Numerical simulation of an intermediate sized bubble rising in a vertical pipe

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Abstract

A Computational Fluid Dynamics (CFD) based front tracking algorithm is applied to investigate the rising behaviour of a single bubble in a vertical pipe with a stagnant or flowing viscous liquid. The ratio of the pipe diameter to the bubble equivalent diameter (D/D_b) is varied within the range of 1.0~10.0. The wall effects on the terminal bubble rising speed (U) and shape are investigated under various flow conditions, which are characterised by the parameters Archimedes number (Ar), Bond number (Bo), and bulk liquid flow speed (U_l). It is found that the terminal bubble rising speed (U) relates to the bubble rising speed in an infinite domain (U_∞) and the pipe diameter by the formula $(U_\infty - U)/U_\infty \propto (D_b/D)^\alpha$, where α is an exponent relating to the bubble deformability, and it is found to be in the range of 1.0~0.7 in this study. In addition, the effects of flowing liquid on the terminal bubble rising speed and shape are also investigated for different sized pipes. It is demonstrated that the bubble rising behaviour is significantly affected by the flowing liquid in the pipe with a small diameter. Moreover, the detailed flow field around the bubble is presented to understand the physics of bubble rising behaviour in a vertical pipe under various flow conditions.

Keywords: bubble rising, wall effect, front tracking method, Taylor bubble.

1 Introduction

The dynamics of a gas bubble rising in a vertical pipe filled with a viscous fluid is of great importance in nuclear and process industries, e.g. petroleum, refining,

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bubble columns and boiling flows. The terminal rising speed and shape of a gas bubble in a large domain without wall effects have been studied by many researchers [1–2]. The correlations obtained from such studies have been widely used in the modelling of bubbly flows [3]. However, in reality, most of the bubbly flow problems occur in a container with walls, e.g. a pipe. Unfortunately, the effects from the nearby stationary wall (pipe wall) and moving wall (neighbouring bubble) are usually neglected due to the lack of the knowledge in this aspect. Extensive experimental and numerical studies have been conducted to investigate the rising of a Taylor bubble (elongated bubble compared to pipe diameter) [4], however, studies of intermediate sized bubble rising in a vertical pipe are quite limited [5], and it is of great importance to understand the physics of the wall effects on bubble rising behaviour in pipes. It is believed that the wall plays critical roles in the regime transitions of multiphase pipe flow. For example, the transition between bubbly flow and slug flow is resulted by the changes of the relative sizes of bubbles in the liquid flows [6].

In order to qualitatively justify the wall effect on the bubble rising dynamics, such as the terminal velocity and shape, detailed studies of the wall proximity are necessary. Uno and Kintner [7] experimentally studied the effect of wall proximity on the velocity of a single air bubble rising in a quiescent liquid contained in a cylinder. Four different liquids were used, namely, distilled water, 61% glycerine, diethylene glycol, and a solution of a surface-active agent. On the basis of the experimental results, a single equation was proposed to express a velocity-correction factor in term of the ratio of the bubble diameter to pipe diameter and an empirical constant, i.e. $U/U_\infty \propto (1-D_b/D)^\alpha$, where D_b is the equivalent bubble diameter, D is the pipe diameter, and α is the exponent with a value of 0.765.

Numerical simulations of a bubble rising freely in viscous liquid have been performed by a number of researchers. Chen et al. [8] used a modified Volume of Fluid method to simulate a bubble rising in a stationary liquid contained in a closed, vertical cylinder. The effect of density and viscosity ratios on the bubble rising is investigated. Hua and Lou [9] proposed an improved front tracking method to simulate the bubble rising in viscous liquid. The simulation results were compared with the available experimental data. Recently, Mukundakrishnan et al. [5] studied the wall effects on a buoyant gas-bubble rising in a liquid-filled finite cylinder using a front tracking finite difference method coupled with a level contour reconstruction of the front. They made detailed simulations on the flow pattern around the bubble and the bubble shape. They presented preliminary discussions about the wall effects on the terminal bubble rising speed. Since it is of great importance to both the scientific researches and engineering applications, further studies are performed here.

