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Deposition of small dust particles on distorted liquid droplets in an electric field

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Abstract

The paper presents a numerical algorithm to simulate the deposition of small dust particles on much larger liquid droplets distorted by the electric field. It involves three steps. First, the droplet distortion in the electric field is determined by balancing the capillary and electric forces (Laplace-Young equations). The electric force is proportional to the square of the electric field intensity and is calculated using the Boundary Element Method. In the second step, the gas flow velocity distribution around the droplet, with a shape as previously determined, is simulated. Finally, the particle trajectories are calculated by numerical integration of the equation resulting from the Newton equation of motion. Numerical results of the deposition efficiency are presented for different values of non-dimensional parameters governing the problem: the electrical Bond, the Stokes, the electrical Stokes and the Reynolds numbers. It has been shown, that the droplet elongation can seriously change the deposition efficiency for the dust particles.

1 Introduction

In numerous applications small solid particles are deposited on much larger liquid droplets. This process is essential, for example, in wet scrubbing to remove dust particles from a flowing gas. Inertial deposition of the dust particles can be substantially improved, if electric forces are employed. When both the droplets and the particles are electrically charged with opposite polarity, the Coulomb force attracts the particles towards a charged droplet and they can be captured. An alternative solution is to charge the dust particles only and to apply an external electric field to the deposition channel.

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This method of electrostatic scrubbing is particularly effective in the micrometer and submicrometer dust size range. The scrubbers utilizing electrostatic deposition require lower water rate, and smaller pressure applied to the deposition channel, operating at the same overall collection efficiency as inertial scrubbers. This is because scrubbers utilizing the electric forces operate at lower relative velocities than those in which the inertial collection is dominant. Existing numerical models for simulation of this process assume the spherical shape of a droplet. This assumption is justified when no electrical field other than that of the droplet is present within the zone of precipitation. However, the external electric field distorts the droplet shape, which changes the deposition conditions.

The first theoretical model for this problem was formulated by Kraemer and Johnstone [1]. They determined the collection efficiency, taking into account the Coulomb, image and Stokes forces as well as the space charge effect. Nielsen and Hill [2] have calculated numerically the collection efficiency taking additionally into account an external electric force and an electric dipole interaction force. Dau [3], and later Schmidt and Loffler [4], solved the Navier-Stokes equations to determine the flow field near the collector. All these solutions have been obtained for a fixed spherical collector. Although the model of Jaworek et al. [5] is the most general, considering the trajectories of particles near a charged spherical droplet in the three dimensional space, and solving simultaneously the differential equations of motion for the particle and the droplet, with the flow field near the droplet obtained from the numerical solution of Navier-Stokes equations, they also assumed the droplet to be ideally spherical.

2 Numerical Model

In real applications liquid droplets fall down in ambient gas, most often perpendicularly to its movement and collect small dust particles. The droplet surface can oscillate, and fluid inside of the droplet often circulates. The droplet is distorted by the flowing gas, the external electric field and presence of other droplets, which can be electrically charged.

The idealized model analysed in this paper assumes a stationary and electrically neutral droplet of radius R, which is conducting and originally spherical. The ambient gas flow is laminar, but it does not distort the droplet shape. A uniform electric field is applied in the direction of the fluid motion. Electrically charged dust particles are introduced upstream and far from the droplet, initially moving with a velocity identical to that of the ambient gas. The particles do not affect neither droplet or gas flow and their mutual interaction is neglected.

The droplet is elongated in the electric field and it distorts both the electric field distribution and the gas flow. Trajectories of moving particles result from the balance of all acting forces, most importantly, inertia, air drag and electrical forces. Some trajectories terminate at the droplet surface, and it is assumed that these particles are deposited. Therefore, a full mathematical simulation of this problem should include the droplet elongation, fluid flow, electric field distribution and determination of the particle trajectories.

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3 Droplet elongation in an electric field

As the droplets are usually conducting, the surface charge is induced and the electric pressure is produced. The capillary force tends to minimize the droplet surface - without any other forces this would lead to a spherical shape - but the electric pressure is non-uniform and a new equilibrium shape is formed. Too large electric field causes droplet break-up and disintegration. The elongation of a single conducting droplets in the electric field was discussed by Adamiak [6].

From the balance of all essential forces, the following equation can be derived for the droplet deformation

$$-Bz + \frac{1}{r_1} + \frac{1}{r_2} + Me_n^2 + K = 0, \qquad (1)$$

where: r_1 and r_2 are the principal radii of curvature, K is nondimensional pressure and e_n is the normal electric field.

