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Adaptive solution of rolling contact using boundary element method

Zi-Li Li & Joost J. Kalker
Delft University of Technology
Subfaculty of Technical Mathematics and Informatics
Faculty of Information Technology and Systems
Mekelweg 4, 2628 CD Delft
The Netherlands
Email: Z.L.Li@math.tudelft.nl
J.J.Kalker@math.tudelft.nl

Abstract

In this paper, the fast and adaptive calculation of large number of consecutive rolling contact problems is discussed. A special BEM method is proposed for the solution of non-Hertz and conformal contact. The attention is paid on the appropriate choice of the potential contact area so that it contains completely the real contact area while it is as small as possible to reduce computing time, on good initial estimation to speed up convergence, and on the adaptability to various possible geometry and loading situations.

1 Introduction

Rolling contact occurs in wheel-rail systems, bearings, gears, printing machines. In rolling contact wear and corrugation analysis, and in rail vehicle system dynamical analysis, large number of frictional contact problems must be solved one after another in consecutive sequence with different contact locations, geometries and loads which may change dramatically, therefore the solution algorithm must

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be very fast, adaptive and robust, and must be able to solve various problems arising from various possible situations.

For three-dimensional rolling contact, analytical solutions are possible only for very simple geometries and few limited loading cases[1]. In most practical situations, numerical solutions are sought. Finite element method is very powerful in the sense that it can solve for many geometries, but it is slow and the meshing is more complicated, which render it impractical for the above purposes. In this respect boundary element method is an attractive choice.

A general formulation is developed by Kalker[2] for rolling contact and implemented in the program CONTACT in a special boundary element method for concentrated non-Hertz contact problems with Boussinesq-Cerruti half-space analytical solution as the influence function. Li and Kalker [3] extended it to conformal contact with finite element results for the influence number.

Explicit multiple point rolling contact, which is the contact with more than one separate sub- contact area found from rigid body geometry, has be applied in vehicle dynamical analysis and wear simulation either by considering each sub- contact area as a Hertz contact problem, solving them separately, and then adjusting iteratively the total loads distribution among the sub-areas until the total force equilibrium is satisfied[4], or by modelling each sub-area as Hertzian and then reduce them to one equivalent contact problem[5]. There are some problems with those methods: firstly the sub-areas are often non-Hertz, secondly the cross-influence of the elastic fields between the sub-areas can not be taken into account, and thirdly the contact points are found from rigid body geometry, hence often missing 'implicit' multiple point contact which may take place due to elastic deformation.

Solution of large number of rolling contact in consecutive sequence has been applied in rail vehicle dynamical analysis, wear and corrugation simulation for some years[6,7], but all the methods are of Hertz type. Though non-Hertz contact is common in reality, it is not employed because of its long computing time and unknown contact area: for Hertz contact the shape and size of the contact area are analytically available and the normal pressure is easily known, which makes the trend of tangential solution predictable and the discretization easy.

In the present paper a mathematical modelling of rolling contact and its discretization in BEM mesh are first briefly introduced, then a solution procedure is described which can automatically find one point contact, explicit and implicit multiple contact, adapt the discretization mesh, initial estimation and solution methods to the various geometries and load conditions, and solve the problems quickly in the non-Hertz manner. Finally some results are given.

2 The Modelling of Rolling Contact

A general variational formulation for rolling contact is proposed by Kalker[2]. It is expressed in the form of the maximisation of complementary energy in elastostatics, without body force, in surface mechanical form:

$$\max_{u,p} C = \sum_{a=1,2} \{-\frac{1}{2}\int_{A_{pa}} \bar{p}_i u_i dS + \frac{1}{2}\int_{A_{ua}} p_i \bar{u}_i dS\}$$

$$-\int_{A_c} (h + \frac{1}{2}u_z)\, p_z\, dS - \int_{A_c} (W_\tau + \frac{1}{2}u_\tau - u'_\tau)\, p_\tau\, dS \qquad (1)$$

$$sub\ p_z \geq 0,\ |p_\tau| \leq f\, p_z \text{ in } A_c,\ g = f\, p_z \text{ is fixed}$$

where p and u are the surface traction and displacement difference, A_{pa} and A_{pu} are the surface areas on body a $(a = 1, 2)$ where traction $(p_i = \bar{p}_i,\ i = x, y, z)$ and displacement difference$(u_i = \bar{u}_i)$ are prescribed. A_c is the potential contact which should include completely the real contact, τ denotes tangential direction and $'$ denotes previous time step, W_τ is the relative displacement due to rigid body microslip, h is the undeformed distance,

$$h = h_0 - q \qquad (2)$$

in which h_0 is the rigid body distance between corresponding elements at their centres when the bodies are in touch but no deformation takes place, and q is the approach(compression).

