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The stress analysis of frictional punch with crack in plane elasticity using boundary element method

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Abstract

In this paper boundary element (BE) formulations for contact and fracture problems are briefly reviewed and the BE formulation using isoparametric quadratic elements is applied to the frictional punch with crack, initiating at each end of the punch. The effects of the present of the crack on the slip-stick conditions, the stresses within the contact interface and deformations are examined. In the present BE formulation, the overall system of equation is obtained by using multi-domain approach in which the contact conditions are taken into account to couple the different system of equation for each body in contact. Several types of contact interface conditions are covered including infinite friction (stick), frictionless and Coulomb friction slip. In order to include fracture in the analysis, singular elements that are restricted to the singular field associated to the presence of the crack are employed.

1 Introduction

This paper covers the application of the BE method to contact and fracture mechanics in which it is well established as an accurate numerical tool particularly well suited for linear elastic problems. The BIE application in this paper represents the plane strain analysis of frictional contact of an elastic punch indenting the elastic foundation of cracks, initiating at each end of the punch.

Hasebe *et al* [1] analyzed the problem in which the crack initiates from one end of a rigid punch. In their work the stress distributions, the stress intensity factors are obtained and the effect of the coefficient of friction on the stress

Boundary Elements XXII, C.A. Brebbia & H. Power (Editors)

intensity factors are examined; however, in their analytical analysis the slip-stick conditions within the contact interface are not taken into account.

2 BE analytical formulation

The basis of the BE formulations is a boundary integral identity for displacements, relating the displacement at an interior point P to the displacements and tractions at a boundary point Q over the surface S, as follows (see the textbooks by Brebbia[2], Becker[3], Banerjee[4]):

$$C_{ij}(P)u_i(P) + \int T_{ij}(P,Q)u_j(Q)dS(Q) = \int U_{ij}(P,Q)t_j(Q)dS(Q) \qquad (1)$$

in which u_i and t_i are the displacement and traction vectors respectively, and U_{ij} and T_{ij} are the displacement and traction kernels respectively, which are functions of the positions of points P and Q and material properties. Note that the above-given equation ignores the effects of non-linear material behaviour and body forces.

3 Formulation of the frictional contact problems

In order to couple the different system of equations obtained from the discretized BI equation for each body in contact, the contact conditions have to be imposed on the contacting node pairs. It should be noted that the nodes outside the contact region have either prescribed displacements or prescribed tractions. As shown in Figure 1, consider node a in domain A and node b in domain B to make a contacting node pair. If these nodes in the contacting surface are not allowed to slip in any direction (stick contact), continuity of displacements and equilibrium conditions require:

$$\text{Compatibility} : u_x^a - u_x^b = x^b - x^a \quad \text{and} \quad u_y^a - u_y^b = y^b - y^a \qquad (2)$$

$$\text{Equilibrium} : t_x^a + t_x^b = 0 \quad \text{and} \quad t_y^a + t_y^b = 0 \qquad (3)$$

If slip condition occurs, referring to the tangential and the normal components of traction and displacements (see Figure 1), the following constraint conditions hold:

$$\text{Frictional law} : t_t^a = \pm\mu \; t_n^a \qquad (4)$$

$$\text{Equilibrium} : t_n^b + t_n^a = 0 \quad \text{and} \quad t_t^b + t_t^a = 0 \qquad (5)$$

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$$\text{Compatibility :} \quad u_n^a = u_n^b \quad \text{and} \quad u_t^a - u_t^b = \delta_t \tag{6}$$

Finally, in the presence of interference (clearance), δ_n, the following compatibility of displacements holds:

$$\text{Compatibility :} \quad u_n^a - u_n^b = \delta_n \tag{7}$$

In the above-given expressions, the subscripts t and n indicate the tangential and the normal directions, respectively and δ_t represents the amount of slip in the tangential direction. The local components of tractions and displacements can be transformed into global components by using the contact angle θ (see Figure 1).

3.1 Contact iterations

It is obvious from the nature of the contact problems that the proper numerical algorithms for contact problems require an iterative and / or incremental procedure.

