Monte Carlo analysis and its application within the valuation of technologies

S. Č. Aguilar, M. Dubová, J. Chudoba & A. Šarman
*Institute of Novel Technologies and Applied Informatics,
Technical University of Liberec, Czech Republic*

Abstract

This work follows the paper entitled The Valuation and Financial Management of (Nano-) Technology in Relation to Sustainable Growth presented at the Third International Conference on Environmental Economics and Investment Assessment (Limassol, Cyprus, 2010), which demonstrated the practical usage of the general economic model on the valuation of a modern and original technology (nano-fibrous carrier) for wastewater treatment applying tailor-made microorganisms with the ability to create natural biofilm. The original general economic model for the valuation of wastewater treatment technologies is structured as follows: cost model wastewater treatment technology, depreciation model of wastewater treatment, cash flow model of wastewater treatment, sensitivity analysis. The authors extended this work on further calculations with the use of the Monte Carlo method, in order to analyze the characteristics of a project's net present value (NPV), the cash flow components that are impacted by uncertainty. These characteristics are modelled, incorporating any correlation, mathematically reflecting their "random characteristics". Then, these results are combined in a histogram of NPV (i.e. the project's probability distribution), and the average NPV of the potential investment into the wastewater treatment technologies – as well as its volatility and other sensitivities – is observed. This distribution allows for an estimate of the probability that the project has a net present value greater than zero (or any other value).
Keywords: Monte Carlo method, risk, net present value, valuation and financial management, general economic model, R&D projects, technology (nano-fibrous carrier) for wastewater treatment, sustainable growth.

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1 Introduction

The actual development of nanotechnology influences a great part of the industrial branches. The application of nanotechnologies represents for certain companies an important step forward. The Institute of Novel Technologies and Applied Informatics, Technical University of Liberec, Czech Republic is in charge of research and application of nanotechnologies. One of the main tasks of the centre is the research and development of nanotechnologies applied to the industrial wastewater treatment branches, in a more concrete way it is concerned about the development of microfibrous biomass carrier in biological wastewater treatment facilities. The research is in charge of a multidisciplinary scientific team which includes disciplines as chemistry, natural sciences, development of textile materials, mathematic modelling and informatics. Last but not least is the integration of ideas coming from the branch of financial management and valuation [1, 2]. This might contribute to answering the question if the technology can be commercially attractive.

The aim if this article is to make an analysis of advantages and disadvantages of the Monte Carlo valuation method and its application to the technology of nanofibrous biomass carrier for purposes of biological wastewater treatment.

This work follows the article entitled The Valuation and Financial Management of (Nano-) Technology in Relation to Sustainable Growth presented in the Third International Conference on Environmental Economics and Investment Assessment (Limassol, Cyprus, 2010) [3], which demonstrated the practical usage of the general economic model on the valuation of a modern and original technology (nano-fibrous carrier) for wastewater treatment applying tailor-made microorganisms with ability to create natural biofilm. The original general economic model for the valuation of wastewater treatment technologies is structured as follows:

- Cost model wastewater treatment technology
- Depreciation model of wastewater treatment
- Cash flow model of wastewater treatment
- Sensitivity analysis [3].

The authors extended this work on further calculations with the use of the Monte Carlo method, in order to analyze the characteristics of a project's net present value (NPV), the cash flow components that are impacted by uncertainty. These characteristics are modelled, incorporating any correlation, mathematically reflecting their "random characteristics". Then, these results are combined in a histogram of NPV (i.e. the project's probability distribution), and the average NPV of the potential investment into the wastewater treatment technologies – as well as its volatility and other sensitivities – is observed. This distribution allows for an estimate of the probability that the project has a net present value greater than zero (or any other value).

2 Risk and technology appraisal – Monte Carlo method

One of the fundamental characteristics of the valuation of investments in research and development of new technologies is its focus on the expected cash flow. The cash flow future values are difficult to predict, therefore it is necessary to include risk management processes in the research. Our research team decided to enhance the actual economic model with software that is able to quantify the risks related to the investment of the developed technology.

One of the fundamental indicators for the valuation of technologies is the net present value (NPV), which helps us to determine if it is worth to invest on certain technology [4–6].

If we want to know the probability at which a project achieves determinate NPV, or at which range will be NPV located, it is necessary to apply other methods that are able to change input parameters in a stochastic way.

In such cases we can apply Monte Carlo methods. These methods are helpful in order to observe the influence in changes in the input variables (NPV). Monte Carlo methods are based on repeated random sampling that translates inputs into uncertainties in model outputs (results). The results of these processes are a set of detailed results that are consequently analyzed.

