velocity of liquid medium is so small, that convective heat transfer can be neglected. The working temperature in heat pipe during thawing process is so low, that the radiant heat transfer can be also neglected. Therefore, the control equation in thawing process is transient heat conduction (parabolic-diffusion) equation. In this case, $\theta = T, a = c = d = 0, b = \dfrac{\rho C}{\lambda}$. T, ρ, C, λ are temperature, density, heat capacity and thermal conductivity, respectively. Equation (1) and corresponding pure boundary integral equation can be simplified.

Initial and boundary conditions are chosen as follow: $T(x, y, 0) = T_m$, where T_m is the thawing temperature of working medium; a constant heat flux is applied on the end of the channel ($x = 0$); in order to correctly capture phase-change interface, Fourier relation is used on phase-change interface:

$$\lambda_k \frac{\partial T_k}{\partial n_k} = \lambda_3 \frac{\partial T_3}{\partial n_3} + \rho_3 Q_c U \tag{5}$$

Where, Q_c and U are the latent heat of phase change and the normal velocity of phase change interface respectively; $k = 1$, or $k = 2$, subscripts 1, 2 and 3 refer to shell, liquid and solid working medium, respectively. On the interface between shell and working medium, the following balance of heat flux is used: $\lambda_1 \partial T_1 / \partial n_1 = \lambda_2 \partial T_2 / \partial n_2$. On other boundary, adiabatic condition is used.

A relaxation iterative scheme is used to iteratively calculate the conditions of the both interfaces - between shell and working medium and between liquid and solid working medium.

The method is used to model the thaw process of the frozen working medium of axial cross section in a modeling liquid channel of the parallel

Transactions on Modelling and Simulation vol 20, © 1998 WIT Press, www.witpress.com, ISSN 1743-355X

channels heat pipe. As shown in Fig. 2, during initially thawing, the process only happens on the neighborhood of the end ($x = 0$) to be applied heat flux. After approximate 80 seconds, the thawing process happens on total interface between shell and frozen working medium, a moving interface as bullet gradually forms. Fig. 3 shows the temperature fields in shell are higher than in liquid region. However, the temperature gradients on shell are less than on liquid region. Main temperature drop happens in liquid region. The same results are also shown in Fig. 4 on the basis of the density and distribution of isothermal lines. In thawing process, there is clear two-dimensional character near both ends. However, except for the ends, the thawing process mainly follows y-axis for a long time. The development of thawing process along x-axis is very slow. These numerical results are accordant with experiment observation results [3] .

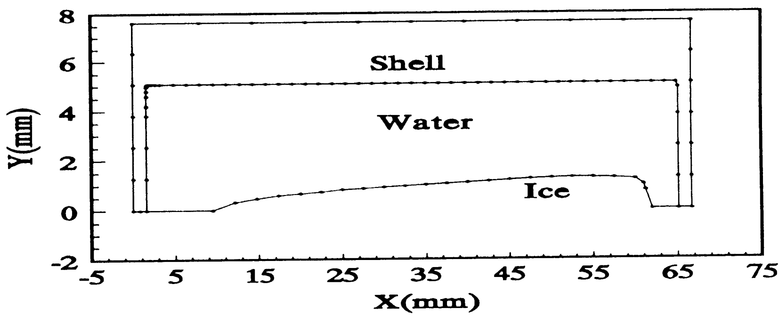


Fig.1 The computational domain of the thawing process in parallel channels heat pipe