One of the iterative procedures used in the present formulation is an efficient automatic iterative scheme in order to perform the contact iterations without load increments. This iterative processes checks the possibilities which are overlap, tensile stress and frictional slip that may arise in determining the proper contact conditions. Since a contact area is based on compressive forces, contacting nodes with normal tensile stresses, which suggest that the contact area is too large for the given load, must be released from the contact in the next iteration. Overlapping nodes indicate that the contact area is too small for the given load. Therefore the associated elements must be included in the contact area for the next iteration in order to prevent any inter-penetration of the bodies in contact. Finally, in the presence of Coulomb friction slip, which occurs when the absolute value of the ratio of the tangential to the normal traction exceeds the coefficient of friction, μ, the related nodes have to be allowed to slip in the next iteration. It should be worth mentioning that all node pairs are assumed to be a sticking stage for the first iteration. The details of such iterative scheme can be found in the textbook by Becker [3].

3.2 Coupling the system of equations

In order to form the overall system of equation for two bodies in contact, the linear algebraic equations obtained from the discretized BI equation for each contacting domain can be formed as follows:

$$\begin{bmatrix} [A]^{(a)} & 0 \\ 0 & [A]^{(b)} \end{bmatrix} \begin{bmatrix} [u]^{(a)} \\ [u]^{(b)} \end{bmatrix} = \begin{bmatrix} [B]^{(a)} & 0 \\ 0 & [B]^{(b)} \end{bmatrix} \begin{bmatrix} [t]^{(a)} \\ [t]^{(b)} \end{bmatrix} \tag{8}$$

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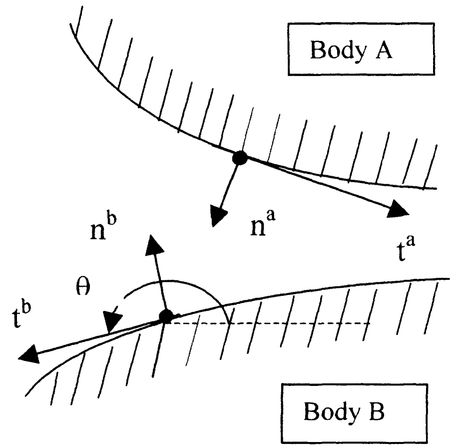

Figure 1; Local and tangential directions

In order to arrive at the solution matrix, both prescribed displacements and prescribed tractions are imposed on the system of equations obtained from the discretized BI equation for each solution domain and the resulting matrices are coupled together according to the contact conditions, thus the following final form of the system of equation can be obtained.

$$\lfloor A^* \rfloor [x] = \lfloor C^* \rfloor \tag{9}$$

where the matrix $\lfloor A^* \rfloor$ is the final coefficients of the overall system of equation, the vector $[x]$ includes the unknown tractions and displacements and $\lfloor C^* \rfloor$ is the known vector.

Implementing non-confirming mesh discretisation of the contacting surfaces can eliminate the need for node-on-node contact within the contact interface. In such mesh discretisation a length mapping procedure or fictitious nodes approach in the contact area can be used [5].

For contact problems, the application of the BIE method has been widely discussed (see ,for example, references[6-9]; however the present BE contact algorithm can be efficiently implemented in elastoplastic frictional contact problems[10].

It is clear that the BE implementation of the contact constraints in the system of equation is more accurate and efficient than the FE approach.

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4 BE fracture formulation

It is well known that the BE approach is a very suitable numerical tool for problems in which stress gradients vary rapidly such as fracture problems. Special numerical procedures such as J-integral calculations, stress and displacement extrapolation developed for finite element method are directly applicable to the BE approach.

4.1 Singular elements

In order to cover the fracture in BE analysis, as in FEM, the quarter-point node-shifting element as means of providing the $r^{-1/2}$ singularity in the stress and strain fields at the crack tip in the solution domain can be used. As shown in Figure 2, in this technique the required $r^{-1/2}$ strain (stress) singularity can be created by shifting the midpoints of isoparametric quadratic element to the quarter-point that is nearer to the crack tip. Thus, these elements are known as singular elements or node-shifted elements. Note that the parameter, δ, in the Figure 2 is the local distance to measure the distance from the crack tip and ℓ is the boundary element length. The crack surfaces are assumed to be very close to each other, thus causing the integral identities to be singular. In order to avoid numerical difficulties subdomain formulation can be employed. The computational performances of the singular elements support the generally accepted belief that the BIE approach is an excellent numerical tool for linear elastic fracture mechanics applications.