The outputs of these simulations can determine for instance:

- the probability that the net present value is lower than the value originally defined,
- distribution function of the model outputs,
- mean values dispersion and dispersion of output indicators.

These mentioned parameters are suitable for the consequent establishment of risks related to the investment of the developed technology. With the aid of the presented results it is possible to infer if it is convenient to pursue the investment. The advantages of this method are the following:

- each sampling has the same level of probability,
- it is possible to change all the inputs within a test,
- it is possible to establish the effect of several variable input parameters,
- it is possible to determine the probability of convenience of the investment. This value can consequently serves as input for the following analyses.

The disadvantages of this method are mainly related to the difficult interpretation of the results and the time demands for the creation of the sets of results with the aid of Monte Carlo.

The basic result from the random outputs is a distribution function (histogram) of the net present value of the investment. From the distribution function it is possible to know other parameters as for example the mean value of the output indicator with the aid of the following model:

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx, \text{ where } f(x) = \frac{dF(x)}{dx} \tag{1}$$

In a similar way it is possible to determine the dispersion value. The Monte Carlo method is based on repeated random trials.

Under this method the estimation of the required values have probabilistic character and are inferred statistically. Practically random trials are substituted by results of certain calculation that is pursued with the application of random numbers.

The level of the method's error related to the calculations is proportional to the value $\sqrt{1/N}$, where N is the quantity of trials. The calculation's error will be therefore, 50% lower with a four times greater quantity of attempts. This error is due to the effect of the central limit theorem. For the estimation of the quantity of simulations it is necessary to know the probability effect that has to be intercepted. It can even occur at the lowest probability.

This probability is identified as p_{min}. The mean value of the estimation that the effect will occur at the lowest probability is:

$$\lambda = p_{\min} \cdot n \tag{2}$$

where n quantity of trials

λ mean value of the effects quantity.

It is recommended that $\lambda > 3$. The problem appears with the assessment of the probability effect at the lowest probability p_{min} (p_{min} can show an assumed probability of investment loss).

The first operation for the establishment of unknown input parameters is the generation of random numbers < 0,1 >, with the usage of standard procedures of software applications.

Afterwards transformation relations help to generate numbers from the intervals to random numbers of the distribution. The most common transformations that can be used for the technology valuation are [7–9]:

- Data from the histogram. The input parameter is given the probability it occurs with and the sum of all the probabilities equals 1. From the basis of these probabilities is created a distribution function.

Data from the distribution within the interval $< a, b >$

$$x = rand \cdot (b - a) + a \tag{3}$$

- Data from the normal distribution (it uses Box Muller transformations).

$$x = \sqrt{-2 \cdot \ln(rand_1)} \cdot \cos(2 \cdot pi \cdot rand_2) \tag{4}$$

- Data from the normal distribution $N(\mu, \sigma^2)$ (μ – mean value, σ – variance.)

$$x = \left(\sqrt{-2 \cdot \ln(rand_1)} \cdot \cos(2 \cdot pi \cdot rand_2) \cdot \sigma\right) + \mu \tag{5}$$

3 Input generation for the software for the valuation with Monte Carlo method

For the application of the Monte Carlo Method it is necessary to create a model, from which it is possible to determine the required parameters for the calculation of NPV. In this paper we present two technologies for industrial waste water treatment as example of the application of this method. The two technologies refer to the wastewater treatment with Anoxkaldnes wheels [10] and with nanofibrous carriers. For the calculation of NPV with Monte Carlo Method it was necessary to define the following parameters:

- market price of DPG material,
- annual increments of the prices of DPG material,
- discount rate k,
- inflation rate,
- tax rate for corporations,
- volume of the investment for each year,
- year of the required investment return,
- acquisition costs,
- annual operation costs.

For the market price of the DPG material it is adequate to apply constant or normal distribution, which has two mean value parameters μ and variance σ. The mean value is presented in our work by the assumed price of the material DPG in CZK/t. The variance is established through the aid of price changes in a certain time period, for instance through the model:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n-1}}, \tag{6}$$

where x_i is the actual price for the last period.

For the annual increment of prices of DPG it is adequate to apply histograms. For example in the case of the nanofibrous carrier DPG was defined an annual value of increment at 2%. With the aid of histograms it becomes feasible to define the following table:

- annual increment value 1% probability 10%,
- annual increment value 1,5% probability 20%,
- annual increment value 2% probability 20%,
- annual increment value 2,5% probability 20%,
- annual increment value 3% probability 20%,
- annual increment value 3,5% probability 10%.

These values were taken based on experimental estimations. It was also possible to apply normal distribution. The parameter μ was 2% and the variance σ might show its value by estimation according to the model above. The discount and tax rates are constant.

