GREEN ROOF PERFORMANCE IN SUSTAINABLE CITIES

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ABSTRACT

In the last few decades, the use of sustainable urban drainage systems is largely spreading and encouraged, because they provide lots of benefits for sewer networks, wastewater treatment plants and the environment. In this context, green roofs can be an effective tool to both delay and attenuate stormwater runoff peaks, reducing runoff at the same time. Their proper design is a key element for stormwater management in highly urbanized cities. The aim of this paper is to propose an analytical probabilistic approach, to evaluate green roof performance in terms of runoff and vegetation's survival without irrigation, to guide planners in choosing proper values for their design parameters. A great advantage of the method is that it can be applied to different sites and climate conditions; moreover, it involves a significant improvement of the typical analytical probabilistic approach, as a chain of consecutive rainfall events was considered, in order to take into account the possibility that storage capacity is not completely available at the beginning of each event, because of pre-filling from previous rainfalls, as typically happens with green roofs. Finally, to verify the goodness of our developed equations, we applied them to a case study.

Keywords: construction, sustainable urban drainage systems, green roof, green architecture, analytical probabilistic modelling, vegetation survival, runoff, stormwater runoff, sustainability.

1 INTRODUCTION

The rapid growth of the urban world's population and the consequent increase of highly urbanised areas, joined to climate changes underway involves a strong imbalance of the natural water cycle with frequent flooding and surface water contamination. In this context, the implementation of sustainable urban drainage systems (SUDSs) can deliver several benefits: relief of the loads in sewer networks, increased efficiency of wastewater treatments plants, reduction of polluted waters discharged into the environment and the increase of biodiversity in urban areas [1]. Among these strategies, green roofs can be an effective countermeasure, because their implementation also entails significant environmental and economic benefits; besides stormwater management and the improvement of quality of receiving water bodies, such as energy savings, the reduction of heat island effects and the enhancement of biodiversity. Moreover, they do not require additional space with respect to the building footprint and can be an effective tool in densely built urban areas, where rooftops constitute about 30–50% of urban impermeable surfaces [2]. With reference to stormwater management, green roofs allow the local disposal of rainwater runoff, the reduction of runoff volumes through evapotranspiration from the vegetation and exposed surfaces, the delay of runoff that starts only after soil saturation, the reduction and delay of runoff peak rates for the infiltration of rainwater into the soil, their temporary storage in the substrate and drainage layer, and the improvement of stormwater quality for effective percolation into the soil.

The first green roofs appeared in Germany in the 1970s [3]; since then, they have spread all over the world, especially in the major modern and advanced cities, where incentive programs were often developed to encourage or even impose their installation. In the last decades, several studies and models deepened green roof performance under different climate and design conditions [4]–[8], often with reference to a specific place or climate [9]–[12] on

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a small spatial and time scale [13], [14]. At present, there is not yet enough scientific evidence to demonstrate the hydrological benefits of green roofs for full-scale installations [15], [16].

The aim of this paper is to guide planners in the design of green roofs for the achievement of a double goal: to guarantee an appropriate water retention capacity and vegetation survival without watering. The use of an analytical probabilistic approach is proposed. This kind of methodology, first proposed by Guo and Adams [17]–[19] and Adams and Papa [20], allows the estimation of the probability distribution functions (PDFs) of the variables of interest from the PDF of input rainfall variables, once the analytical relation and the variables characterizing the process are defined. One of the main advantages of the analytical probabilistic approaches is that of combined simplicity of "design storm" methods and the probabilistic reliability of continuous simulations. Moreover, the resulting equations are not calibrated on a specific place, but can be applied to different basins and climates all over the world. In recent years, the analytical probabilistic approach has been applied by different authors also to SUDS such as rain water harvesting systems [21]–[24], infiltration trenches [25], [26], permeable pavements [27], bioretention systems [28], green roofs [29]–[31] and stormwater detention facilities [32]–[37]. In this paper, the PDF of runoff from green roofs and of water content into growing medium at the end of dry periods have been estimated, with the dual purpose of driving planners in towards the choice of best green roof layer thickness, to guarantee a good retention capacity and survival of vegetation without irrigation.