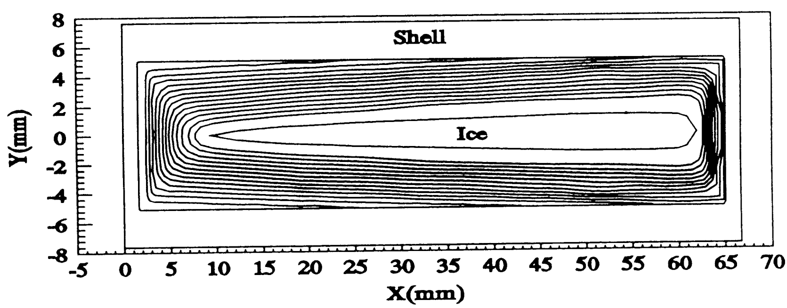


Fig.2 The variation of phase-change interface in the thawing process

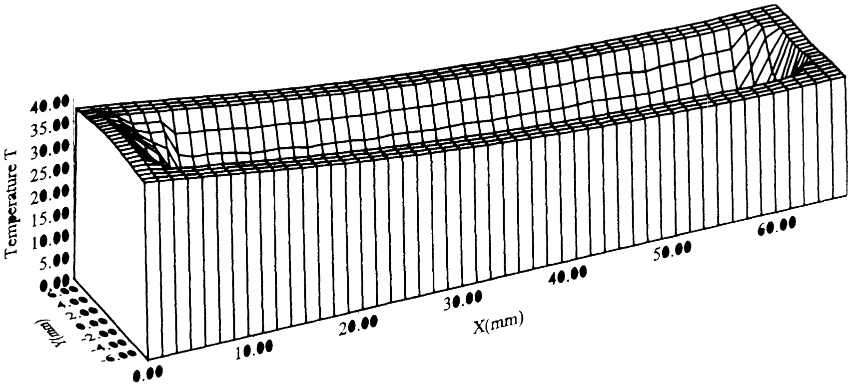


Fig. 3 The temperature distribution at 386 seconds in the thawing process

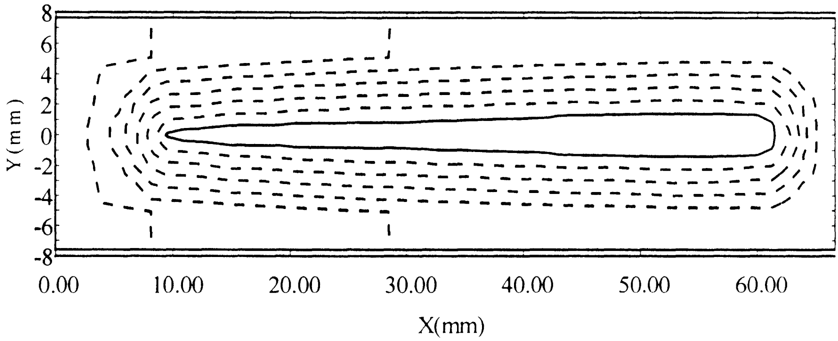


Fig. 4 Isothermal map at 386 seconds in the thawing process (the temperature difference between both isothermal line is 6℃)

Thermal wave propagation in biological tissues

For the thermal wave propagation in spatial and in biological tissue, a very few investigations were conducted until now. In this paper the DRBEM is extended to simulate the two-dimensional thermal wave propagation in biological tissues. In this case we assume the elevated temperature above initial temperature fields as $\theta(r,t) = T(r,t) - T(r,0)$, in equation (1), $a = \tau / \alpha_o$, $b = (\rho C + \tau W_b C_b)/K$, $c = \dot{W}_b C_b / K$, $d = -(Q_r + \tau \partial Q_r / \partial t)/K$. Where, ρ, C, and K denote the density, the specific heat and thermal conductivity of the tissue, respectively; C_b denotes the specific heat of blood; W_b blood perfusion rate; Q_r heat generations due to spatial heat source, respectively; T tissue temperature, and t time. τ is the thermal relaxation time. The above-established numerical DRBEM algorithm is used to solve equation (1).

The bioheat transfer problem in a two-dimensional square domain (Fig.5) is taken as the studying object. In the calculation, typical thermal parameters of living tissues are taken as $\rho = 1000 kg/m^3$, $K = 0.5 W/m \cdot K$, $C = C_b = 4200 J/Kg \cdot ℃$, $W_b = 0.5 kg/m^3 \cdot s$.