The inflation rate can be estimated through histograms or normal distribution. The investment volume for each year is not possible to be implemented into the same model. For the simulation of these inputs it is necessary to describe different variations of the model. The results are then compared and the best possibility is established. A similar process is pursued for the "year of required return". The acquisition operation costs can be constant (the values are determined based on analyses) or they can be established through histograms.

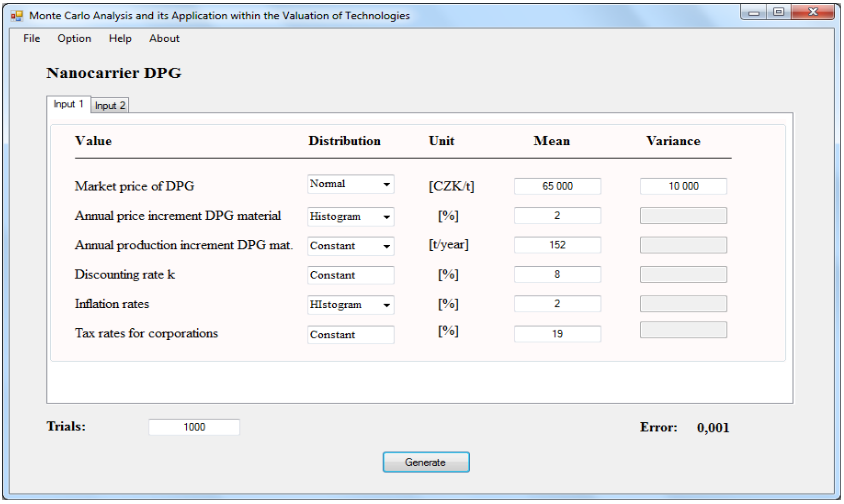

Figure 1: Input Values I. Detail.

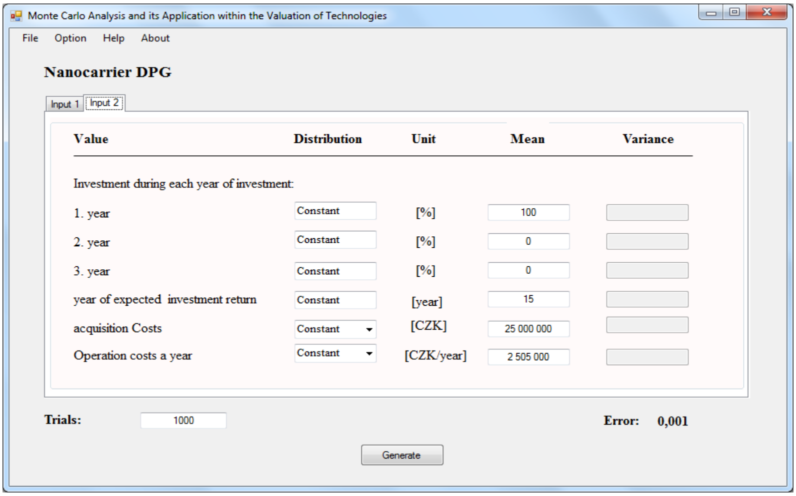

Figure 2: Input Values II. Detail.

In figures 1 and 2 detailed parts of the software are shown for the calculation of NPV with the Monte Carlo method. For certain input parameters it is possible to choose different types of distribution.

4 Simulation process of the Monte Carlo method for technology valuation

The Monte Carlo Method is based on a repeated random trial with different input parameters, therefore the input parameters have to be stochastic.

The market price for the DPG material for the Anoxkaldnes technology and for the technology based on nanofibrous carrier, will have the following parameters, which were obtained through experimental estimations: μ = 65000 CZK/t a $\sigma = 10000$ CZK/t. Similarly to "the annual increment of prices for the DPG material" it is possible to describe it through the histogram that is shown above. On the following tables there are presented inputs, for which it was pursued the calculation of NPV through Monte Carlo methods. The results of the analysis are presented through a distribution function for the correspondent technology.

Table 1: Input data for the analysis (Nanofibrous Carrier).

Input Value	Unit	Value	Distribution
Market Price DPG material	[CZK/t]	**65 000,00**	Normal
Market prices DPG. annual increment	[%]	**2**	Histogram
Annual production increment DPG material	[year/t]	**158**	Constant
Discounting rate k	[%]	**8**	Histogram
Inflation rate	[%]	**2**	Histogram
Tax rates for corporations	[%]	**19**	Constant
Investment during each year of investment:	x	**X**	X
1.year	[%]	**100**	Constant
2. year	[%]	**0**	Constant
3. year	[%]	**0**	Constant
Year of expected investment return	[year]	**15**	Constant
Acquisition Costs	[CZK]	**25 000 000,00**	Constant
Operation costs a year	[CZK/year]	**2 505 000,00**	Constant

Table 2: Input Values for the analysis (Anoxkaldnes).