Rolling contact is a local phenomenon, we drop in (1) $\bar{p}_i$ and $\bar{u}_i$.

3 Discretization

Discretize A_c into a mesh of equal rectangles with MX rows and MY columns, each element with dimension Dx, Dy and area dS, (1)

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can be reduced into the normal problem:

$$A_{IzJz}\, p_{Jz}\, dS + h_I + A_{IzJ\tau}\, p_{J\tau}\, dS = e_I \tag{3}$$

and the tangential problem:

$$A_{I\tau J\alpha}\, p_{J\alpha}\, dS + W_{I\tau} - u'_{I\tau} + A_{I\tau Jz}\, p_{Jz}\, dS = -w_I\, p_{I\tau}/|p_{I\tau}| \tag{4}$$

where A_{IiJj} is the influence number[2,3], e_I is the deformed distance and w_I is the microslip distance.

4 The Solution Procedure

The normal and the tangential problems are solved separately in an iterative process. Kalker's active set algorithm [2] is employed.

4.1 The Search for Initial Contact Points

In the rolling direction, the surface of one of the contacting bodies in the vicinity of the contact areas is circular or quadratic and the other can be regarded as flat, they are of arbitrary profiles in the lateral direction. The bodies are subject to translational and rotational motions in space, but their relative motion is constrained by each other. The initial contact points are found by searching for the minimal distance between them with the requirement that at these points the outer normals of the contacting surfaces should coincide. A direct application of this requirement to the search would be time consuming since the search is carried out two dimensionally.

However if we fix the coordinate system on the surface that is flat in the rolling direction with the x axis pointing in the rolling direction, z being the outer normal and y pointing laterally, an analytical representation of the curve on which the outer normal of the surface is parallel to the yz plane can be found on the surface of the other body and contact can only take place on this curve. In this way the search dimension is reduced to one and for continuous smooth surfaces their normals will coincide at the minimal distance points on this curve.

4.2 The Various Situations

There are various geometry and loading situations, choice is made according to their nature during the analysis.

Concentrated Contact

In concentrated contact the surfaces of the contacting bodies are flat or almost flat in the vicinity of the contact area and the size of the contacting parts of the bodies are large with respect to the contact area. Half-space influence number can be employed with satisfactory results.

Conformal Contact

For conformal contact, half-space influence number may not be applicable and analytical influence function may not be available. Finite element solution is employed [3].

The discretization and the computation by FEM appears much more time consuming than the complete solution of a contact problem by BEM. But since the geometry of rolling contact bodies do not change much during their life of use, once the influence number is ready, it can be used for all the conformal contact problems of such bodies.

Multiple Point Contact

The search in section 4.1 is carried out with the bodies being taken as rigid. The result may be one point, multiple points or a line. The multiple-point contact thus found is called explicit. In many applications, for instance in wheel-rail contact, explicit multiple contacts exist theoretically only for special relative positions of the contact bodies, it needs very short search interval to find them, while implicit contacts occur due to elastic deformation around the explicit multiple contact locations in large regions. Hence it is more important to find and include the possible implicit multiple contacts. In this work, it is achieved by an appropriate choice of the potential contact area. The multiple contact problem is solved as non-Hertz in one potential contact area, the cross-influence is automatically taken into account.

Loading Manners

The loads can be given in different combinations of total forces components, rigid body microslip components and displacement. When total force components are prescribed, the additional corresponding total force equilibrium equations to (3) and (4) must be satisfied:

$$\Sigma p_{Ix}\, dS = F_x \tag{5}$$

$$F_y = p_{Iy}\, cos\delta_I\, dS - p_{Iz}\, sin\delta_I\, dS \tag{6}$$

$$F_z = p_{Iz} \, cos\delta_I \, dS + p_{Iy} \, sin\delta_I \, dS \tag{7}$$

where δ_I is the angle between the normal of element I and F_z.