In this paper, a Computational Fluid Dynamics based front tracking algorithm is applied to examine the rising behaviour of a single bubble in a vertical pipe with a stagnant or flowing viscous liquid. The ratio of the bubble equivalent diameter to the pipe diameter is varied in the range from 0.1 to 1.0. The simulation is started with the basic cases of single bubble rising in a large domain without wall effects in different regimes, where different terminal bubble

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shapes such as spherical, oblate ellipsoidal, dimpled ellipsoidal, skirted bubble and spherical cap, can be obtained. Then, further simulations are performed to investigate the wall effects by reducing the pipe diameter. The improved front tracking method by Hua and Lou [9] is employed for this study, as this method is extensively validated against a number of experiments on bubble rising in various flow regimes. Our simulation results about the wall effects on the bubble terminal velocity agree reasonably with Uno and Kintner's [7] correlation when the Archimedes number (Ar) and Bond number (Bo) are relatively higher. However, we found that the exponent varies with the flow regimes. And a new correlation for bubble terminal rising speed is proposed to take into account the wall effect $(U_\infty - U)/U_\infty \propto (D_b/D)^\alpha$. The effects of pipe wall on the bubble shape are also qualitatively investigated. The proximity of the cylinder wall tends to elongate the bubble in the pipe axial direction. But in the surface tension dominated flow regime, the bubble shape will remain spherical.

2 Mathematical formulation and computational method

2.1 Physical problem

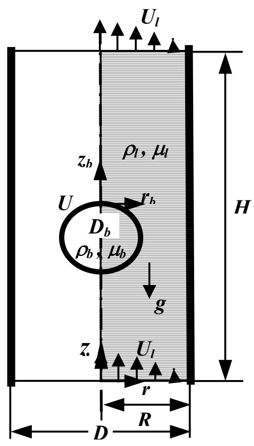


Figure 1: Schematic illustration of a gas bubble rising in a vertical pipe with a flowing viscous liquid.

The physical problem of a gas bubble rising in a pipe with a flowing viscous liquid is illustrated in Figure 1. The gas bubble has an equivalent diameter $D_b = \sqrt[3]{6V/\pi}$, density ρ_b , viscosity μ_g and rising speed U . The bubble volume is assumed to be V . The pipe has diameter D or radius R and height H . The liquid inside the pipe has density ρ_l , viscosity μ_l , and the bulk speed of liquid flow is assumed to be U_l . The flow inside the pipe is assumed to be fully developed laminar flow with a parabolic distribution: $u_l(r) = 2.0 \cdot U_l \cdot \left[1 - (r/R)^2\right]$.

The gravitational acceleration is assumed to be g. Two coordinate systems are employed in the current analysis: a stationary cylindrical system (r,z) used for solving the governing equations; and a moving cylindrical system (r_b, z_b) with the origin located on the bubble nose used for post-processing and analysing the flow field around the bubble.

2.2 Mathematical formulation and numerical method

In this study, it is reasonable to assume that both the gas and liquid phases are incompressible. The governing equations for the multiphase fluid flow system can be expressed as

$$\nabla \cdot \mathbf{u} = 0 , \tag{1}$$

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot \rho \mathbf{u}\mathbf{u} = -\nabla p + \nabla \cdot [\mu(\nabla \mathbf{u} + \nabla \mathbf{u}^T)] + \int \sigma \kappa_f \, \mathbf{n}_f \, \delta(\mathbf{x} - \mathbf{x}_f) ds_f + (\rho - \rho_l)\mathbf{g} , \tag{2}$$

where $\mathbf{u}$ is the fluid velocity, p denotes pressure, ρ and μ stand for the density and viscosity, respectively, $\mathbf{g}$ is the gravitational acceleration, s stands for the arc length measured on the interface, κ_f denotes the curvature of the interface, σ is the surface tension coefficient and is assumed to be a constant, $\mathbf{n}_f$ stands for the unit normal vector on the interface, $\mathbf{x}_f$ is the position vector on the interface, and $\delta(\mathbf{x} - \mathbf{x}_f)$ stands for the delta function that is non-zero only when $\mathbf{x} = \mathbf{x}_f$.

