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System identification by the analog equation method

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Abstract

In this paper the Analog Equation Method, a BEM-based method, is employed to establish the differential equation governing the behaviour of a physical system for which the source (input) and its response (output) are known. We come across to such cases in the effort to identify constitutive laws of materials including constant or spatially varying parameters (non-homogeneous materials), or constitutive laws depending non-linearly on the unknown field function and/or its derivatives. Certain example problems are presented which validate the efficiency of developed procedure. Among them, temperature distribution problems in non-homogeneous bodies or in bodies with temperature dependent thermal conductivity as well as non-linear steady-state flow problems governed Burgers' equation.

1 Introduction

The Analog Equation Method (AEM), developed recently by Katsikadelis [1], has been successfully employed to solve a series of problems arising in engineering discipline, such as linear and non-linear potential problems in non-homogeneous or non-linear bodies, linear and non-linear plate problems [2,3], non-linear vibrations of plates [4], plane stress problem in bodies with constant [5] or variable thickness [6] and finite deformation of cables and cable structures [7]. In this paper AEM is employed to establish the differential equation governing the behaviour of a physical system for which the source (input) and the response (output) of the system are known. The dominant part of the differential operator or at least its order is known from general physical principles, while the remaining part of the operator or some of its coefficients, constant or spatially varying may be unknown. We deal with such problems when we try to establish a mathematical model predicting the behaviour of a system or to determine the constitutive laws of new materials or when we are interested to transfer experimental results from the model to the real structure in non-linear responses (scaling in non-linear mechanics). It must be emphasised here that the recently developed highly efficient computational methods, FEM and BEM, can solve the differential equations describing the behaviour of a system with great accuracy. However, the question that arises is whether the differential equation describes the behaviour of the system reliably. Thus, the identification of a physical system plays an important role in the investigation of its actual behaviour. The establishment of a physical law from experiments on specimens do not always give reliable data. If, for example, the boundary conditions (support and loading) of the specimen are altered, the obtained results may be different. On the other hand, if the experiments are performed on the system, the obtained physical law is more reliable, as it is derived from its global response. It is in this context that this method is proposed. The presented numerical examples validate the efficiency of the method.

2 Problem statement and analysis

Consider the boundary value problem

$$N(u) = 0 \quad \text{in } \Omega \tag{1}$$

$$B(u) = \bar{u} \quad \text{on } \Gamma \tag{2}$$

where $N(u)$ is an elliptic differential operator and $B(u)$ are admissible boundary conditions.

If $N(u)$ is of the second order with linear dominant part, then Eqs(1,2) may be written as

$$\alpha_1 u_{xx} + \alpha_2 u_{xy} + \alpha_3 u_{yy} + f(u, u_x, u_y) = 0 \quad \text{in } \Omega \tag{3}$$

$$u = \bar{u} \quad \text{on } \Gamma_1 \quad \text{and} \quad u_n = \bar{u}_n \quad \text{on } \Gamma_2 \tag{4}$$

where $\alpha_i = \alpha_i(x,y)$, $(i = 1,2,3)$ and $f(u, u_x, u_y)$ are unknown functions to be specified from a given set of solutions (as many as they will be required) of the problem (3,4) obtained perhaps experimentally.

The function $f(u, u_x, u_y)$ is in general non-linear and can be sought in the form of a polynomial series, i.e.

$$\begin{aligned} f(u, u_x, u_y) = b_{000} &+ b_{100}u + b_{010}u_x + b_{001}u_y \\ &+ b_{200}u^2 + b_{020}u_x^2 + b_{002}u_y^2 \\ &+ b_{110}uu_x + b_{101}uu_y + b_{001}u_x u_y + \ldots \end{aligned} \tag{5}$$

where $b_{ijk} = b_{ijk}(x,y)$ are unknown coefficient functions.