Input Value	Unit	Value	Distribution
Market Price DPG material	[CZK/t]	**65 000,00**	Normal
Market prices DPG. annual increment	[%]	**2**	Histogram
Annual production increment DPG material	[year/t]	**968**	Constant
Discounting rate k	[%]	**8**	Histogram
Inflation rate	[%]	**2**	Histogram
Tax rates for corporations	[%]	**19**	Constant
Investment during each year of investment:	x	**X**	X
1.year	[%]	**100**	Constant
2. year	[%]	**0**	Constant
3. year	[%]	**0**	Constant
Year of expected investment return	[year]	**15**	Constant
Acquisition Costs	[CZK]	**46 024 000,00**	Constant
Operation costs a year	[CZK/year]	**526 600,00**	Constant

Course of the first sampling:

First, there are generated two random numbers, which are necessary for the description of the parameter "Market price of the DPG material". Through the transformation $x = \left(\sqrt{-2 \cdot \ln(rand_1)} \cdot \cos(2 \cdot \mathrm{pi} \cdot rand_2) \cdot \sigma\right) + \mu$ the actual value of the parameter is determined, that is introduced to the program within one sampling.

The first will be used for the indicator "Annual increment of the price for DPG material" and the second for the "Inflation rate". For the generated number is the parameter's value determined by a distribution. For example generated number is 0,7654, and then the "annual increment of the DPG material" is 3%.

For all these generated input indicators there are established all the outputs from the software and the result is registered in a vector.

This procedure is repeated for all the samplings. The resulting vectors are presented in ascending order and for each element it is given the correspondent probability according to the model

$$p_i = \frac{i - 0,5}{n}, \quad (7)$$

where $i \in \langle 1, n \rangle$. A detail of a resulting vector is shown in Table 3.

Table 3: Output vector, nanofibrous carrier detail.

Probability	*NPV*
0,0005	*-12344000*
0,0015	*-7336000*
0,0025	*-705000*
0,0035	*713000*
0,0045	*2516000*
0,0055	*4079000*
0,0065	*4639000*
0,0075	*4864000*
0,0085	*5228000*
0,0095	*5319000*
0,0105	*5927000*
0,0115	*6089000*
0,0125	*6104608*
0,0135	*6126000*
0,0145	*6161000*

In Figures 3 and 4 there is a detail of the distribution function NPV for all the given values of the selected parameters. From the resulting vector presented in Table 3 and the distribution functions can be inferred the following outputs:

- The probability at which NPV might be lower than a certain value – NPV might be lower than 15.10^6 CZK with the probability of 5,8% (nanofibrous carrier); 260.10^6 CZK (Anoxkaldnes technology)
- Distribution function fractile – with 20% of probability will NPV be lower than 23 275 000 CZK (nanofibrous carrier), 321 900 000 CZK (Anoxkaldnes technology)
- Probability of a negative NPV – for nanofibrous carrier is lower than 0,004%; for Anoxkaldnes technology 0,001%. The number was observed at 2000 samplings.
- The mean value, median (50% fractile), quartiles (25% and 75% fractile), interquartile interval.

The results from different production strategies (volume of the investment for each year, year of expected return) can be compared also with the aid of box diagrams.

From the estimations we can infer that one can expect positive values of NPV for both technologies. For the nanofibrous carrier technology is the probability of a negative NPV lower than 0,004%; for Anoxkladnes is lower than 0,001%. These results seem to be positive for potential investors in research and development for both technologies. From the results we can also observe that the mean value NPV for the expected year of return (15 years) is higher with the Anoxkaldnes technology.

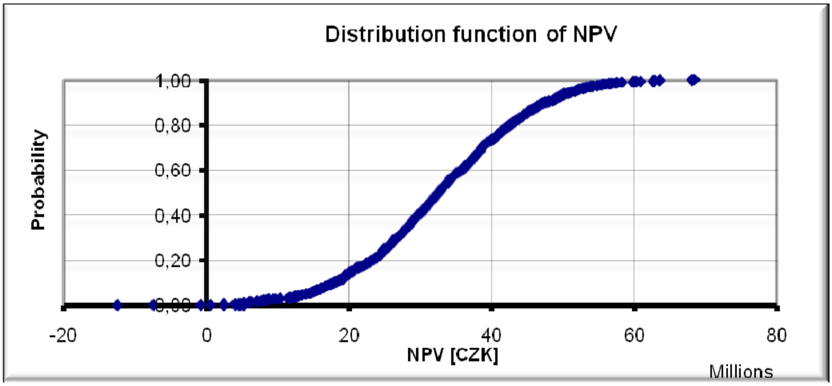

Figure 3: Distribution function for nanofibrous carrier.