An important innovation of our proposed manuscript is that it allows us to consider pre-filling from a chain of previous rainfall events, that gives a possibility that the retention capacity is not completely available at the beginning of the considered rainfall. This aspect is particularly relevant for SUDS like green roofs, characterised by low outflow rates, limited to the degree of evapotranspiration from vegetation and soil [32], [35]. The method allows an optimum green roof design, as the resulting expressions relate the growing medium thickness to the average return interval (ARI). To validate our developed equations, an application to a case study in Milano, Italy is proposed.

2 HYDROLOGICAL MODELLING

Green roofs are engineered multi-layered structures, with a vegetated upper surface, working in shallow systems without connection to natural ground. A typical green roof is composed of the vegetation layer, the growing media layer, a blend of mineral material enriched with organic material where water is retained and in which vegetation is anchored, the filter fabric; the drainage layer, generally constituted of plastic profiled elements that store water for the plants' sustainment during dry periods, evacuating excess water in roof drains; the root resistant membrane and the mechanical protection geotextile. Fig. 1 shows the conceptual reference scheme considered in this paper for green roof modelling.

The volumes in Fig. 1 must be intended as specific for the unit area. Rainfall is first intercepted by the vegetation, then infiltrated into the growing medium layer, where it is retained, used by the roots and released back into the atmosphere through evapotranspiration. The excess is stored in the underlying drainage layer, equipped with an overflow to the urban drainage system that activates when the retention capacity is filled.

Green roof design includes the selection of the thickness of each layer and the choice of plant cover. Rainwater stored in a green roof can vary between zero, when it is completely dry and w_{max} if the water content in the green roof is at its maximum, as in the case of two very close heavy rainfall events. Interception by vegetation generally equals just a few millimetres and the drainage layer capacity generally is between 5–10 cm; the focus here is

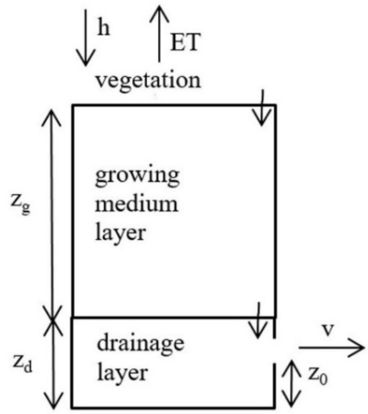

Figure 1: Green roof conceptual scheme considered for our modelling. h: rainfall depth; ET: evapotranspiration; v: runoff; z_g: growing medium thickness; z_d: drainage layer thickness; z_0: overflow height.

on the growing medium layer that can vary between zero and $\phi_f \cdot z_g$, where ϕ_f represents the growing medium moisture content at saturation, when the storage volume is full. In the estimation of the probability of runoff from green roofs, the term w_{max} refers to the sum of the maximum retention capacity in the three layers (vegetation, growing medium and drainage), while in the estimation of the probability of having a minimum water content enough to allow vegetation survival without irrigation, only the maximum retention capacity of the growing medium is considered. Extended rainless periods, especially occurring during the hot season, can result in the soil moisture falling to a "wilting point", with subsequent death of the plant cover. The death of the vegetation nullifies transpiration, but progressive soil desiccation, on the other hand, is initially in some way positive, in terms of increased capacity to buffer runoff.

The evapotranspiration rate, that is, the amount of water released to the atmosphere from the plants' transpiration and soil evaporation, strictly depends on rainfall depth and the water content of the growing medium from previous rainfalls. In the modelling, it is assumed always equal to potential evapotranspiration, that is its maximum value, the worst condition for vegetation.

Water content in the growing medium is estimated at the end of the rainfall event (subscript u) for the estimation of the probability of runoff and at the end of the dry period (subscript e) for the estimation of the probability to have a minimum water content that would allow vegetation survival without irrigation. Considering a variable number $i = 1, \dots, N$ of chained rainfall events, final water depth in the growing medium is calculated by eqn (1):

$$h_{gu,i-1} = \begin{cases} h_{ge,i-1} + h_{i-1} - Et \cdot \theta_{i-1} & 0 \le h_{ge,i-1} + h_{i-1} - Et \cdot \theta_{i-1} < \phi_f \cdot z_g \\ \phi_f \cdot z_g & h_{ge,i-1} + h_{i-1} - Et \cdot \theta_{i-1} \ge \phi_f \cdot z_g \\ 0 & otherwise. \end{cases} \quad (1)$$