Thus, Eq. (3) is written as

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$$a_1 u_{xx} + a_2 u_{xy} + a_3 u_{yy} + b_{000} + b_{100} u + b_{010} u_x + b_{001} u_y$$
$$+ b_{200} u^2 + b_{020} u_x^2 + b_{002} u_y^2 \tag{6}$$
$$+ b_{110} u u_x + b_{101} u u_y + b_{001} u_x u_y + \ldots = 0$$

If the function $f(u, u_x, u_y)$ is approximated by a finite number of terms in Eq. (6), the functions $\alpha_i(x,y)$ and $b_{jk\ell}(x,y)$ can be established as following.

Let n be the total number of the unknown coefficient functions $\alpha_i, b_{jk\ell}$. Suppose that we have n solutions of the problem (3),(4), u^p, $p = 1,2,\ldots n$ corresponding to n different sets of the boundary quantity $\bar{u}$. Then, substitution of these solutions into Eq. (6) yields a system of equations from which the unknown coefficients can be established.

The solution u^p is given as a set of values at the nodal points of a mesh on Ω. Subsequently, the problem is to express all the derivatives involved in Eq. (6) in terms the of values of the field function u. This is achieved us by the Analog Equation Method (AEM) as presented in the next sections.

3 The analog equation method

Let $u = u(x,y)$ be the sought solution of Eq. (3) subjected to the boundary conditions (4). If the Laplacian operator is applied to this function, we obtain

$$\nabla^2 u = q(x,y) \tag{7}$$

Eq. (7) indicates that the solution of the problem described by Eqs(6) and (4) could be obtained as the solution of this equation subjected to the boundary condition (4), if the source density function $q(x,y)$ were known. The establishment of this unknown source density function is one of the essential ingredients of AEM. This can be accomplished using BEM as following.

The solution to the Laplace Eq. (7) is given in integral form as

$$\varepsilon u(P) = \int_\Omega v q d\Omega - \int_\Gamma (v u_n - v_n u) ds \tag{8}$$

where

$$v = \frac{1}{2\pi} \ell n r, \quad r = |P - Q|, \quad P:\{x,y\}, \; Q:\{\xi, \eta\} \tag{9}$$

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is a singular solution of

$$\nabla^2 v = \delta(P-Q) \tag{10}$$

and $\varepsilon = 1,\ 1/2$ or 0 depending on whether $P \in \Omega,\ P \in \Gamma$ or $P \notin \overline{\Omega}$, respectively. The boundary is assumed smooth at point P. $\overline{\Omega}$ is the closure of the domain.

Differentiation of Eq. (8) with $\varepsilon = 1$ yields

$$u_x(P) = \int_\Omega v_x q d\Omega - \int_\Gamma (v_x u_n - v_{nx} u) ds \tag{11a}$$

$$u_y(P) = \int_\Omega v_y q d\Omega - \int_\Gamma (v_y u_n - v_{ny} u) ds \tag{11b}$$

$$u_{xx}(P) = \int_\Omega v_{xx} q d\Omega - \int_\Gamma (v_{xx} u_n - v_{nxx} u) ds \tag{11c}$$

$$u_{yy}(P) = \int_\Omega v_{yy} q d\Omega - \int_\Gamma (v_{yy} u_n - v_{nyy} u) ds \tag{11d}$$

$$u_{xy}(P) = \int_\Omega v_{xy} q d\Omega - \int_\Gamma (v_{xy} u_n - v_{nxy} u) ds \tag{11e}$$

Theoretically, the boundary integral Eq. (8) $(P \in \Gamma)$ together with the boundary conditions (4) permit the establishment of u and u_n on the part of the boundary where they are not specified in terms of q. Subsequent substitution into Eq. (8) $(P \in \Omega)$ permits the establishment of u in terms of q. After that, Eqs(11) yield the desired derivatives in terms of q. The foregoing procedure involves the solution of boundary as well as domain integral equations which are solved numerically using the D/BEM as following.