The governing equations can be further non-dimensionalized using characteristic length (the equivalent bubble diameter D_b) and speed ($\sqrt{gD_b}$).

$$\nabla \cdot \mathbf{u}^* = 0 , \tag{3}$$

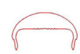
$$\begin{aligned} \frac{\partial(\rho^* \mathbf{u}^*)}{\partial \tau^*} + \nabla \cdot \rho^* \mathbf{u}^* \mathbf{u}^* &= -\nabla p^* + \frac{1}{Ar} \nabla \cdot [\mu^* (\nabla \mathbf{u}^* + \nabla^{\mathrm{T}} \mathbf{u}^*)] \\ &\quad + \frac{1}{Bo} \int_{\Gamma} \kappa^* \mathbf{n} \delta(\mathbf{x}^* - \mathbf{x}^*{}_f) ds + (\rho^* - 1)\mathbf{g}^* \end{aligned} \tag{4}$$

where, $\mathbf{x}^* = \frac{\mathbf{x}}{D_b}$, $\mathbf{u}^* = \frac{\mathbf{u}}{\sqrt{gD_b}}$, $\tau^* = \sqrt{\frac{g}{D_b}} t$, $\rho^* = \frac{\rho}{\rho_l}$, $p^* = \frac{p}{\rho_l g D_b}$, $\mu^* = \frac{\mu}{\mu_l}$, $\kappa^* = D_b \kappa$, $\mathbf{g}^* = \frac{\mathbf{g}}{g}$. And the Archimedes number (Ar) and Bond number (Bo) are defined as,

$$Ar = \frac{\rho_l g^{1/2} D_b^{3/2}}{\mu_l} \text{ and } Bo = \frac{\rho_l g D_b^2}{\sigma} .$$

The Archimedes number (Ar) denotes the importance of the buoyancy force over viscous force, and the Bond number (Bo, also called Eotvos number) represents the relative importance of the buoyancy and surface tension forces. Hence, the problem of bubble rising can be specified by the following flow parameters such as density ratio ($\eta = \rho_g / \rho_l$), viscosity ratio ($\lambda = \mu_g / \mu_l$), Archimedes number, Bond number, and the ratio of the bubble diameter to the pipe diameter (D_b / D), and the bulk speed of the liquid flow inside the pipe (U_l).

Table 1: Flow conditions for the simulation cases and the predicted terminal bubble shape.

Simulation No.	Ar	Bo	U_l	D/D_b			
				1	2	4	8
1	10	5	0	----			
2	10	50	0				
3	50	10	0				
4	100	50	0				
5	100	100	0				
6	10	50	0.2				----
7	10	50	-0.2				----

The numerical method used in this study is based on the finite volume / front tracking method developed by Hua and Lou [9], which has been extensively validated for simulating single bubble rising freely in a quiescent viscous liquid. In the current numerical simulation, a bubble is released at a short distance above the bottom of the pipe (as shown in Fig. 1), and is initially assumed to be spherical or elliptical (depending upon the pipe diameter) with a dimensionless equivalent diameter of one. The pipe is filled with liquid, and the bubble will be accelerated and move upwards due to buoyancy force. The vertical pipe height is long enough to allow the bubble to reach the steady state. In order to study the effects of the pipe wall proximity on the bubble rising, the pipe diameter is varied within the range of $1.0 \cdot D_b \sim 10.0 \cdot D_b$, while other parameters and the fluid properties are kept constant for the simulations. Ar and Bo are varied in order to investigate the wall effects on bubble rising in different flow regimes (spherical, dimpled ellipsoidal, ellipsoidal, spherical cap, skirted bubble) while the density ratio and viscosity ratio are kept constant for all simulations ($\rho_b / \rho_l = 0.001$; $\mu_b / \mu_l = 0.01$) in this study. The flow conditions for the different simulation cases reported in this paper are listed in Table 1.

