

# A simple wetting and drying method for shallow water flow with application in the Vitória Bay estuary, Brazil

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## Abstract

This paper presents a simple and effective method to model wetting and drying processes in natural water bodies. The proposed method has been nicknamed “rough-porous layer”, or RPL. It bears similarities with the so called marsh porosity method, but is simpler to implement, mass conserving and does not affect wave celerity in wet areas. The method can be easily implemented in finite element models, what makes it useful in numerical modeling of natural water bodies with intricate topographies. A standard testing case of a basin with variable slope is used for validations and comparison with other methods. The effectiveness of the RPL is better evaluated in modeling the complex Vitória Bay estuary (Brazil) with its extensive mangrove areas.

*Keywords: wetting and drying, Vitória Bay estuary, mangrove swamps, finite element method.*

## 1 Introduction

Coastal water bodies such as bays and estuarine systems quite often present mangrove swamps, marsh and other wetlands. These regions play an important role in ecosystem productivities contributing to both territorial and marine biodiversity. These wetlands are maintained mainly by periodic drying and wetting processes [4]. Thus, the ability to represent such processes is essential in



environmental hydrodynamics modeling [1]. However, an adequate inclusion of wetting and drying processes is a nontrivial task for most existing numerical models.

Ideally, shallow water models should be able to account for the wetting and drying processes by smoothly varying the modeled domain as the water level changes. However, the existing techniques that adopt variable domains, e.g. inclusion and exclusion of cells, are either far from smooth or too expensive for practical engineering purposes. In the last decades a variety of approaches have been proposed to represent wetting and drying processes in finite volume, finite differences and finite element numerical methods for shallow water flows [2, 3, 5, 8–11].

Most variable domain schemes work fine in idealized domains with simple geometry. However, in natural complex water bodies, numerical instabilities, spurious oscillations and the like are often a serious challenge, due to changing boundaries, and difficulties to deal with very shallow drying zones, cf. [12, 13].

This paper presents a simple and efficient method to represent wetting and drying processes in finite element numerical models for shallow water flows, herein called RPL, for “rough-porous layer”. RPL has similarities with other porosity approaches, such as the Marsh Porosity Method (MPM) used in the RMA2 model [6]. These methods do not modify the horizontal domain, but may modify the vertical domain, accounting for subsurface fluxes occurring through thin layers, as depicted in Figure 1.

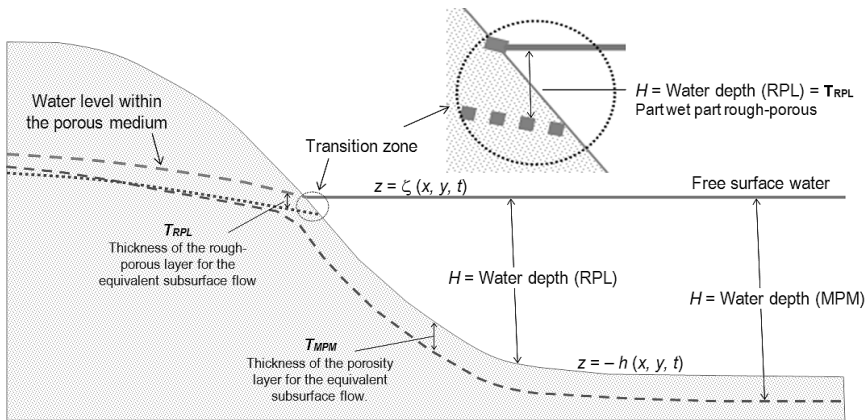


Figure 1: Approximate representation of change in flowing section of the proposed method. For the sake of visualization, the layer thicknesses are exaggerated.

Referring to Figure 1, while the MPM considers a subsurface porosity layer, with variable thickness  $T_{MPM}$ , throughout the horizontal domain, in the RPL method, the rough-porous layer, for the equivalent subsurface flow, only exists in places where and when the water depth is smaller than the specified thickness

$T_{RPL}$ . It is evident that in the wetted area, the water depth in the MMP is incremented, affecting wave celerity. The same does not occur in the RPL.

The equivalent roughness amplitude in the RPL is typically from 1 to 2.0 times the layer thickness  $T_{RPL}$ , leading to a highly restricted rough flow with Chézy coefficients in the range of 8 to 14. The method is implemented in the 3D finite element hydrodynamic model of SisBaHiA<sup>®</sup> (*Base System for Environmental Hydrodynamics* in Portuguese – cf. [17])

The remainder of this paper is organized as follows. The shallow water governing equations are introduced in Section 2. Next, Section 3 describes the RPL wetting and drying method and the model in which it is implemented. Section 4 presents numerical examples to illustrate the performance of the method. Concluding and final remarks are given in the last section.