4 Numerical implementation

The D/BEM is employed. For this purpose the boundary is discretized in *N boundary elements* while the domain into *M cells*. The discretized equations are obtained from Eqs(11a)-(11e) using the boundary element technique for the boundary integrals and the finite element technique for the domain integrals. In the example problems studied in this paper, constant boundary and domain elements have been employed. The singular and hyper-singular domain integrals are evaluated using the technique presented in [1]. The kernel integrals are evaluated by Gauss integration. The discretized counterparts of Eqs(8),(11a)-(11e), when applied at the nodal points, yield

$$\{u\}_N = [\overline{A}]\{q\}_M + [\overline{B}]\{u\}_N + [\overline{C}]\{u_n\}_N \tag{12}$$

$$\{u\}_M = [A]\{q\}_M + [B]\{u\}_N + [C]\{u_n\}_N \tag{13a}$$

$$\{u_x\}_M = [A_x]\{q\}_M + [B_x]\{u\}_N + [C_x]\{u_n\}_N \tag{13b}$$

$$\{u_y\}_M = [A_y]\{q\}_M + [B_y]\{u\}_N + [C_y]\{u_n\}_N \tag{13c}$$

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$$\{u_{xx}\}_M = [A_{xx}]\{q\}_M + [B_{xx}]\{u\}_N + [C_{xx}]\{u_n\}_N \quad (13d)$$
$$\{u_{yy}\}_M = [A_{yy}]\{q\}_M + [B_{yy}]\{u\}_N + [C_{yy}]\{u_n\}_N \quad (13e)$$
$$\{u_{xy}\}_M = [A_{xy}]\{q\}_M + [B_{xy}]\{u\}_N + [C_{xy}]\{u_n\}_N \quad (13f)$$

where

$\{u\}_N$, $\{u_n\}_N$: are $N \times 1$ matrices including the boundary nodal values of u and u_n.

$\{q\}_M$ is an $M \times 1$ matrix including the values of $q(x,y)$ at the domain nodal points.

$\{u\}_M, \{u_x\}_M, \{u_y\}_M, \ldots, \{u_{yy}\}_M$: are $M \times 1$ matrices including the domain nodal values of $u, u_x, u_y, \ldots, u_{yy}$.

$[\bar{A}], [\bar{B}], \ldots, [A], [B], \ldots [C_{yy}]$: are known coefficients matrices originating from the integration of the kernels along the boundary elements and on the domain cells.

Eqs(12) together with the boundary conditions (4) permit the evaluation of the not prescribed boundary quantities. Subsequently, substituting these values into Eqs(13a) to (13f) yields

$$\{u\} = [G]\{q\} + \{D\} \quad (14a)$$
$$\{u_x\} = [G_x]\{q\} + \{D_x\} \quad (14b)$$
$$\{u_y\} = [G_y]\{q\} + \{D_y\} \quad (14c)$$
$$\{u_{xx}\} = [G_{xx}]\{q\} + \{D_{xx}\} \quad (14d)$$
$$\{u_{yy}\} = [G_{yy}]\{q\} + \{D_{yy}\} \quad (14e)$$
$$\{u_{xy}\} = [G_{xy}]\{q\} + \{D_{xy}\} \quad (14f)$$

where $[G], [G_x], \ldots, [G_{yy}]$ are $M \times M$ coefficient matrices and $[D], [D_x], \ldots, [D_{yy}]$ constant $M \times 1$ column matrices. The subscript M has been released from these matrices as they all refer to domain nodal values.

If the vector $\{u\}$ is known at the nodal values, then the vector $\{q\}$ can be established in terms of $\{u\}$ on the basis of Eq. (14a). Therefore, all derivatives can be expressed in terms of the vector $\{u\}$.

For each $\{u^p\}$ $(p = 1,2,\ldots n)$ we obtain a corresponding vector $\{u_x^p\}, \{u_y^p\}, \ldots$.