Result Possibilities

Equations (1) are formulated for transient rolling. If in the calculation of u'_τ the traction is set the same as that of the current time step, while the corresponding influence number is for the previous step, the solution is for steady-state. The solution can also be required for normal, tangential problems.

4.3 The Solution of The Normal Problem

The normal equations are linear, they can be solved by Gauss elimination with LU decomposition.

4.3.1 The Choice of The Potential Contact A_c

The potential contact area should include the real contact area completely, at the same time it should be as small as possible to reduce computing time.

A choice of potential contact for individual non-Hertz problem can be facilitated by test runs, which is not possible during a simulation.

A usual way is to assign directly an area around the initial contact points as the potential contact. It may be made to a reasonable size according to experience, as is often the case in test run, and/or according to knowledge of relationship between the sizes of contact areas and the loads from previously computed cases. This still has the possibility to exclude some sub- contact areas when they are relatively widely distributed while attempt is made to minimize A_c.

In the present work, the undeformed distance h is used: the area where $h \leq h_c$ is chosen as A_c, where h_c is obtained from information of previously computed cases.

In this way, when approach q is given, the choice of A_c is simple. More often is that the normal force is prescribed. In such cases, q is an unknown, the normal equations are solved iteratively for it. To start the iteration an initial estimation of q must be made. On the other side, the first q obtained from solving the normal equations is inaccurate if the estimation is not good. Because q has very sensitive effect on the size of the contact area, the contact area thus obtained can often be much greater than the real one. As the computing time is $O(N_c^3)$ (N_c is the total number of elements in the contact area), this leads to significant increase in computing cost. To overcome this, the following method is employed:

q is estimated from $q = k_q\ (F_z)^{\frac{2}{3}}$, where k_q can be obtained from previously calculated cases. This q is then used to determine the extent c_y of the contact area in the lateral direction together with the undeformed distance h. On the other hand, the extent of contact area in x and y direction c_x, c_y are proportional to $(F_z)^{\frac{1}{3}}$, since now c_y is determined, c_x can also be determined approximately in a similar manner as q. The potential contact area thus decided in most cases is very close to the real contact area, and since it is decided on the basis of undeformed distance, it includes the real contact area completely.

4.3.2 The Initial Estimation

Good initial estimation can significantly accelerate the computation. For Newton-Raphson process when it is close to the solution, quadratic convergence is expected. To use the previous case as the estimation, the discretization is kept unchanged whenever possible.

For some cases, say for the first case, or during transition from one point contact to multiple contact or vice versa, good estimation from previous case is not possible, then multi-grid method is employed, the program start with a coarse grid for an estimation.

4.3.3 The Minimization of Total Number of Elements

The accuracy of the solution depends on N_c. To keep a balance between the computing time and the required accuracy, effort is made to limit N_c around a certain empirical number N_{c0}, which can be decided based on experience or test runs. When N_c deviates from N_{c0} too far, the discretization is modified and the normal problem is solved again. This though costs some additional time, it is necessary and beneficial because either the accuracy requirement is retained or more time is saved in solving the tangential problem and in the following cases.

It is noticed that for conformal contact, larger N_c is necessary to achieve the same accuracy.

4.4 Solution of The Tangential Problem

The tangential equations are non-linear when some elements are in slip due to friction law which requires that the direction of slip is opposite to that of the tangential traction and that the magnitude of the tangential traction should not be greater than the normal pressure times the local coefficient of friction. Two methods are employed for the solution.

The first is Gauss elimination with LU decomposition and with Newton-Raphson linearization. The timing is $O(N_c^3)$. It is robust, with rare divergence. Should it occur, perturbation of the traction, the discretization and the input almost always works.

The other way is to use Gauss-Seidel method. Each time the two unknowns p_{Ix}, p_{Iy} in one element are solved with Newton-Raphson linearization. The timing is $O(N_c^2)$. It is faster, but less robust.

Most of the cases can be solved with the Gauss-Seidel method, whenever it does not converge, the Gauss elimination method is invoked.