There are two non-dimensional numbers, which affect the droplet elongation: the Bond number B and electric Bond number M. The first one expresses the ratio of gravitational and capillary forces, the second one - electrical and gravitational forces. In the current analysis it has been assumed that $B=0$.

Eqn (1) is a nonlinear differential equation with unknown function, describing the droplet shape and it has been solved iteratively. Beginning with an undisturbed droplet, the electric field is calculated, using the Boundary Element Method (BEM) in a version explained below, and the electric pressure is evaluated. This is sufficient to obtain the first approximation for the droplet elongation. Then the electric field is updated and the process continues until the final solution is reached.

Two examples of the droplet elongation are shown in Fig.1. The stronger electric field, the larger electric Bond number and larger droplet deformation. When M exceeds some critical value, there is no stable solution, what is treated as the droplet break-up.

4 Electric field calculation

The electric field distribution is governed by the well-known Laplace equation. The external electric field is uniform and the droplet surface is equipotential, which can be used to formulate the Dirichlet boundary conditions. As the domain is unbounded and the space is homogeneous without any electric charges, the BEM seems to be a natural choice for solving the problem. Using a simple layer potential the Fredholm integral equation of the first kind can be formulated for the droplet surface charge, as shown by Brebbia at al. [7]

$$\int_{\Gamma} \sigma(P) G(P, Q) d\Gamma = E_0 z, \qquad (2)$$

where $G(P,Q)$ is the Green function and E_0 - intensity of the external electric field.

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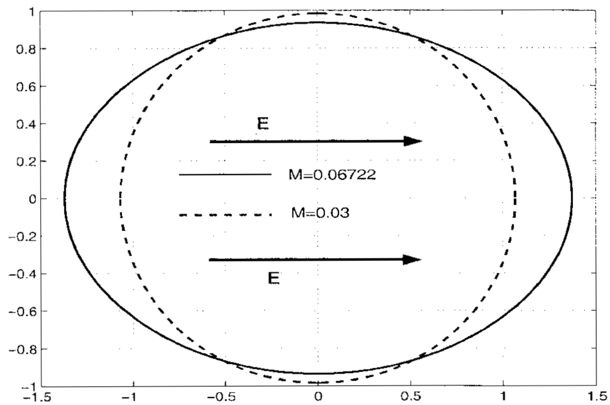

Figure 1 Elongation of a conducting droplet in the external electric field

A rather conventional technique is used to solve this equation: the droplet surface is divided into some number of the boundary elements and the solution over each element is interpolated by a linear function. This procedure leads to an algebraic set of equations with unknown surface charge density on the droplet surface. As there is no electric field inside of the droplet, this density is also equal to the normal component of the electrostatic displacement vector, so the solution of eqn (2) can be directly used in the eqn (1) for the droplet elongation.

5 Air flow distribution

Flowing gas is another factor affecting the particle trajectories. Distortion of the originally uniform velocity distribution was predicted for the viscous laminar flow model, which is governed by the Navier-Stokes equations. As the problem is two-dimensional in the cylindrical set of coordinates, it can be simplified by introducing the stream function - vorticity formulation, Jaworek at al. [5]

$$\frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} = -r\omega , \tag{3}$$

$$\frac{Re}{r}\left(-\frac{\partial \psi}{\partial z}\frac{\partial \omega}{\partial r} + \frac{\partial \psi}{\partial z}\frac{\omega}{r} + \frac{\partial \psi}{\partial r}\frac{\partial \omega}{\partial z}\right) = \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r}\frac{\partial \omega}{\partial r} - \frac{\omega}{r^2} + \frac{\partial^2 \omega}{\partial z^2}, \tag{4}$$

where: *Re* is the Reynolds number, ψ - stream function and ω - vorticity. This equation has been solved using the Finite Element Method for the triangular grid. As the domain is open, it must be truncated as some distance far from the droplet. The position of these artificial boundaries was determined empirically.

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6 Particle trajectories

The trajectory of a particle of density ρ_p, radius r_p and charge q near the neutral droplet of equivalent radius R is governed by the Newton vector differential equation, which can be presented in a dimensionless form as follows:

$$St\frac{d\boldsymbol{v}}{dt} + \boldsymbol{v} - \boldsymbol{v}_0 + Ste\boldsymbol{E} = 0, \tag{5}$$

where: $\boldsymbol{v}$ is the particle velocity, $\boldsymbol{v}_0$ - velocity of ambient gas, $\boldsymbol{E}$ - electric field intensity, St - Stokes number and Ste - electric Stokes number.

$$St = \frac{2r_p^2\rho_p v_0^2 C_m}{9\mu R} \text{ and } Ste = \frac{qE_0 C_m}{6\pi\mu r_p^2},$$

where: C_m is the Cunningham factor and μ - air kinematic viscosity.