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3 Results and discussion

3.1 Simulation of transient bubble rising in vertical pipe

The temporal variations of bubble shape and position while it is rising in the vertical pipes with different diameters are shown in Figure 2. The liquid is initially stagnant. As the pipe diameter is reduced, the bubble inside the pipe is elongated in shape due to the constraints of the pipe wall, and the bubble rising speed is also reduced significantly. To investigate the effect of pipe wall on the bubble rising speed, the detailed flow field around the terminal bubble is shown in Figures 3(a), (b) and (c). Here, the reference system is located on the nose of the rising bubble. It shows that a falling liquid film is formed between the bubble and the pipe wall when the pipe diameter is decreased to the same order as the bubble equivalent diameter. Flow dynamics in the falling liquid film has significant effect on the bubble rising. The velocity profiles along the radial direction crossing the bubble and liquid film inside the pipes with different diameters of $1.0D_b$, $1.6D_b$ and $2.0D_b$ are shown in Figure 3(d). Here the reference system is located on the stationary pipe. As the pipe diameter becomes smaller, the liquid film thickness decreases, and the length of the liquid film increases with the bubble being elongated. A stable bubble rising speed can be obtained when the buoyancy force acting on the bubble is balanced by the drag force including the viscous shear force from the flow in the falling liquid film. It can be seen from Figure 3(d) that the liquid film thickness decreases as the pipe diameter is reduced, and the maximum downward speed in the falling liquid film increases. A higher velocity gradient is induced in the liquid film, and results in a higher viscous shear stress. Hence, the terminal bubble rising velocity becomes lower as the pipe size decreases.

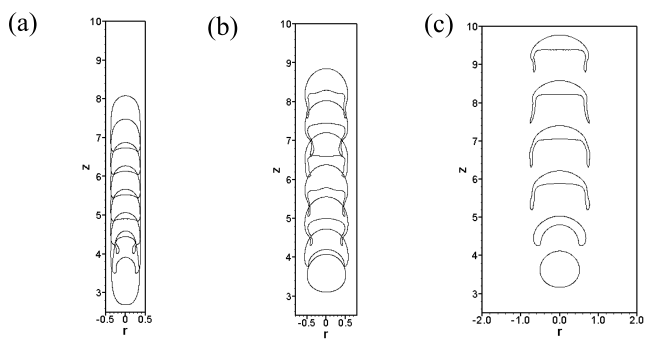

Figure 2: Bubble rising behaviour in vertical pipes with different diameters (a) $D = 1.0D_b$; (b) $D = 1.6D_b$; (c) $D = 4.0D_b$, $Bo = 100$, and $U_l = 0$.