Collocation of Eq. (6) at the nodal points inside Ω and substituting $\{u^p\}$ and its derivatives on the basis of Eqs(14a) through (14f), we obtain

$$
\begin{aligned}
&[u_{xx}^p]\{\alpha_1\}+[u_{xy}^p]\{\alpha_2\}+[u_{yy}^p]\{\alpha_3\} \\
&+[I]\{b_{000}\}+[u^p]\{b_{100}\}+[u_x^p]\{b_{010}\}+[u_y^p]\{b_{001}\} \\
&+\{(u^p)^2\}\{b_{200}\}+\{(u_x^p)^2\}\{b_{020}\}+\ldots=0
\end{aligned}
\tag{15}
$$

Eq.(15) for $p=1,2,\ldots n$ yields a set of $M\times n$ linear algebraic equations which permit the establishment of $\{\alpha_1\},\{\alpha_2\},\{\alpha_3\}$ and $n-3$ coefficients of the polynomial series (5).

5 Numerical examples

On the basis of the procedure described in previous sections some example problems have been studied to illustrate the efficiency of the proposed method.

Example 1. Burgers' Equation

We consider experimental data of the velocity field with convection-diffusion structure and containing the full non-linearity of the one-dimensional flow. The steady-state situation of this flow with unit value of Reynolds' number ($Re = 1$) is governed by the Burgers' equation, which is derived from the Navier-Stokes equations by imposing certain simplifying assumptions on them and has the form [8]

$$\nabla^2 u - u u_x = 0 \tag{16}$$

u is the velocity component in x-direction.

We consider a square with side length $a = 1$. Its boundary is discretized into 60 constant elements (15 on each side), while the interior is discretized into 10x10 constant square cells. The values of the velocity are measured on the nodal points. They are obtained from *numerical experiments*.

We assume that the flow is governed by an equation having the form

$$\nabla^2 u + f(u, u_x, u_y, u_{xx},) = 0 \tag{17}$$

and we require to establish the function $f(u,u_x,u_y,u_{xx})$ from the experimental data. This function is approximated as

$$f(u,u_x,u_y) = (b_o + b_1 u)u_x + b_2 u_{xx}^2 + b_3 u_{xy}^2 + b_4 u_{yy}^2 \tag{18}$$

The coefficients b_k ($k = 0,1,2,3,4$) are obtained from the data of five *numerical experiments* by solving numerically Eq. (16), using AEM for

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non-linear potential problems [1], under the following five different sets of boundary conditions.

a. $u(0,y)=2.0$ $u(1,y)=3.0$ $u(x,0)=u(x,1)=2.0+x$
b. $u(0,y)=2.0$ $u(1,y)=3.5$ $u(x,0)=u(x,1)=2.0+1.5x$
c. $u(0,y)=2.5$ $u(1,y)=3.0$ $u(x,0)=u(x,1)=2.5+0.5x$
d. $u(0,y)=2.5$ $u(1,y)=4.0$ $u(x,0)=u(x,1)=2.5+1.5x$
e. $u(0,y)=3.0$ $u(1,y)=3.5$ $u(x,0)=u(x,1)=3.0+0.5x$

This procedure yields the following set linear algebraic equations for the evaluation of coefficient b_k.