The trajectory of an airborne particle in the vicinity of a droplet can be determined numerically by solving eqn (5) with the flow field determined from the numerical solution of the Navier-Stokes equations and the electric field solved from the Laplace equation.

7 Results

7.1 Particle trajectories

Some number of dust particles was introduced far from the droplet in the upstream direction and their trajectories were simulated for different values of all non-dimensional numbers affecting the process. The results for a significantly distorted droplet (M=0.06722, what is close to the static stability limit), zero Reynolds number and without electric forces (Ste=0.0) are shown in Figs. 2, 3 and 4.

For a very small Stokes number (Fig. 2), the particle inertia is negligible and the particle trajectories practically follow the air flow lines. Only if the starting point of the trajectory is very close to the axis of symmetry does the particle eventually collide with the droplet. For all other starting points the particle is not deposited.

With increasing value of the Stokes number more and more trajectories ends up at the droplet surface indicating larger deposition efficiency (Fig. 3). This is a result of increasing particle inertia, which is comparable with the air drag. For a very large Stokes number (Fig. 4), inertia makes trajectories practically linear. The deposition efficiency is obviously much better in this case - it is much easier to collect heavier particles.

Introduction of the electric forces completely changes this pattern. Relatively strong electric interaction (Ste=0.2) even for a very small Stokes number attracts a lot of particles towards the droplet (Fig. 5). Electric charging of the dust particles is especially effective in the improvement of the collection efficiency of small particles. Even stronger electric forces for heavier particles do not change the

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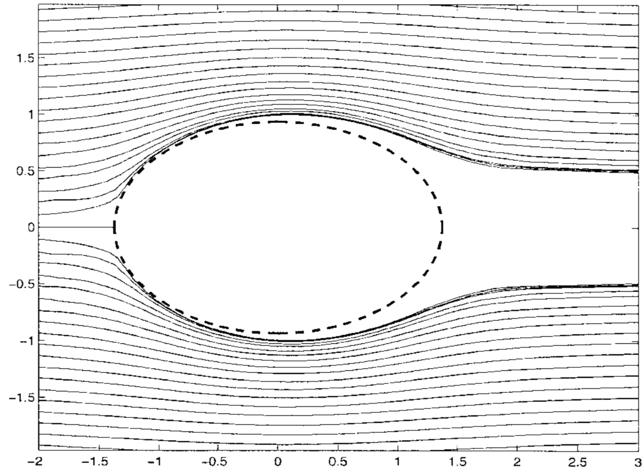

Figure 2 Particle trajectories for a small Stokes number and without electric forces (St=0.1, Ste=0, Re=0, M=0.06722)

trajectories so drastically (Fig. 6). Inertia is still an important factor for these particles.

7.2 Collection efficiency

Even if the trajectories show the effect of different factors affecting the particle motion well, they do not precisely show the efficiency of deposition. A simple definition of the collection efficiency is used in this paper [1]

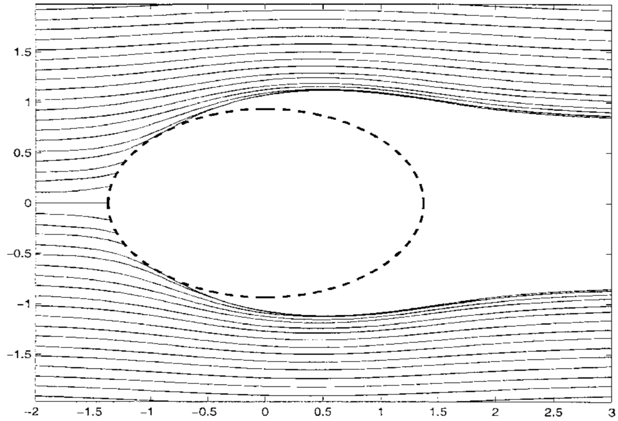

Figure 3 Particle trajectories for an intermediate Stokes number without electric forces(St=0.5, Ste=0, Re=0, M=0.06722)