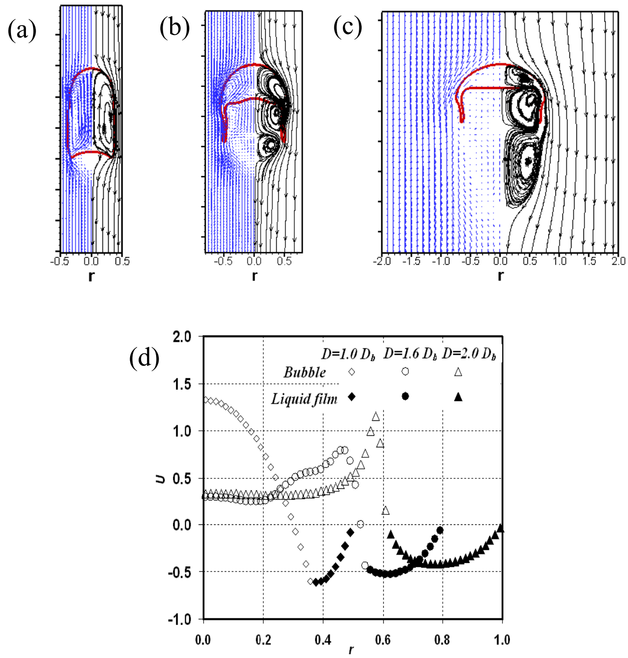

Figure 3: Detailed flow field around the terminal rising bubble in different sized pipes, (a) $D = 1.0D_b$; (b) $D = 1.6D_b$; and (c) $D = 4.0D_b$, and (d) shows the velocity profiles in the bubble and liquid film across the pipe.

3.2 Wall effects on terminal bubble shape and rising speed

It is understood that both terminal bubble shape and rising speed can be significantly affected by the relative size of the bubble to the pipe (D_b / D). Table 1 shows the terminal bubble shapes under different flow conditions. When the pipe diameter is large enough ($D / D_b > 4.0$), the pipe wall has much less effects on the terminal bubble shape for most simulation cases. Significant effect of pipe wall on the terminal bubble shape occurs when the pipe size is small or the bubble size is large with $D / D_b < 2.0$. Generally, when a pipe wall exists in the proximity of a bubble, the bubble will be elongated along the axial direction of the pipe. The bubble rising speed is reduced as a result of the increase of the resistance from the liquid. The variations of bubble shape and terminal speed with the relative size of bubble to pipe behave differently at different flow regimes.

In the surface tension dominated regime ($Ar < 5.0$ and $Bo < 5.0$), the strong surface tension will always keep the bubble in spherical shape. Even when the pipe diameter is of the same order of magnitude of the bubble, the spherical bubble will not deform significantly as it moves very slowly in the pipe. With slightly increase of both Archimedes number ($Ar > 5.0$) and Bond number ($5.0 < Bo < 50.0$), an oblate ellipsoidal bubble will be observed in liquid without wall effects. When the pipe diameter becomes smaller, the bubble will be elongated and become prolate shaped with spherical cap at both head and tail. With a further increase in the Bond number ($Bo > 50.0$), the role of surface tension becomes less important in determining the bubble shape, and the balance of inertial and viscous force becomes more important, and the bubble wake starts to affect the shape of the bubble bottom. When the Archimedes number is small ($5.0 < Ar < 20.0$ and $Bo > 50.0$), the bubble bottom becomes dimpled. When the Archimedes number is large ($Ar > 100.0$ and $Bo > 50.0$), the strong bubble wake flattens the bubble bottom, creating a spherical cap bubble. Within the intermediate range Archimedes number ($20.0 < Ar < 100.0$ and $Bo > 50.0$), skirted bubbles can be observed. When the bubble is rising in a small pipe ($D / D_b \propto O(1)$) with $Bo > 50.0$, the bubble will be elongated, and a falling liquid film will be formed between the bubble and pipe wall. The bubble head normally has a semi-spherical shape, and the bubble bottom may be dimpled, skirted or flattened, depending upon the Archimedes number.