## 2 Governing equations

In order to focus in the wetting and drying issues, only a simplified version of the depth averaged module of the 3D hydrodynamic model of SisBaHiA<sup>®</sup> is presented. The complete equations can be seen in [17]. As is well known, the shallow water equations are derived from the incompressible Navier-Stokes equations by assuming that the pressure is in hydrostatic balance, and by averaging the equations along the vertical direction [2]. For the purposes of this paper, the shallow water equations can be written as follows, using the conventional indicial notation, with  $i, j = 1, 2$ :

$$\frac{\partial H}{\partial t} + \frac{\partial H u_i}{\partial x_i} = 0, \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -g \frac{\partial \zeta}{\partial x_i} + \frac{1}{\rho H} \left( \frac{\partial}{\partial x_i} (H \tau_{ij}^T) + \tau_i^S - \tau_i^B \right) + a_i, \quad (2)$$

where  $u_i$  is velocity component in  $i$ -direction,  $z = \zeta$  is the free surface elevation and  $z = -h$  the bottom elevation from a reference level.  $H = \zeta + h$  is the water depth, and  $\rho$  is the water density. The term  $a_i$  stands for the Coriolis term,  $g$  is the gravitational acceleration;  $\tau^S$  and  $\tau^B$  are surface and bottom stress vectors, respectively;  $\tau_{ij}^T$  is the turbulent stress tensor parameterized by filtering techniques, cf. [14, 16, 17].

Furthermore to obtain a well-posed problem appropriated boundary conditions have to be imposed along the boundary of the domain, and an initial state need to be provided as well.

The shallow water equations (1) and (2) are solved by finite element method in SisBaHiA<sup>®</sup>, cf. [16]. The system is available in [17].



### 3 Drying-wetting approach

One of the major issues of the shallow-water governing equations in natural water bodies modeling is their inability to deal with drying and wetting areas, where the water depth can reach zero value. And the aim of a wetting and drying approach is to allow for the appearance and disappearance of these areas [1].

As mentioned, the RPL method has similarities with the marsh porosity method considered in [6]. Both keep the same horizontal domain, and represent subsurface flows through a thin layer in areas where the fluid top layer is below ground level. But the RPL employs simpler ways of calculating the modified bathymetry and roughness within the subsurface flow layer.

The RPL method considers that the water level surface can drop below ground level. The dry mesh nodes are not removed from the computational domain, but enter in a state of restricted subsurface flow. In the method there are three flow zones: above ground flow, transitional range flow, and subsurface flow. The upper bound of the transition zone,  $z_{TS}$ , is defined by:

$$z_{TS} = -h + T_{RPL} \quad (3)$$

for each flow zone, the modified “rough-porous” bathymetry,  $h_R$ , and amplitude of the equivalent roughness,  $\varepsilon_R$ , are calculated as follows:

- Above ground flow:  $\zeta(x, y, t) > z_{TS}(x, y, t)$

$$\begin{aligned} h_R &= h(x, y, t) \\ \varepsilon_R &= \varepsilon(x, y) \end{aligned} \quad (4)$$

- Transitional range flow:  $-h \leq \zeta \leq z_{TS}$

$$\begin{aligned} h_R &= -\zeta + T_{RPL} \\ \varepsilon_R &= \frac{\varepsilon - rT_{RPL}}{T_{RPL}}(\zeta + h) + rT_{RPL} \end{aligned} \quad (5)$$

- Subsurface flow:  $\zeta > -h$

$$\begin{aligned} h_R &= -\zeta + T_{RPL} \\ \varepsilon_R &= rT_{RPL} \end{aligned} \quad (6)$$

The roughness factor,  $r$ , typically ranges from 1 to 2. The bottom stress  $\tau_i^B$  depends on the amplitude of the bottom roughness  $\varepsilon_R$ , and is computed as:

$$\tau_i^B = \rho\beta u_i \quad (7)$$

where

$$\beta = \frac{g}{C_h^2} \sqrt{u^2 + v^2} \quad (8)$$



and the Chézy coefficient  $C_h$  is given by

$$C_h = 18 \log_{10} \left( \frac{6H}{\varepsilon_R} \right) \tag{9}$$

One can verify that in the rough-porous layer occurs a restricted rough flow with Chézy coefficients in the range of 8 to 14. For depth averaged models of natural water bodies, typical Chézy coefficients are above 40.