For $i = 1$, the growing medium is considered empty at the beginning of the event ($h_{ge,0} = 0$). Water depth in the growing medium at the beginning of a generic rainfall (always considering $i = 1, \dots, N$) results in eqn (2):

$$h_{ge,i} = \begin{cases} h_{gu,i-1} - Et \cdot d_{i-1} & h_{gu,i-1} - Et \cdot d_{i-1} > 0 \\ 0 & otherwise. \end{cases} \tag{2}$$

Runoff at the end of a generic event v_i is expressed by eqn (3):

$$v_i = \begin{cases} h_{gu,i-1} - Et \cdot d_i + h_i - Et \cdot \theta_i - w_{max} & Condition_1 \\ h_i - Et \cdot \theta_i - w_{max} & Condition_2;\ Condition_3 \\ w_{max} - Et \cdot d_i + h_i - Et \cdot \theta_i - w_{max} & Condition_4 \\ 0 & Otherwise, \end{cases} \tag{3}$$

$Condition_1$: $h_{gu,i-1} \le w_{max}$; $h_{gu,i-1} > Et \cdot d_i$; $h_{gu,i-1} - Et \cdot d_i + h_i - Et \cdot \theta_i > w_{max}$,
$Condition_2$: $h_{gu,i-1} \le w_{max}$; $h_{gu,i-1} \le Et \cdot d_i$; $h_i - Et \cdot \theta_i > w_{max}$,
$Condition_3$: $h_{gu,i-1} > w_{max}$; $w_{max} \le Et \cdot d_i$; $h_i - Et \cdot \theta_i > w_{max}$,
$Condition_4$: $h_{gu,i-1} > w_{max}$; $w_{max} > Et \cdot d_i$; $w_{max} - Et \cdot d_i + h_i - Et \cdot \theta_i > w_{max}$.

For $i = 0$, that is v_0, runoff results, as seen in eqn (4):

$$v_0 = \begin{cases} h_0 - Et \cdot \theta_0 - w_{max} & h_i - Et \cdot \theta_i > w_{max} \\ 0 & Otherwise. \end{cases} \tag{4}$$

With reference to eqn (4):

- Condition 1 expresses the case that there is no runoff at the end of event $i - 1$, there is pre-filling from event $i - 1$ at the beginning of event i and there is runoff from the green roof at the end of event i;
- Condition 2 expresses the case that there is no runoff at the end of event $i - 1$, there is no pre-filling from event $i - 1$ at the beginning of event i and there is runoff from the green roof at the end of event i;
- Condition 3 expresses the case that there is runoff at the end of event $i - 1$, there is no pre-filling from event $i - 1$ at the beginning of event i and there is runoff from the green roof at the end of event i;
- Condition 4 expresses the case that there is runoff at the end of event $i - 1$, there is pre-filling from event $i - 1$ at the beginning of event i and there is runoff from the green roof at the end of event i.

3 PROBABILISTIC MODEL

The aim of the proposed probabilistic model is to give an estimate of the growing medium thickness to be considered in green roof designs, in order to have, with an assumed probability, a minimum water content enough to allow vegetation survival without irrigation and limited runoff. Hydrological variables considered in our modelling are: rainfall depth h, rainfall duration θ and inter-event time d; these are assumed to be independent and exponentially distributed rainfall variables. To isolate independent rainfalls from a continuous record of events, a minimum inter-event time (IETD) was defined [38]. If the inter-event time between two consecutive rainfalls was smaller than IETD, they were joined into a single event, otherwise they were considered as distinct and independent.

The assumption of the exponential PDF for rainfall variables is usually considered acceptable to satisfy the need of an easier mathematical tractability [39]–[41]. To overcome the bias due to the use of the exponential PDF, some authors suggest using the Weibull PDF [42] or the double-exponential PDF [34]. Although a better fitting of the observed frequencies of rainfall records can be achieved with these alternative PDFs, the improvement in terms of model accuracy seems negligible, compared with the significant increase of the mathematical complexity in the development of equations.