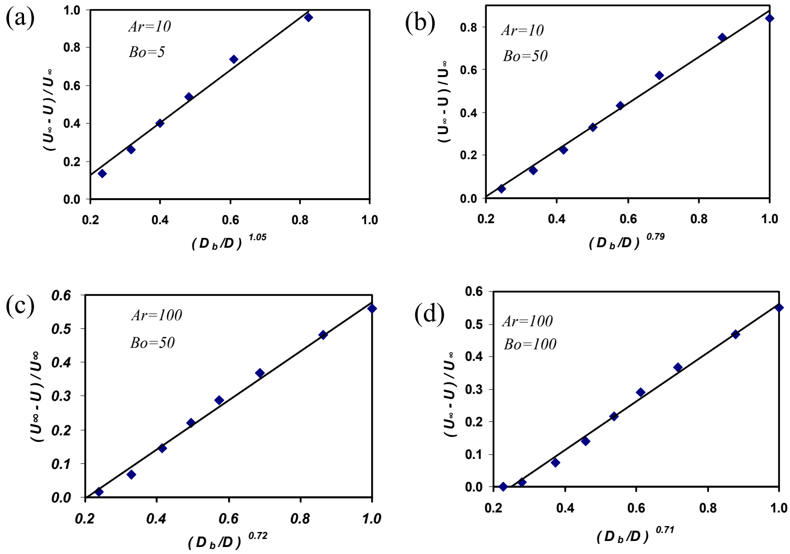

Figure 4: The effects of pipe wall on the terminal bubble rising speed under different flow regimes.

The wall effects on the bubble rising is investigated in comparison with the corresponding case of bubble rising in an infinite liquid without a wall. Hence, it is reasonable to express the relative change of the bubble terminal velocity ($(U_\infty - U)/U_\infty$) as a function of the ratio of bubble size to the pipe diameter

(D_b / D). Figure 4 shows the correlation between $(U_\infty - U)/U_\infty$ and D_b / D under different flow regimes. It can be concluded that the wall effects on the terminal bubble rising speed follows the correlation $(U_\infty - U)/U_\infty \propto (D_b / D)^\alpha$, where α is an exponent depending on the flow regime. It is found that the exponent α is about 0.7~0.8 when $Bo > 50.0$.

3.3 Effect of pipe flow on bubble rising

Pipes are generally used to transport gas and liquid. Therefore, investigation of the effect of pipe flow on the bubble rising behaviour is of great engineering interest. Figure 5 shows the detailed flow field around the bubble in different sized pipes with liquid flowing upwards ($U_l = 0.2$) or downwards ($U_l = -0.2$). When the pipe diameter is large enough, the terminal bubble shape is not affected by the upward flow or downward flow in the pipe. Figure 6(a) shows the velocity profile along the radial direction crossing the bubble and at the far field away from the bubble ($D/D_b = 4.0$). From Figure 6(a), it can be concluded that the terminal bubble rising velocity can be estimated by adding the bubble rising speed in stagnant liquid and the pipe flow speed. On the contrary, when the pipe diameter becomes small enough, a liquid film is formed between the bubble and pipe wall. The liquid flow inside the pipe starts to affect the bubble shape and bubble moving speed. As shown in Figure 5, the upward liquid flow enhances the bubble rising speed when $D/D_b = 2.0$, and the dimple at the bubble bottom becomes deeper. On other hand, the downward liquid flow reduces the bubble rising speed significantly and the dimple at the bubble bottom disappears. When the pipe diameter become smaller $D/D_b = 1.0$, the pipe flow affects the bubble shape and movement more significantly. The upward liquid flow helps

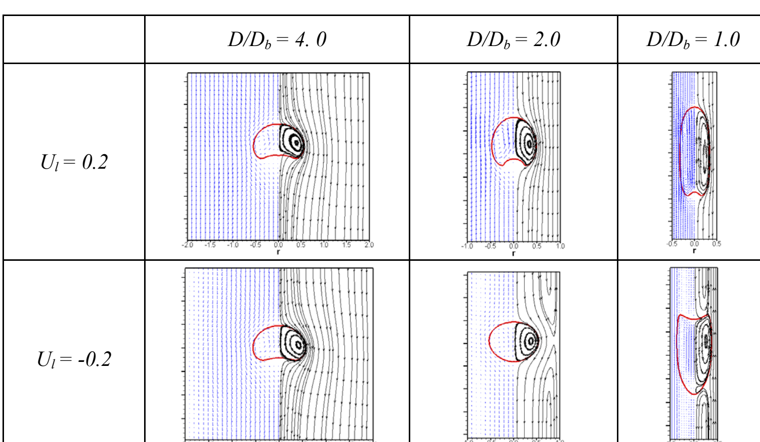

Figure 5: The detailed flow field around the rising bubble in different sized pipes with upward or downward flows. $Ar = 10.0$, $Bo = 50.0$.