$$\begin{bmatrix} [u_x^{(1)}] & [u^{(1)}u_x^{(1)}] & [u_{xx}^{(1)}u_{xx}^{(1)}] & [u_{xy}^{(1)}u_{xy}^{(1)}] & [u_{yy}^{(1)}u_{yy}^{(1)}] \\ [u_x^{(2)}] & [u^{(2)}u_x^{(2)}] & [u_{xx}^{(2)}u_{xx}^{(2)}] & [u_{xy}^{(2)}u_{xy}^{(2)}] & [u_{yy}^{(2)}u_{yy}^{(2)}] \\ [u_x^{(3)}] & [u^{(3)}u_x^{(3)}] & [u_{xx}^{(3)}u_{xx}^{(3)}] & [u_{xy}^{(3)}u_{xy}^{(3)}] & [u_{yy}^{(3)}u_{yy}^{(3)}] \\ [u_x^{(4)}] & [u^{(4)}u_x^{(4)}] & [u_{xx}^{(4)}u_{xx}^{(4)}] & [u_{xy}^{(4)}u_{xy}^{(4)}] & [u_{yy}^{(4)}u_{yy}^{(4)}] \\ [u_x^{(5)}] & [u^{(5)}u_x^{(5)}] & [u_{xx}^{(5)}u_{xx}^{(5)}] & [u_{xy}^{(5)}u_{xy}^{(5)}] & [u_{yy}^{(5)}u_{yy}^{(5)}] \end{bmatrix} \begin{Bmatrix} \{b_o\} \\ \{b_1\} \\ \{b_2\} \\ \{b_3\} \\ \{b_4\} \end{Bmatrix} = \begin{Bmatrix} \{q^{(1)}\} \\ \{q^{(2)}\} \\ \{q^{(3)}\} \\ \{q^{(4)}\} \\ \{q^{(5)}\} \end{Bmatrix} \quad (19)$$

where $[u_x^{(1)}]$, $[u^{(1)}u_x^{(1)}],\ldots,[u_{yy}^{(5)}u_{yy}^{(5)}]$ are diagonal matrices with dimensions 100x100.

The solution of Eqs (19) is presented in Table 1. As it was anticipated the coefficients are $b_o \approx b_2 \approx b_3 \approx b_4 \approx 0$ while $b_1 \approx -1$. Thus

$$f(u,u_x,u_y,u_{xx}) = -uu_x \quad (20)$$

Table 1. *Computed values of the coefficients b_k in Example 1 at 10 nodal points along the line $x=y$.*

Point Number	b_o	b_1	b_2	b_3	b_4
1	.156-06	-1.000	-.139-06	.577-07	-.159-06
12	.138-05	-1.000	-.446-05	.113-05	.325-05
23	.157-05	-1.000	.510-06	.171-04	-.874-05
34	.570-06	-.999	.890-06	-.506-05	-.217-06
45	-.169-05	-.999	.315-05	-.991-04	-.101-05
56	-.119-05	-.999	.469-05	.218-03	-.351-05
67	-.192-05	-.999	-.422-06	.107-03	.378-06
78	-.271-06	-.999	-.143-06	.106-05	.113-06
89	.263-06	-1.000	.119-06	-.204-06	-.700-07
100	.117-06	-1.000	.620-07	-.183-07	-.264-07

Example 2. Determination of the thermal conductivity function in non-linear bodies.

We consider a body in which the conductivity depends on the temperature, i.e.

$$k = k(u) \tag{21}$$

The steady-state heat transfer in absence of heat sources is described by the following differential equation resulting from Fourier's Law

$$k\nabla^2 u + k_x u_x + k_y u_y = 0 \tag{22}$$

In this example, we want to establish the conductivity $k(u)$ from experimental data in a square two-dimensional body having side length $a = 1$. We use the same discretization as in *Example 1.* The values of temperature u are measured on the nodal points. They are obtained from *numerical experiments* on a homogeneous body with

$$k(u) = -2 + 0.01u, \tag{23}$$

In this case, Eq.(22) becomes

$$\nabla^2 u + \frac{1}{u-200}(u_x^2 + u_y^2) = 0 \qquad 0 \le x, y \le 1 \tag{24}$$

We assume the following approximation for $k(u)$

$$k(u) = b_o + b_1 u + b_2 u^2 + b_3 u^3 + b_4 u^4 \tag{25}$$

Since the body is homogeneous b_k are constant. Then Eq.(22) becomes

$$\begin{gathered}(b_o + b_1 u + b_2 u^2 + b_3 u^3 + b_4 u^4)\nabla^2 u + (b_1 + 2b_2 u + 3b_3 u^2 + 4b_4 u^3)u_x^2 \\ +(b_1 + 2b_2 u + 3b_3 u^2 + 4b_4 u^3)u_y^2 = 0\end{gathered} \tag{26}$$