Exponential PDFs of rainfall depth, rainfall duration and inter-event time are expressed as, respectively, eqns (5)–(7):

$$f_h = \xi \cdot e^{-\xi \cdot h}, \tag{5}$$

$$f_\theta = \lambda \cdot e^{-\lambda \cdot \theta}, \tag{6}$$

$$f_d = \psi \cdot e^{-\psi \cdot (d - IETD)}, \tag{7}$$

where $\xi = 1/\mu_h$; $\lambda = 1/\mu_\theta$; $\psi = 1/(\mu_d - IETD)$ and μ_h, μ_θ and μ_d are respectively the mean values of rainfall depth, rainfall duration and interevent time.

The probability of having a runoff exceeding a given threshold $\bar{v}$ and the probability that water content in the growing medium exceeds a minimum threshold w is estimated, setting $h = h_i = h_{i+1}$, $\theta = \theta_i = \theta_{i+1}$, $d = d_i = d_{i+1}$ in eqns (1)–(4): this involves the deletion of Condition 2. Runoff PDF is estimated to distinguish two different conditions: maximum emptying time, that is the time needed to empty the retention capacity when it is full, lower (Case 1) and higher (Case 2) than $IETD$; for Case 1 pre-filling from previous rainfalls is excluded and the full storage capacity is available, while for Case 2 the possibility that the retention volume is partially filled from previous rainfalls was considered.

Case 1: $w_{max}/Et \leq IETD$:

$$P_{v1} = P(v > \bar{v}) = \int_{h=w_{max}+\bar{v}+Et\cdot\theta}^{\infty} f_h \cdot dh \int_{\theta=0}^{\infty} f_\theta \cdot d\theta = \gamma \cdot e^{-\xi\cdot(w_{max}+\bar{v})}, \tag{8}$$

where $\gamma = \frac{\lambda}{\lambda + Et\cdot\xi}$.

Case 2: $w_{max}/Et > IETD$:

$$\begin{aligned} P_{v2} = P(v > \bar{v}) = & \int_{\theta=0}^{\infty} f_\theta \cdot d\theta \int_{d=IETD}^{\infty} f_d \cdot dd \int_{h=w_{max}+\bar{v}+Et\cdot\theta}^{\infty} f_h \cdot dh \\ & + \sum_{i=2}^{N} \left[\int_{\theta=0}^{\infty} f_\theta \cdot d\theta \int_{d=IETD}^{\frac{w_{max}+\bar{v}}{Et}} f_d \cdot dd \int_{h=\frac{w_{max}+\bar{v}+(i-1)\cdot Et\cdot d}{i}+Et\cdot\theta}^{\frac{w_{max}+\bar{v}+(i-2)\cdot Et\cdot d}{i-1}+Et\cdot\theta} f_h \cdot dh \right] \\ = \gamma \cdot \Big\{ & e^{-\xi\cdot(w_{max}+\bar{v})} + \psi \cdot \textstyle\sum_{i=2}^{N} \Big[-(i-1)\cdot\beta_i \cdot e^{-\xi\cdot Et\cdot IETD\cdot\left(\frac{i-2}{i-1}\right) - \frac{\xi}{i-1}\cdot(\bar{v}+w_{max})} - i\cdot\beta_i^* \cdot \\ & e^{-\frac{\xi}{i}\cdot[Et\cdot IETD\cdot(i-1)+(\bar{v}+w_{max})]} - \xi\cdot Et\cdot\beta_i\cdot\beta_i^*\cdot e^{\psi\cdot IETD-(\bar{v}+w_{max})\cdot\left(\frac{\psi}{Et}+\xi\right)} \Big] \Big\}, \end{aligned} \tag{9}$$

with: $\beta_i = \frac{1}{\xi\cdot Et\cdot(i-2)+\psi\cdot(i-1)}$; $\beta_i^* = -\frac{1}{i\cdot\psi+(i-1)\cdot\xi\cdot Et}$.

To estimate the probability for green roof vegetation to survive without irrigation, a minimum water content w is considered. The condition for which water content can be different from zero at the end of a dry period, that is when pre-filling from previous events is considered, results to be: $(z_g \cdot \phi_f - w)/Et > IETD_g \cdot \phi_f$.