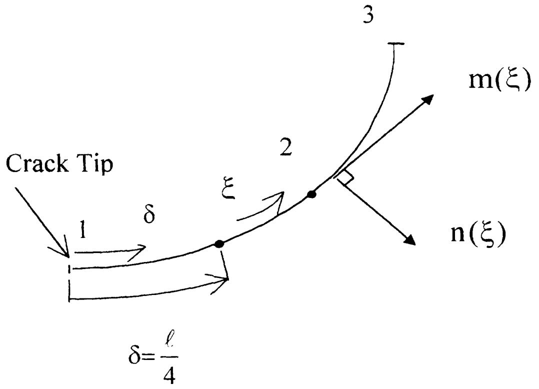

Figure 2: Node-shifted singular element for two-dimensional fracture applications.

Another approach for BE fracture analysis is dual boundary element [see, for example, reference [11-14]), which do not require subdomain discretisation of

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the solution domain; however it cause a significant increase in programming and analytical work in order to deal with singular integrals arising in traction boundary integral identities.

It should be worth mentioning that the amount of the mesh refinement at crack tips is much less than that needed in FE analysis.

5 Frictional punch problem

This BIE application represents the plane strain analysis of the frictional contact of a punch of height H_p and half-width W_P indenting the foundation of height H_f and half-width W_f. The subscripts p and f refer to the punch and foundation respectively. Boundary and loading conditions are given in Figure 3. The relevant data for dimensions employed in this analysis are H_P/W_P=2, W_f/W_p =4, H_f / W_p =4. The relevant crack length is assumed to be W_p/C_L,= 0.5 in the crack case analysis. BE mesh design is given in Figure 4. The fracture is covered in the analysis by creating artificial contact in which contact conditions are assumed to be perfectly glued, as shown in Figure 5. Note that the line of the crack and the edge of the punch aligns. The poison's ratio, ν, and the young's modules E are assumed to be both at 0.3 and 109.10^6 Pa respectively. In this analysis the coefficient of friction, μ, and the applied load P_O are assumed to be at 0.2 and 1.10^6 Pa respectively.

It is obvious that the problem involves singularities at the crack tip and singularities at the edge of the punch. However, it is clear from the literature that relatively finer mesh division, depending on the amount of the engineering judgement, is enough to obtain the proper contact condition, unlike fracture applications.

It is possible to obtain acceptable results by using finer mesh division at the crack tip region, particularly when the crack tip is remotely located from the edge of the punch. However it was experienced that the amount of the mesh refinement necessary to eliminate inconsistency in stress levels and deformations at the crack tip were greatly reduced by using singular elements.

Figure 6 shows the stress distribution in the y-direction, while figure 7, represents stress levels in the x-direction. As expected, there is a smooth increase of the contact stresses towards the end of the contact interface, but for the case involving crack, the contact stresses start to increase sharply in the vicinity of the edge of the punch. This is because of the ratio of the applied load to the elastic modulus employed in this analysis. Figure 8 shows the expected rate of the increase in the tangential friction stresses for the case not involving the crack. In this figure it is clear that the effect of the present of the crack on the stick-slip conditions is significant: The sticking area gets significantly larger, while the slipping area gets smaller.

The deformed shape of the foundation is given in figure 9 when magnification-scaling factor is 2. Figure 10 represents the deformed shape of the foundation in the present of the crack with the same magnification-scaling factor. As expected, the effect of the present crack on the deformations is significant.

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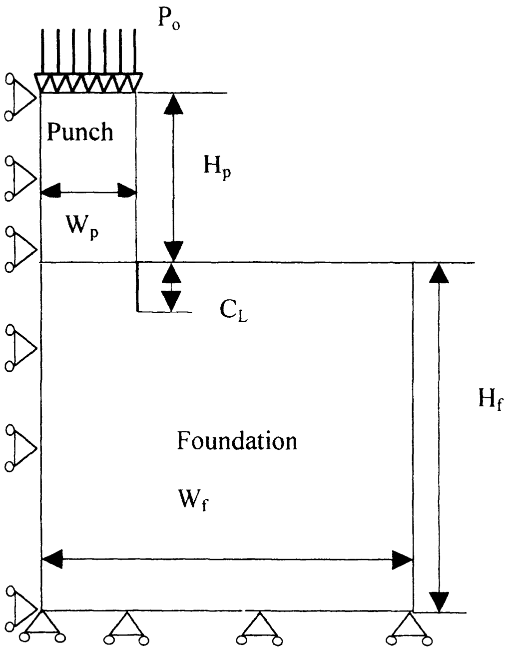

Figure 3: Frictional punch problem with crack.