In Eq.(26), the coefficient b_o is arbitrary. Thus only the ratios $\bar{b}_1 = b_1 / b_o$, $\bar{b}_2 = b_2 / b_o$, $\bar{b}_3 = b_3 / b_o$, $\bar{b}_4 = b_4 / b_o$ can be determined. Thus, after rearranging Eq.(26), we have

$$\begin{aligned}(u\nabla^2 + u_x^2 + u_y^2)\bar{b}_1 + u(u\nabla^2 u + 2u_x^2 + 2u_y^2)\bar{b}_2 \\ +u^2(u\nabla^2 u + 3u_x^2 + 3u_y^2)\bar{b}_3 \\ +u^3(u\nabla^2 u + 4u_x^2 + 4u_y^2)\bar{b}_4 = -\nabla^2 u\end{aligned} \tag{27}$$

The coefficients $\bar{b}_k$ $(k = 1, 2, 3, 4)$ are obtained from the data of four *numerical experiments* by solving numerically Eq.(24), under the following four sets of boundary conditions.

a. $u(0,y) = 300,\ \ u(1,y) = 400,\ \ u(x,0) = u(x,1) = 300 + 100x$
b. $u(0,y) = 300,\ \ u(1,y) = 420,\ \ u(x,0) = u(x,1) = 300 + 120x$
c. $u(0,y) = 320,\ \ u(1,y) = 450,\ \ u(x,0) = u(x,1) = 320 + 130x$
d. $u(0,y) = 350,\ \ u(1,y) = 480,\ \ u(x,0) = u(x,1) = 350 + 130x$

Table 2. Computed values of the coefficients $\bar{b}_k$ in Example 2 at10 nodal points along the line $x = y$.

Point Number	$\bar{b}_1$	$\bar{b}_2$	$\bar{b}_3$	$\bar{b}_4$
1	-.500-02	.516-11	.105-13	.810-17
12	-.500-02	.132-10	.260-13	.192-17
23	-.500-02	.498-11	.933-14	.652-17
34	-.499-02	-.109-10	.202-13	-.140-17
45	-.499-02	-.181-10	.322-13	-.214-17
56	-.499-02	-.143-10	.248-13	-.159-17
67	-.499-02	-.665-11	.110-13	-.689-17
78	-.499-02	-.790-12	.121-14	-.694-17
89	-.500-02	-.137-11	-.222-14	.135-17
100	-.500-02	.768-12	-.120-14	.705-17

Collocation of Eq.(27) at the interior nodal points yields a system of $4M$ ($M = 100$) linear algebraic equation for $\bar{b}_k$ similar to that of Eqs(19), from which the values the coefficients $\bar{b}_k$ are obtained. The values of $\bar{b}_k$ at certain points are given in Table 2. As it was expected $\bar{b}_1 = -0.005$, $\bar{b}_2 \approx \bar{b}_3 \approx b_4 \approx 0$. That is

$$k = b_o(1 - 0.005u) \tag{28}$$

Example 3. Determination of the thermal conductivity in non-homogeneous bodies

We consider a body in which the conductivity varies spatially, i.e. for a two-dimensional body, we have

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$$k = k(x,y) \tag{29}$$

The steady-state heat transfer in absence of heat sources is described by Eq. (22), which may also be written as

$$\nabla^2 u + (\ell nk)_x u_x + (\ell nk)_y u_y = 0 \tag{30}$$

In this example, we want to establish the conductivity function $k(x,y)$ in square two-dimensional body having side length $\alpha = 1$. We use the same discretization as in *Example 1.* The required experimental data are the values of the temperature at the boundary and interior nodal points as well as the values of thermal flux on the boundary. They are obtained from *numerical experiments* on a material with

$$k(x,y) = e^{-0.1(x-0.5)^2} \tag{31}$$

In this case Eq. (30) becomes

$$\nabla^2 u - 0.2(x-0.5)u_x = 0 \quad 0 \le x, y \le 1 \tag{32}$$

We set

$$\ell nk = \kappa(x,y) \tag{33}$$

and we assume the following approximation in order to include possible non-linearity.