Two different cases are analyzed: a single rainfall ($i = 1$) and a series of chained (N) rainfall events ($i > 1$).

For $i = 1$, that is when a single rainfall event is considered, the exceedance probability to have, at the end of the dry period between two consecutive rainfalls, a minimum water volume in the growing medium, to ensure vegetation survival, results in eqn (10):

$$P_{w1} = P\left(h_{ge} > w\right) = \int_{h=w+Et\cdot(d+\theta)}^{\infty} f_h \cdot dh \int_{d=IETD}^{\frac{\emptyset_f \cdot z_g - w}{Et}} f_d \cdot dd \int_{\theta=0}^{\infty} f_\theta \cdot d\theta$$

$$= \gamma \cdot \beta \cdot \left[e^{-\xi\cdot(Et\cdot IETD+w)} - e^{\psi\cdot\left(IETD+\frac{w}{Et}\right)-\phi_f\cdot z_g\cdot\left(\xi+\frac{\psi}{Et}\right)}\right]. \quad (10)$$

For $i > 1$, that is if a series of chained rainfalls is considered, the probability was as given by eqn (11):

$$P_{wN} = P\left(h_{ge} > w\right)$$

$$= \int_{\theta=0}^{\infty} f_\theta \cdot d\theta$$

$$\cdot \left\{ \int_{d=IETD}^{\frac{\emptyset_f \cdot z_g - w}{Et}} f_d \cdot dd \cdot \left[\int_{h=\emptyset_f\cdot z_g+Et\cdot\theta}^{\infty} f_h \cdot dh \cdot + \int_{h=\frac{w}{i-1}+Et\cdot(\theta+d)}^{\frac{\emptyset_f\cdot z_g+Et\cdot d\cdot(i-2)}{i-1}+Et\cdot\theta} f_h \cdot dh \right] \right.$$

$$\left. + \int_{\frac{\phi_f\cdot z_g+Et\cdot d\cdot(i-1)}{i}+Et\cdot\theta}^{\frac{\phi_f\cdot z_g+Et\cdot d\cdot(i-2)}{i-1}+Et\cdot\theta} f_h \cdot dh \int_{d=IETD}^{d=\frac{\emptyset_f\cdot z_g\cdot(i-1)-i\cdot w}{Et\cdot(i-1)}} f_d \cdot dd \right\} =$$

$$= \gamma \cdot \left\{ e^{-\xi\cdot\emptyset_f\cdot z_g} \cdot \left[1 - e^{\psi\cdot\left(IETD+\frac{w}{Et}-\frac{\phi_f\cdot z_g}{Et}\right)}\right] + \frac{2\cdot(1-\beta)\cdot\beta_i}{i-1} \cdot e^{-\left(\frac{\psi}{Et}+\xi\right)\cdot(\phi_f\cdot z_g-w)+\psi\cdot IETD-\frac{\xi\cdot w}{i-1}} - \right.$$
$$\beta_i \left[2 \cdot e^{-\frac{\xi}{i-1}\cdot[\phi_f\cdot z_g+Et\cdot IETD\cdot(i-2)]} + e^{\frac{\xi\cdot w\cdot i\cdot(i-2)}{(i-1)^2}-\phi_f\cdot z_g\cdot\left(\frac{\psi}{Et}+\xi\right)+\psi\cdot IETD+\frac{\psi\cdot i\cdot w}{Et\cdot(i-1)}}\right] + \beta \cdot$$
$$\left. e^{-\frac{\xi\cdot w}{i-1}-\xi\cdot Et\cdot IETD} + \beta_i^* \cdot \left[e^{-\phi_f\cdot z_g\cdot\left(\frac{\psi}{Et}+\xi\right)+\psi\cdot IETD+\frac{\psi\cdot i\cdot w}{Et\cdot(i-1)}} - e^{-\frac{\xi}{i}\cdot[\phi_f\cdot z_g+Et\cdot IETD\cdot(i-1)]}\right]\right\}. \quad (11)$$

The quantities γ, β, β_i and β_i* are equal to:

$$\gamma = \frac{\lambda}{\lambda+\xi\cdot Et},$$

$$\beta = \frac{\psi}{\psi+\xi\cdot Et},$$

$$\beta_i = \frac{(i-1)\cdot\psi}{(i-1)\cdot\psi+\xi\cdot Et\cdot(i-2)},$$

$$\beta_i^* = \frac{\psi\cdot i}{\xi\cdot Et-i\cdot(\xi\cdot Et+\psi)}.$$

Eqns (10) and (11) can be used to estimate the growing medium thickness z_g required for a green roof design, once variables are defined characterising rainfall, vegetation and exceedance probability.