### 4 Numerical results

The following numerical simulations illustrate the performance of the RPL method.

#### 4.1 Case 1: standard test in a simple geometry basin with variable slope

This is a standard test case used in many papers, e.g. [4, 5], to reproduce the water front movement driven by a wave in a variable slope shoreline. The basin domain comprises a rectangular channel with a length of 500 m and width 25 m and variable slope. The domain was discretized by a mesh of 70 biquadratic quadrilateral elements, cf. Figure 2.

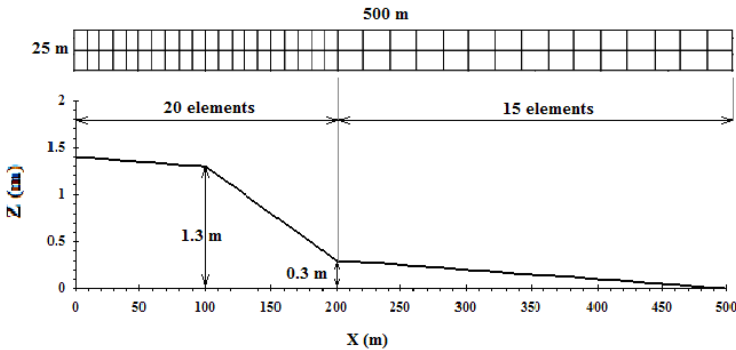


Figure 2: Top view of the finite element mesh, and longitudinal section showing the variable slope bathymetric data. In the mesh, all elements are 9 nodes biquadratic, but only the vertices are shown.

Normal velocity was set to zero on all land boundaries. At the open boundary, located at  $x = 500$ , the water level varies as:

$$\zeta = 1.0 + 0.75 \cos \left( \frac{2\pi t}{3600} \right) \tag{10}$$

Initial conditions are  $\zeta = 1.75\text{m}$  and  $u_i = 0$  everywhere. The time step for the test was 9.0s. The rough-porous layer parameters were:  $T_{RPL} = 0.1\text{m}$ ,  $r = 1.0$ , and  $\varepsilon = 0.03\text{ m}$ . Wind and Coriolis forces were not considered. Eddy viscosity was set to  $0.5\text{ m}^2/\text{s}$ , to follow [4, 5].

Figure 3 shows velocities and elevations at different times, compared with results presented in [4]. One sees at  $t = 12$  min a significant acceleration of the flow near the point  $x = 100$  m, which is due to rapid changes in the bathymetry and very shallow conditions around the point. It is evident that the results with the simple RPL method are quite similar to the ones presented in [2, 4, 5].