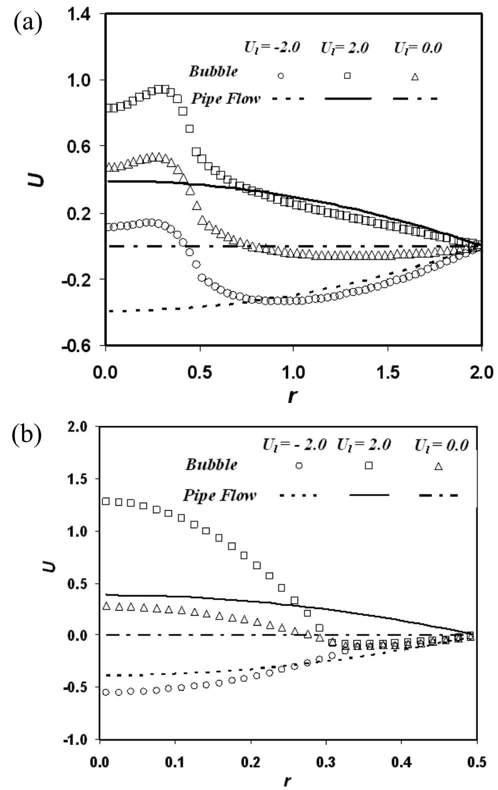

Figure 6: Comparison of velocity profiles crossing the rising bubble and the pipe flow for different sized pipe (a) $D/D_b = 4.0$ and (b) $D/D_b = 1.0$. $Ar = 10.0$, $Bo = 50.0$.

the rising of the elongated bubble, creating a high velocity gradient in the pipe and builds a dimple at the bubble bottom. On the contrary, the downward flow pushes the "bullet" shaped bubble downwards. A comparison of the velocity profiles across the bubble under different pipe flow conditions in the small pipe ($D/D_b = 1.0$) is shown in Figure 6(b). It is interesting to find that the velocity profile across the liquid film is independent of the pipe flow conditions for the current study cases. This is maybe due to the high viscosity of the liquid when the Archimedes number is low ($Ar = 10.0$), and the boundary layer along the pipe wall determines the velocity profile in the liquid film.

4 Conclusion

A Computational Fluid Dynamics (CFD) based front tracking algorithm has been applied to investigate the rising behaviour of a single bubble in a vertical pipe with a stagnant (in the far field) or flowing viscous liquid. It is found that the relative size of the bubble and pipe has significant effect on the terminal bubble rising velocity (U) and shape. When $D/D_b > 4.0$, the wall effect on the terminal bubble shape and rising speed can be neglected. A strong wall effect on bubble shape and terminal velocity can be observed when $D/D_b < 2.0$. In general, a small pipe diameter elongates the bubble in the axial direction and reduces the bubble rising velocity. It is found that the bubble rising velocity can be affected by the pipe diameter as $(U_\infty - U)/U_\infty \propto (D_b/D)^\alpha$, where α is an exponent relating to the bubble deformability. The exponent α has a value about 0.7~0.8 when $Bo > 50.0$. The liquid flow may also produce significant effects on the bubble moving behaviour in a vertical pipe. It is found that the pipe flow has an effect on the bubble shape and moving pattern only when the pipe diameter is small. In a large pipe, the bubble shape is not significantly affected by the pipe flow, and the bubble moving speed is the sum of the pipe flow speed and its rising speed in the stagnant liquid.

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