4 APPLICATION TO A CASE STUDY

Eqns (8)–(11) were applied to a case study in Milan, Italy, using statistics from the series of rainfall events recorded at the Milano-Monviso gauge station in the period 1971–2005. To assess the PDFs of runoff and water content for growing medium, an IETD = 10 hours was assumed. Table 1 reports the mean and coefficient of variation of rainfall depth h, rainfall duration θ and inter-event time d.

Table 1: Mean and coefficient of variation of rainfall variables.

	IETD = 10 hours	
	μ (mm)	V (–)
h (mm)	18.49	1.15
θ (hours)	14.37	1.03
d (hours)	172.81	1.30

As already discussed [35], the assumption of exponential distribution for rainfall variables is not the best fitting choice for these data. Table 2 reports the correlation indexes among rainfall variables.

Table 2: Correlation indexes among rainfall variables.

	IETD = 10 hours
$\rho_{h,d}$ (–)	0.11
$\rho_{\theta,h}$ (–)	0.62
$\rho_{d,\theta}$ (–)	0.11

Inter-event time results to be only weakly correlated to other two variables, while correlation between rainfall depth and duration was significant. The effects of this correlation on final results have been already discussed by the authors [33].

Runoff probability P_v is estimated by varying maximum retention capacity w_{max}. It results from the sum of the maximum retention capacity of vegetation, growing medium and drainage layer. Maximum retention capacity of vegetation is of few millimetres; maximum retention capacity in growing medium can generally vary between 0 and 1,000 mm, considering both extensive green roofs (with a thickness of the layer equal to few centimetres) and intensive green roofs (with a thickness of the layer of some hundreds of centimetres; the maximum retention capacity of the drainage layer usually varies from 0–150 mm, so that the maximum retention capacity of the whole roof can vary between 0 and 1,250 mm.

In the calculation, an extensive green roof has been considered and the maximum retention capacity was varied between 0 and 200 mm. Water content at saturation is assumed equal to ϕ_f = 0.58 (–) [43]. The growing medium thickness z_g is assumed variable between 20 mm and 500 mm. Different studies in the literature collected results of field measurements, trying to define a range of reasonable values of green roof evapotranspiration rates: experimental estimations range from 0.69 to 6–9 mm/day, with typical values of 1–6 mm/day [44], [45]. In this paper, a value of evapotranspiration rate equal to Et=0.125 mm/hour was used. In the calculation of both probability P_v and P_w threshold values $\bar{v}$ and w were set equal to zero. Exceedance probabilities calculated by eqns (8)–(11) are compared with cumulative frequencies obtained by the continuous simulation of recorded rainfalls; the number of considered chain events to achieve a good fit between P and F is equal to i = 5.

Fig. 2 shows the water content PDF for vegetation survival without irrigation; the probability increases with the thickness of the growing medium.

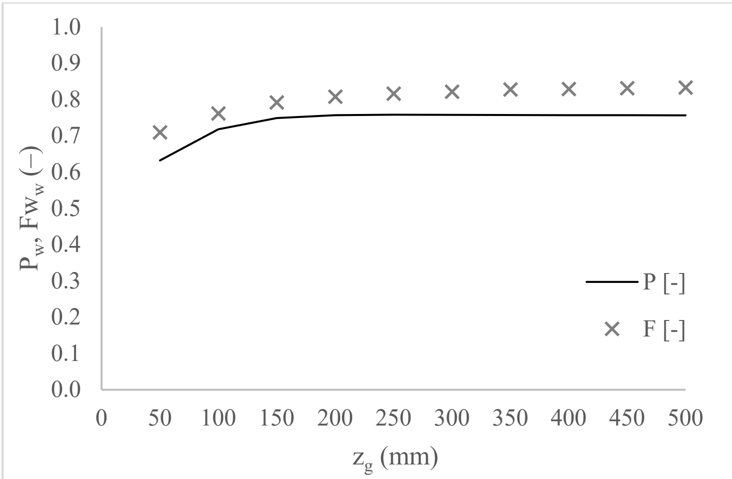

Figure 2: Probability (P) and frequency (F) distribution functions of water content into growing medium layer varying its thickness (z_g).