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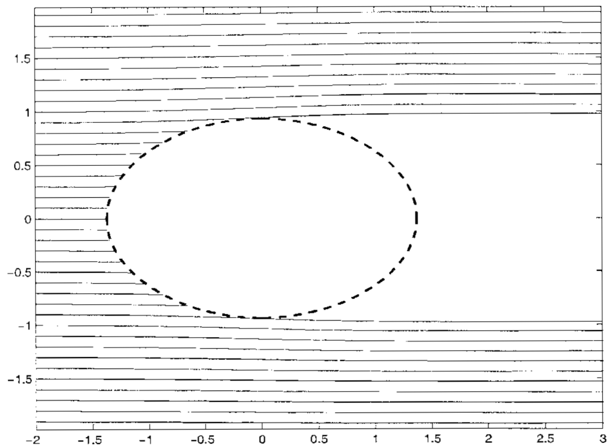

Figure 4 Particle trajectories for a large Stokes number (St=10, Ste=0, Re=0, M=0.06722)

$$\eta = y_c/R,$$

where: y_c is a width of the particle beam which is collected, and R - droplet radius.

Using this definition, Fig. 7 has been prepared and it shows the effect of both the Stokes and electric Stokes numbers on the particle deposition. The deposition efficiency increases from practically zero for a very small St to almost 100% for a

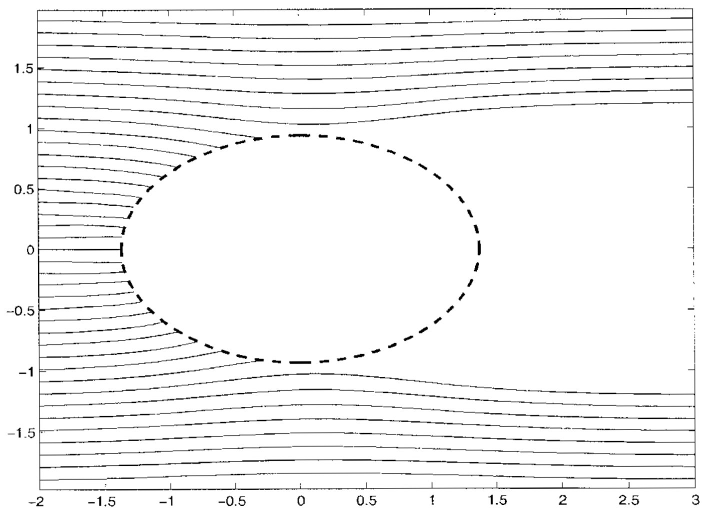

Figure 5 Particle trajectories for a small Stokes number and strong electric forces (St=0.1, Ste=0.2, Re=0, M=0.06722)

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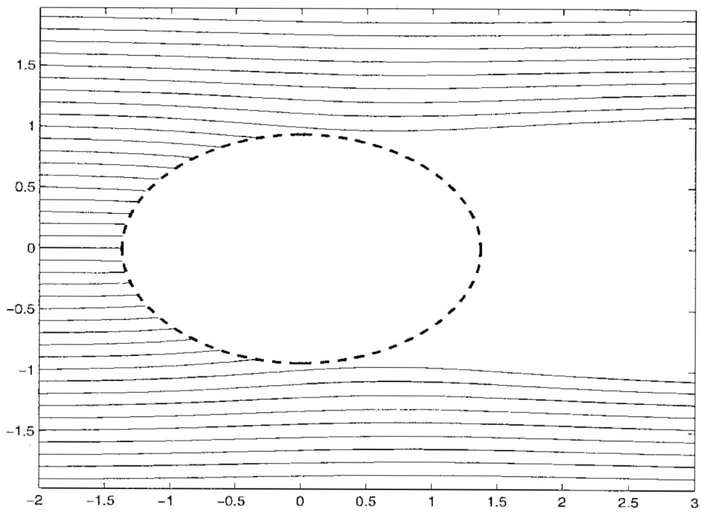

Figure 6 Particle trajectories for a large Stokes number and strong electric forces (St=1.0, Ste=0.5, Re=0, M=0.06722)

very large St. The electric forces always improve collection, although this effect is much stronger for a small St, where the deposition efficiency can be increased even a few times. For a large St, there is still some improvement, but it is not very significant.