For the tangential estimation, if the discretization is the same as the previous case, the state of the elements (eg. in slip or adhesion) and their tangential tractions are used.

5 Results

A typical wheel-rail contact is chosen for multiple point contact (fig.1). The rail is fixed and the wheelset displaces laterally from its central position $y = 0.0$. When the rail bottom cant is 0.0, the explicit two point contact takes place at lateral displacement $y = 6.91\ mm$, but implicit two point contact occurs between $y = 6.45 \sim 7.3\ mm$. The results shown are computed from $F_z = 10^5\ N$, with the rigid body microslip due to the geometry only. Single line denotes adhesion areas, double line denotes slip areas.

The proposed solution procedure is implemented. To maintain the error below 10 percent, an average of 0.8 second is needed on HP712 workstation for the solution of one complete steady-state rolling contact problem including the search of the contact location.

This method is applied to wheel-rail wear simulation. For 1 mm of material wear on the wheel profile, simulation by the proposed method takes only one hour for high accuracy . The application of this method to vehicle dynamical simulation is under way.

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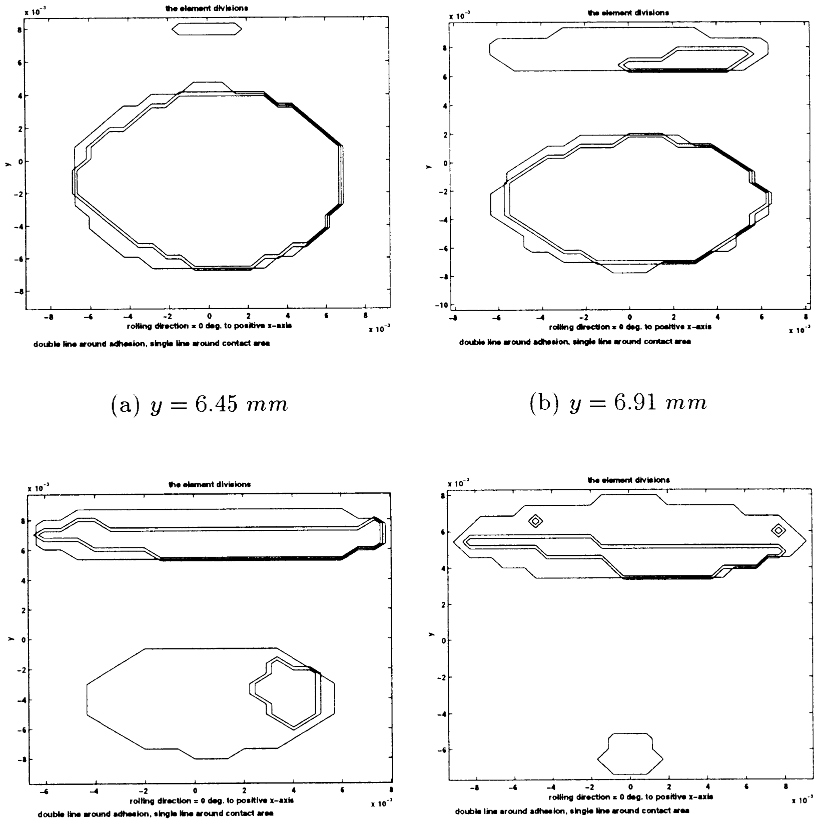

(a) $y = 6.45\ mm$ (b) $y = 6.91\ mm$

(c) $y = 7.10\ mm$ (d) $y = 7.30\ mm$

Figure 1: Transition of implicit two-point contact

6 Conclusion

In wear and corrugation analysis, and in rail vehicle system dynamical analysis, large number of rolling contact problems must be solved with different contact locations, geometries and loads, the solution method must be very fast, robust and adaptive to the various possible situations. Boundary element method is suitable for this purpose.

The proposed solution procedure incorporates different methods for various geometries, loads and initial estimation, and takes advantage of the information from previously computed cases for better choice of the potential contact areas, and for initial estimation. The solution methods are non-Hertz. The choice of potential contact area by way of undeformed distance ensures that the real contact area is completely included, even in the case of widely distributed implicit multiple sub- contact areas, while the potential contact can be reasonably small to reduce computing time. Application of it to wear simulation shows that it can adapt to the various situations, and is fast and robust.

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