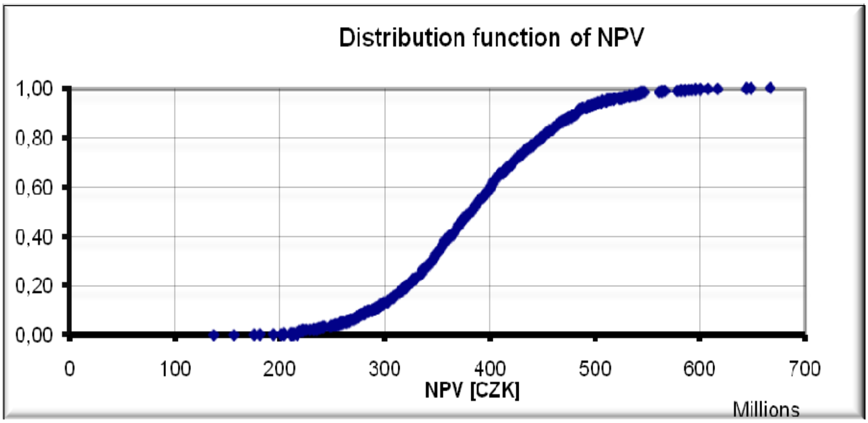

Figure 4: Distribution function for Anoxkaldnes carrier.

5 Conclusion

This paper presented an extension of the actual economic model with software based on the Monte Carlo Method. The benefit of this application for its users is the quantification of risks designated to the probability of which project might achieve certain net present value (NPV), in order to ease the decision making process of the investment and consequent commercialization of determinate developed technology. Other advantages of the Monte Carlo method are mainly:

- each sampling has the same level of probability,
- it is possible to change all the inputs within the correspondent test,
- it is possible to establish the effect of several variable input parameters,

- it is possible to determine the probability of convenience of the investment. This value can consequently serve as input for the following analyses.

The disadvantages of this method are mainly related to the difficult interpretation of the results and the time demands for the creation of the sets of results with the aid of Monte Carlo.

The research team plans to test this modified economic model in other developed technologies developed by the research team and to modify the economic model with the application of other sophisticated methods.

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References

[1] Křiklavová, L. Technologický návrh biofilmového reaktoru s nanovlákenným nosičem pro čištění průmyslových odpadních vod [diploma project]. Liberec: Technická univerzita v Liberci – Fakulta mechatroniky, informatiky a mezioborových studií, 2009.

[2] Nanotechnologie v ČR – praktické aplikace 2006. [online], Pavel Houser. [cit. 2010-02-01]. Available at www: <http://businessworld.cz/ostatni/ nanotechnologie-v-cr-prakticke-aplikace-3595

[3] Aguilar, C. S., Dubová, M., Mucsková, E. The Valuation and Financial Management of (Nano-) Technology in Relation to Sustainable Growth. *Waste Management and Environment V*. WIT Press. Southampton 2010. 13 p. ISBN978-1-84564-460-4.

[4] Arnold, G. *Corporate Financial Management,* 3rd. ed. UK: Pearson Education Limited, 2005. 1200 s. ISBN 0-273-68726-3.

[5] Boer, F.P. *Oceňování technologií*, 1. ed. Brno: Zooner Press, s.r.o., 2007. ISBN 978-80-86815-66-4.

[6] Brealey, R., A., Myers, S., C. *Principles of Corporate Finance,* New York: McGraw-Hill Companies, Inc., 1996. ISBN 0-07-114053-0.

[7] Virius, M. Aplikace matematické statistiky: Metoda Monte Carlo. 3rd ed. Praha: ČVUT, 1998. 168 s. ISBN 80-01-01779-6.

[8] Fabian, F, Kluibert Z. Metoda Monte Carlo a možnosti jejího uplatnění. 1st ed. Praha: Prospektrum, 1998. 148s. ISBN 80-7175-058-1.

[9] Jäckel, P. Monte Carlo methods in finance. 1st ed. Chichester: John Wiley and Sons, 2002. 219s. ISBN 0-471-49741-X.

[10] Homepage of the technology AnoxKaldnes, http://www.anoxkaldnes.com/ Eng/c1prodc1/mbbr.htm,http://www.veoliawaterst.com/mbbr/cz/application s.htm