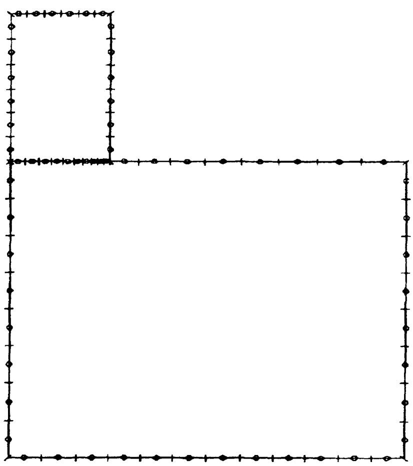

Figure 4: BE mesh design for punch problem.

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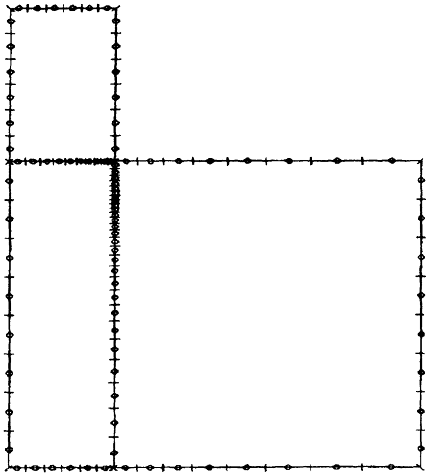

Figure 5: BE mesh design for frictional punch with crack.

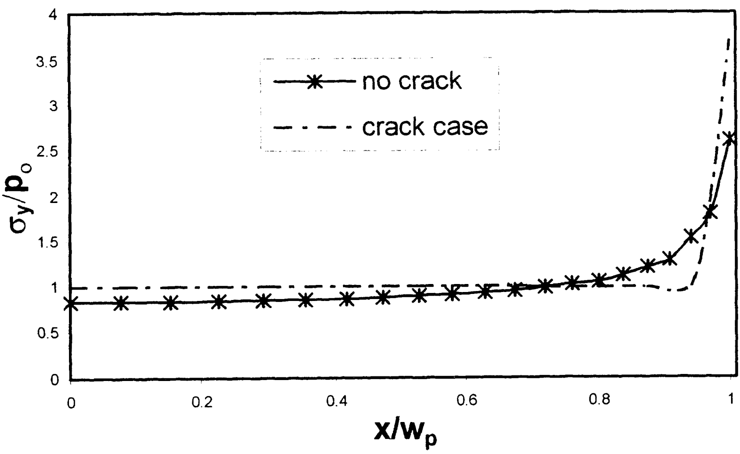

Figure 6: The stress, σ_y, distribution.

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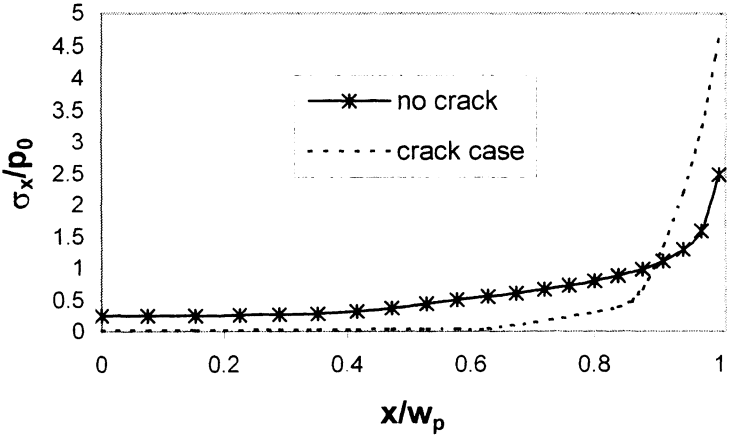

Figure 7: The stress, σ_x, distribution.