Table 3: Analysis results on the whole period of records.

z_g (mm)	50	100	150	200	250	300	350	400	450	500
P (mm)	0.63	0.72	0.75	0.76	0.76	0.76	0.76	0.76	0.76	0.76
T (years)	3	4	4	4	4	4	4	4	4	4

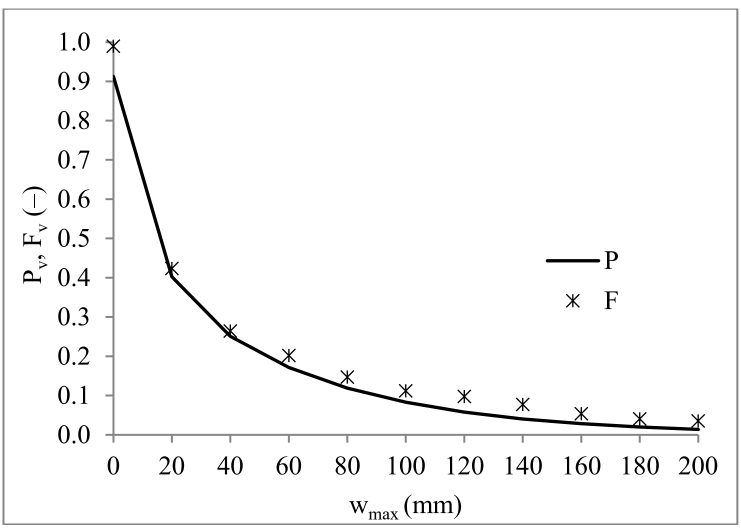

Figure 3: Probability (P) and frequency (F) distribution function of runoff from growing medium varies the maximum water content (w_{max}).

Expressing the probability of vegetation survival in terms of average return intervals, ARI = 1/(1-P), it varies from 3 years with z_g = 50 mm, to 4 years with z_g = 200 mm (Table 3); also increasing till five times the growing medium thickness, from 100 mm to 500 mm ARI did not improve. To achieve higher ARIs, an additional water supply to the green roof, by way of an irrigation system, is needed.

Fig. 3 shows the runoff PDF varying green roof maximum retention capacity.

Assuming that the maximum retention capacity by vegetation is of a few millimetres, the maximum retention capacity of the drainage layer is equal to 50 mm, and that with the growing medium thickness z_g = 100 mm, the maximum retention capacity of the layer was 58 mm, the maximum retention capacity resulted equal to about 110 mm; for this value the probability of runoff is very low, and it can be considered optimum to guarantee a good performance of the green roof, both in terms of stormwater control and fulfilment of water demand by the non-irrigated vegetation.

5 CONCLUSIONS

The proposed method allows one to estimate the probability of runoff from green roofs and the probability of survival of the vegetation cover without irrigation. The equations developed enable designers to link these probabilities with the thickness of growing medium, considering both the vegetation type and the climate features of the site.

An important improvement of the proposed method is that it allows us to consider the effects of chained rainfall events in the evaluation of the probabilities, without the need for continuous simulation of the hydrological processes. This makes the results more realistic and reliable, and the application easier and cheaper, in terms of time and data needs. The developed equations can be a valid aid for green roof design, as they allow us to define the thickness of the growing medium for different levels of risk.

The example of application to a case study in Milan, Italy showed a good fit of the results obtained by the proposed formulas and continuous simulation of observed data. An interesting result was that for both the probability of runoff and the probability of having minimum water content in the growing medium for vegetation to survive without irrigation, increasing growing medium thickness beyond a certain threshold does not provide significant advantages, in terms of green roof performance, but only economical disadvantages. Upon application to our case study, the optimal value was around 100 mm. Although the probabilistic model is of general value and applicability, numerical results were related and limited to the climatic features of the case study. Future developments of the proposed model will then also consider the application to other case studies, in different climatic contexts.

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