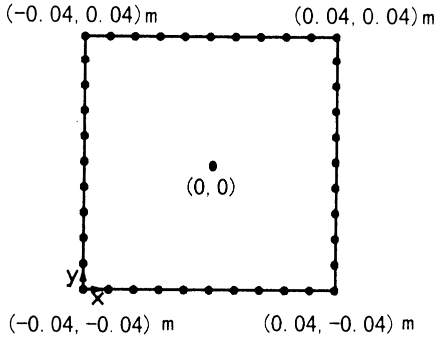
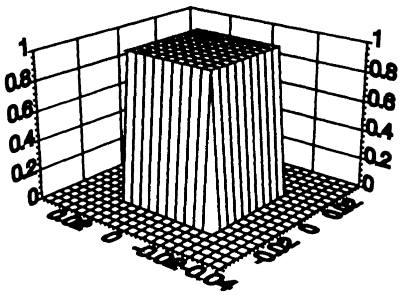


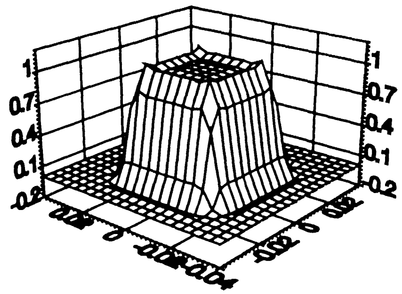
Fig.5 DRBEM discretization domain

The boundary and initial conditions defined as $\theta(x, y \in \Gamma; t) = 0\ ^oC$; $\theta(x, y; 0) = 1\ ^oC$ for $-0.02m \le x, y \le 0.02m$, $\theta(x, y; 0) = 0$ for other x, y; $\dot{\theta}(x, y; 0) = \vartheta(x, y, 0) = 0$.

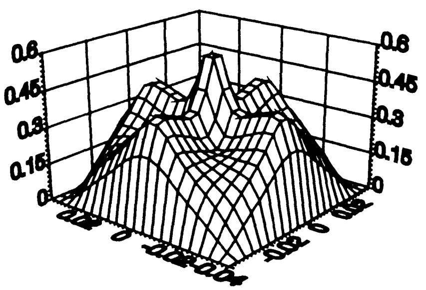
We first take the case with especially large $\tau = 1E4s$, which is high limit of thermal relaxation time reported in some literature, and $W_b=0$, $Q_r=0$. Fig. 6 shows the evolution case of a square temperature wave in biological tissue. It is quickly resolved into four rhythmical thermal waves with equal amplitude, and they further propagate reversibly. At time t=5.796E3s, each wave has traveled a few distances but their later parts still superpose together (Fig.6(c)) thus the core temperature is still the highest. At this time, the four waves begin to strike the four sides of the boundary. Calculations of Fig. 6(d)-6(e) show that, reflected from the rigid wall (i.e. $\theta = 0$), the four waves will change their signs of θ, showing a phase jumping. These figures also show that due to the interaction of the reflected waves and the incident waves, those results in the acute wave peak in body core as shown in Fig. 6(e). Later, the negative four waves will superpose on the negative value to produce a large negative acute peak wave in the core zone as shown in Fig. 6(f). Finishing superposition, waves will transfer across each other, resolution and again travel toward the surrounding area. The last figures depict the further reflection, superposition and resolution of thermal wave. It can be seen that, the initially positive thermal wave can change to the negative one and vice versa. The above results can be applied to interpret an interesting temperature changing phenomenon observed by Divrik et al [4] and some other earlier authors.