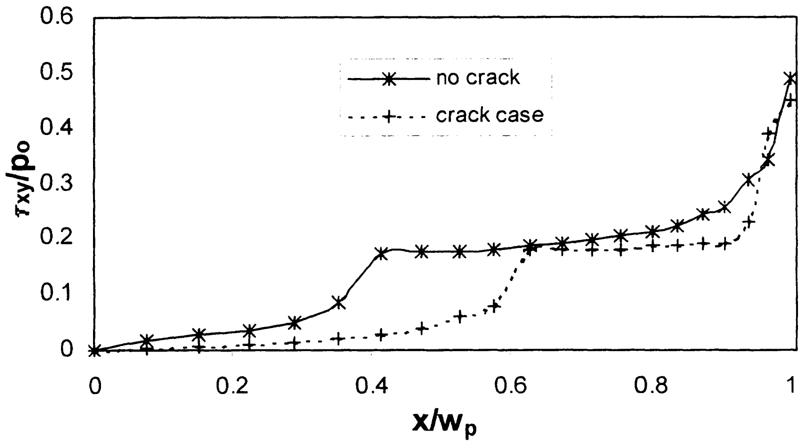

Figure 8: The shear, τ_{xy}, distribution.

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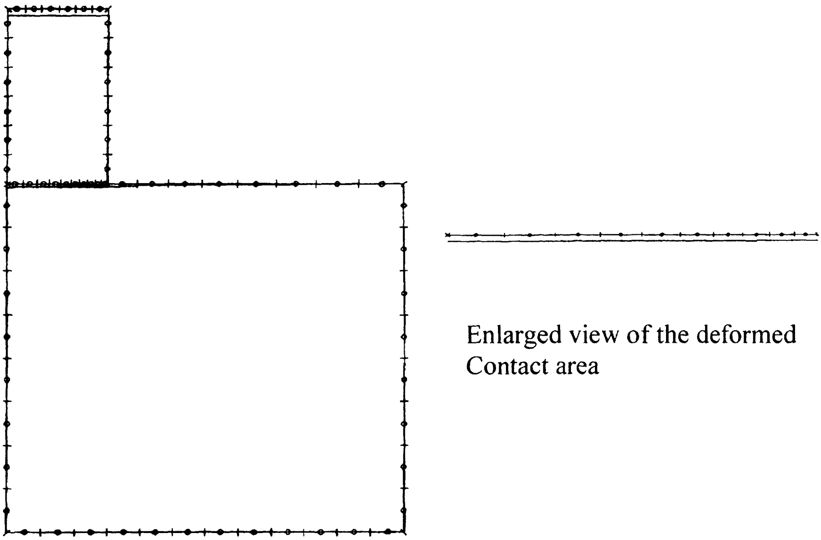

Figure 9: The deformed view of the frictional punch
when magnification-scaling factor is 2.

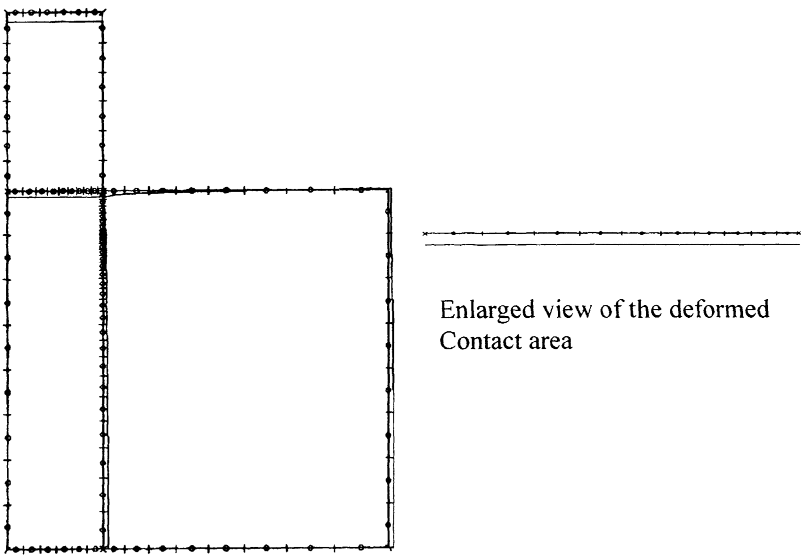

Figure 10: The deformed view of the frictional punch with crack
when magnification- Scaling factor is 2.

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6 Conclusion

In this work boundary element (BE) formulations for contact and fracture problems are briefly reviewed and the BE formulation using isoparametric quadratic elements is applied to the frictional punch with cracks, initiating at each end of the punch. The effects of the presents of the crack on the slip-stick conditions, on the stresses in the contact area and the deformations are examined.

The stress intensity factors can be obtained by means of extrapolation of the computed crack face displacements and the effects of the coefficient of friction on the stress intensity factors can be investigated.

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