$$\kappa(x,y) = b_o + b_1 u + \ldots + b_n u^n \tag{34}$$

Since the material is non-homogeneous the coefficients b_k depend on the position x, y, that is $b_k = b_k(x,y)$. Substituting Eq. (34) into Eq. (30) yields

$$\begin{aligned} &\nabla^2 u + (b_{0,x} + b_{1,x} u + b_1 u_x + \ldots + b_{n,x} u^n + n b_n u^{n-1} u_x) u_x \\ &+ (b_{0,y} + b_{1,y} u + b_1 u_y + \ldots + b_{n,y} u^n + n b_n u^{n-1} u_y) u_y = 0 \end{aligned} \tag{35}$$

The temperature u and its derivatives are established *from n numerical experiments* by solving numerically Eq. (32). Note that the derivatives $b_{k,x}$ and $b_{k,y}$ appear in Eq. (35). They can be expressed in terms of b_k using AEM as presented in Sections 3 and 4. That is

$$\nabla^2 b_k = \bar{q}_k \quad \text{in} \quad \Omega \qquad k = 1,2,\ldots n \tag{36}$$
$$b_k = \text{prescribed on } \Gamma$$

where $\bar{q}_k$ is a fictitious sources. The boundary values of b_k can be established from the corresponding boundary values of the thermal flux. For the simplicity of computation, the numerical results have been obtained for $n = 0$, that is using only the first term in Eq. (34). On the basis of Eqs(14), Eq. (36) yields

$$\{b_o\} = [G_o]\{\bar{q}_o\} + \{D_o\} \tag{37a}$$
$$\{b_{o,x}\} = [G_{o,x}]\{\bar{q}_o\} + \{D_{o,x}\} \tag{37b}$$
$$\{b_{o,y}\} = [G_{o,y}]\{\bar{q}_o\} + \{D_{o,y}\} \tag{37c}$$

Substituting Eqs(37) into Eq. (35) yields

$$([u_x][G_{o,x}] + [u_y][G_{o,y}])\{\bar{q}_o\} = -(\{\nabla^2 u + [u_x]\{D_{o,x}\} + [u_x]\{D_{o,y}\}) \tag{38}$$

The experimental data are obtained from the numerical solution of Eq. (32) subjected to the following boundary conditions

$$u(0,y) = 1, \quad u(1,y) = 2, \quad u(x,0) = u(x,1) = 1 + x$$

The fictitious source $\{\bar{q}_o\}$ is evaluated from Eq. (38). Then $\{\kappa\} = \{b_o\}$ is computed from Eq. (37a). The computed values of κ are presented in Table 3 as compared with the exact ones. They are in good agreement.

Table 3. Computed values of the conductivity $\kappa = \ell nk$ along the line $y = 2.25$

Point Number	Computed	Exact
41	-.2025-01	-.2025-01
42	-.1224-01	-.1225-01
43	-.6252-02	-.6250-02
44	-.2246-02	-.2250-02
45	-.2494-03	-.2500-03
46	-.2494-03	-.2500-03
47	-.2246-02	-.2250-02
48	-.6252-02	-.6250-02
49	-.1224-01	-.1225-01
50	-.2025-01	-.2025-01

6 Conclusions

In this paper the AEM has been employed in a class of problems, in which the excitation and the field function describing the ensuing response of the system are given and it is required to establish the governing differential equation. The proposed method can solve many interesting problems in engineering practice. Among them the establishment of the mathematical model of systems, the constitutive laws of materials as well as the development of structural analysis methods suitable to solve control and optimisation problems in structural design.

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