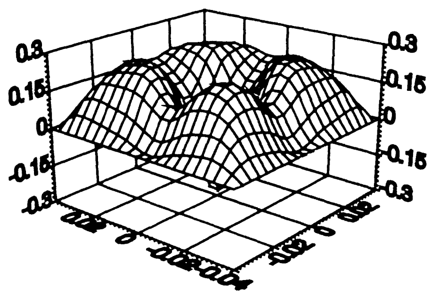
(a)t=0



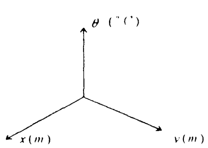
(b)t=579.6s



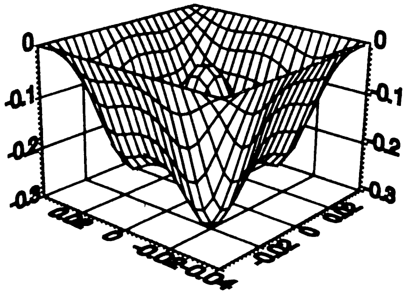
(c) t=5796.6s



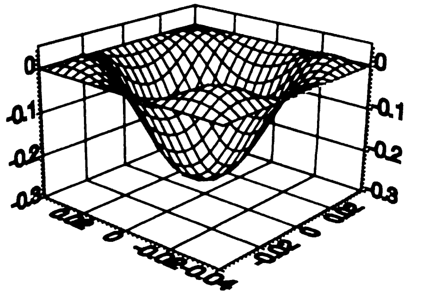
(d)t=8694.8s



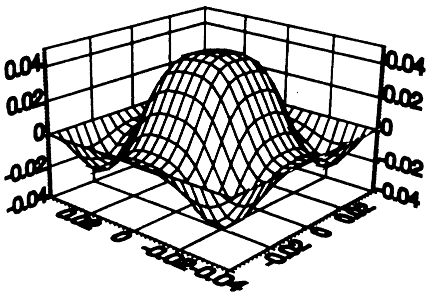
Transactions on Modelling and Simulation vol 20, © 1998 WIT Press, www.witpress.com, ISSN 1743-355X



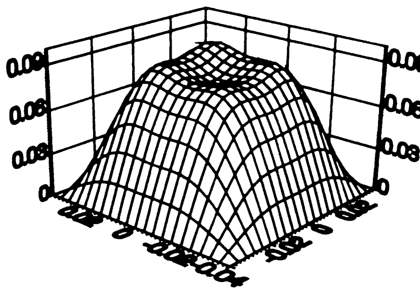
(e)t=1.449E4s



(f)t=2.319E4s



(g)t=2.898E4s



(h)t=3.768E4s

Fig.6 Square wave evolution in tissue when $\tau = 1e4s$, $W_b = 0$

In other examples, the low limit of thermal relaxation time reported in references $\tau = 20s$ is chosen. The both case - $Q_r = 0$ and the changing heating power are also accounted except for $W_b = 0$, respectively. Our numerical computations show the thermal wave effect in tissue will be weak to be observable when $Q_r = 0$. The greater parts of energy are propagated such as the form of diffusion. However, when heating power is changed, the heat transfer inside the body still shows distinctive wave properties.

Conclusions

The thawing process of frozen working medium in the modeling liquid channel of the heat pipe gradually forms an axisymmetric moving interface as bullet. Main temperature drop happens in liquid region.

The higher the thermal relaxation time is, the stronger the thermal wave effect will be in biological tissues. Under changing heat source, the effect of thermal wave becomes more obvious.

The DRBEM is an efficient method for solving the heat conduction with phase change (parabolic-diffusion equation) and the propagation of thermal wave in biological tissues (hyperbolic-wave equation). The DRBEM is a pure boundary integral method without domain integral and can be developed as an universal BEM used to simulate more complex problems.

Acknowledgment

The National Natural Science Foundation of China and the Key Project of Academia Sinica support this research.

References

1.P.W. Partridge, C.A. Brebbia and L.C. Wrobel, 1992, *The Dual Reciprocity Boundary Element Method*, Elsevier Applied Science, London.

2.Wen-Qiang Lu and Shufen Li, 1995, The Boundary Element Method for Analyzing Transient Temperature Fields with Phase-Change Moving Interface in the Heat pipe, *Journal of Engineering Thermophysics*, vol. 16, pp349-353, (in Chinese).

3.Yuntao Zeng, 1997, The boundary element analysis of the thaw and freeze characters of microgravity heat pipes, Master Thesis, The Graduate School at Beijing, USTC, Academia Sinica, Beijing, China.

4.A.M. Divrik, R.B. Roemer and T.C. Cetas, 1984, Inference of Complete Tissue Temperature Fields from a Few Measured Temperatures: an Unconstrained Optimization Method, *IEEE Transactions on Biomedical Eng.*, BME- vol. 31, pp. 150-160.