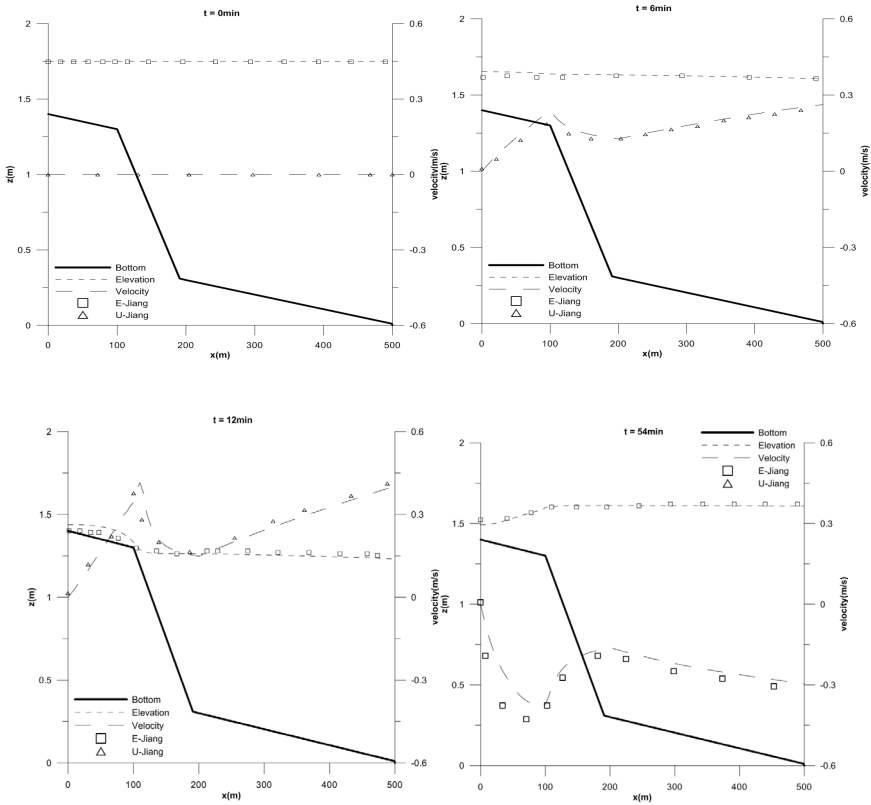


Figure 3: Basin with variable slope: numerical simulation results for velocity and elevation using the RPL wetting and drying method compared to the capillarity results of [4].

#### 4.2 Case 2: application to Vitória Bay estuary

The Vitória Bay estuary on the State of Espírito Santo (Brazil) shelters large ports, besides having one of the largest mangrove swamps located in an urban area in Brazil.



To simulate this estuary the main channels and the entire mangrove areas were included in the model. The Vitória Bay estuary is a complex region with narrow channels and wide areas, with depths varying from 34 m to  $-1$  m in respect to mean sea level, MSL, at the Vitória Port. As shown in Figure 4 the mangrove areas top limit are indicated by  $-1$  m in the legend. The tides of the estuary are classified as microtides with tidal height less than 2 m cf. [15]. As the winds are usually mild and river flows are relatively small, the hydrodynamics of this complex region is mainly governed by tidal effects. Due to the extensive mangrove areas, modeling the Vitória Bay estuary hydrodynamics requires a consistent wetting and drying method. However, the main purpose of this simulation is to qualitatively illustrate the capability to represent wetting and drying capabilities of the RPL method in a real case.

The mesh of the computational domain is presented in Figure 5, it has 4052 elements with  $Q_2$ - $Q_1$  interpolation and 17886 nodes. The mesh is refined to capture the tidal channels within the mangrove swamps. No-slip and impermeability conditions were set along the close boundaries. Open boundary conditions are regulated by tidal variations generated from data measured at Port of Tubarão. As initial conditions, velocities were set to zero and  $\zeta = \text{constant}$ .

For this test,  $\varepsilon = 0.04\text{m}$  in mangrove areas and  $0.03\text{m}$  elsewhere. Wind and Coriolis forces were neglected. The parameters for the RPL method were  $r = 1.0$ , and  $T_{RPL} = 0.25\text{m}$ . For the present test, the time step was 10s, giving a Courant number of up to 10.5.

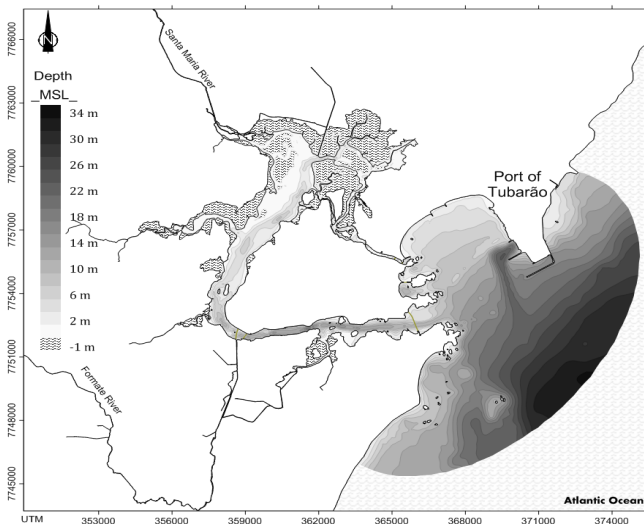


Figure 4: Bathymetry of Vitória Bay estuary.