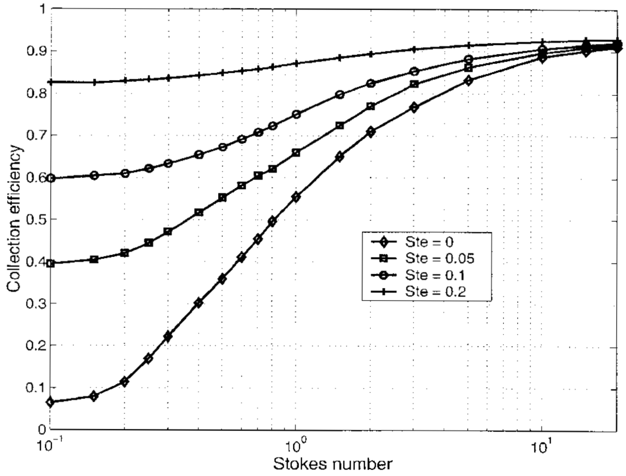

Figure 7 Effect of the Stokes and the electric Stokes numbers on the deposition efficiency (Re=0, M=0.06722)

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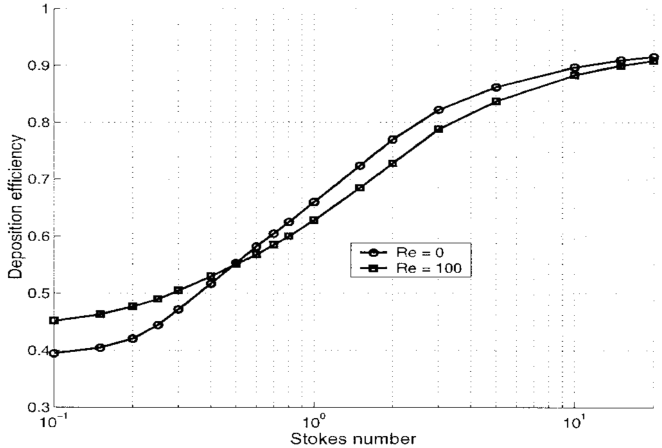

Figure 8 The deposition efficiency versus Stokes number for different Reynolds numbers (Ste=0.05, M=0.06722)

The deposition efficiencies of slightly charged particles as a function of the Stokes number for two different Reynolds numbers are compared in Fig. 8. For low St, the deposition efficiency is better for higher Re, but for a higher St a smaller Re would yield better results. This can be explained by analysis of the distribution of the airflow lines near the droplet.

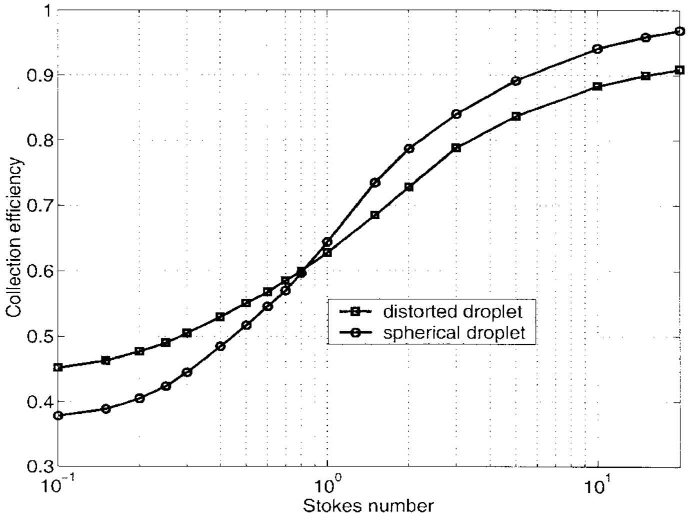

Figure 9 The deposition efficiency for spherical and distorted droplets (Ste=0.05, Re=100)

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Finally, the deposition on distorted and spherical droplets are compared in Fig.9. A distorted droplet produces stronger concentration of the electric field near the tip, therefore, it gives higher deposition efficiency at small St, when the electric effects dominate. Assuming a larger St, the collection efficiency for distorted droplets is smaller than for spherical ones, as the vertical dimension of the elongated droplet is reduced.

8 Conclusions

The presented numerical algorithm makes it possible to accurately predict the deposition of dust particles on distorted water droplets. The essential part of the numerical algorithm is based on the Boundary Element Method, which is used for predicting the electric field distribution. Knowledge of this field is necessary for determining the droplet elongation and electric forces acting on the dust particles. Especially for the first problem there is a need for very accurate calculations, as any irregularities or lack of smoothness in the field distribution would trigger instabilities of calculations. This is particularly important for the elongations which are close to the maximum one.

The results of calculations show details of the deposition process and allow for calculations of the deposition efficiency. The examples included in this paper present effect of all relevant non-dimensional parameters: Stokes, electric Stokes, electric Bond, and Reynolds numbers. Only the last one is relatively insignificant, all the others play an important role in the process.

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