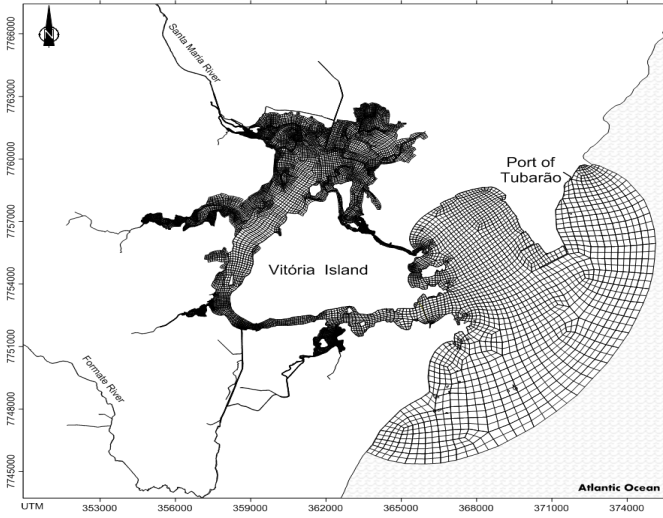


Figure 5: Mesh of discrete domain model made up of 4052 elements for the Vitória Bay estuary simulation.

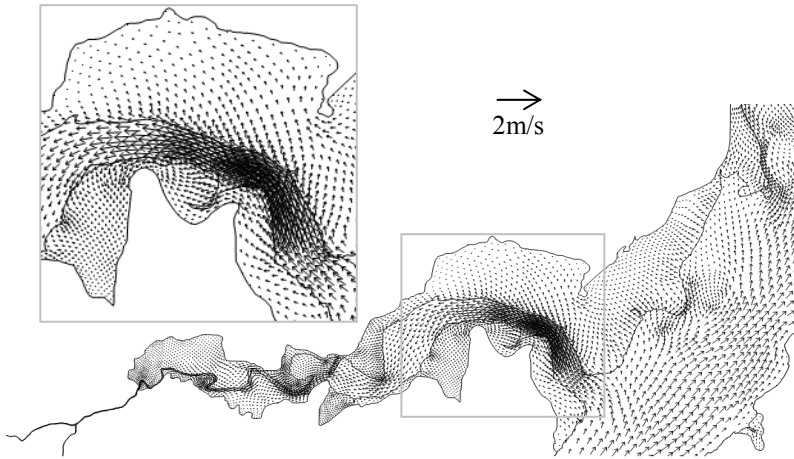


Figure 6: Demonstration of the velocity vectors and the flow pattern at flood tide in one of the channels of Vitória Bay estuary.

Figure 6 shows a computed flow pattern during mean flood tide. One may note that during this period there are relatively few dry mangrove swamps areas because of the relatively higher water level. In the other extreme, one can observe that near the end of the ebb tide period shown in Figure 7, the water level drops into the narrow channels within the mangrove areas, and wide dry areas show up. In these areas one sees the vanishing velocity vectors in the restricted subsurface flow inside the rough-porous layer.



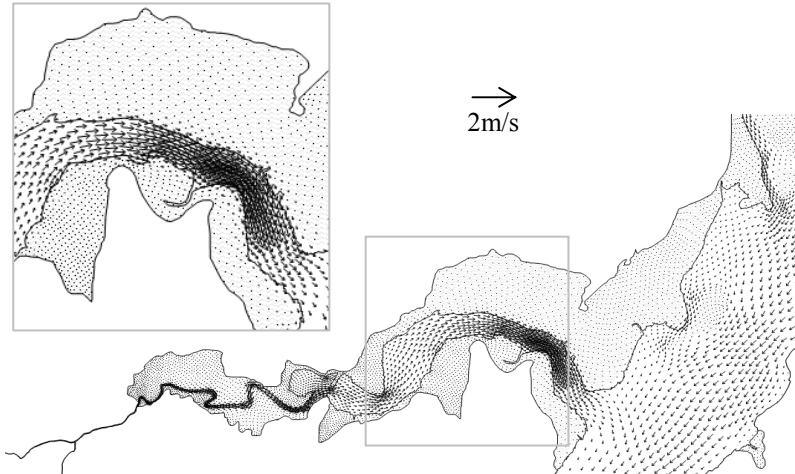


Figure 7: Demonstration of the velocity vectors and the flow pattern at ebb tide in one of channels of the Vitória Bay estuary.

These results on water level and velocities show the ability of the RPL method to well represent wetting and drying processes in real cases with complex bathymetry and intricate geometry.

## 5 Conclusions

The rough-porous layer, RPL, wetting and drying method is very simple to implement, and gives results that are quite similar to other methods validated by comparison in a standard slope varying basin. The method is effective and robust as shown by the good results obtained in the test in a real water body with convoluted geometry, complex bathymetry and wide mangrove swamps.

The method has been applied successfully in modeling very large reservoirs in the Amazon region, with water level variation of about 10m during a typical hydrologic year [18].

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