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Representation of partial wave reflection and transmission for rubble mound coastal structures

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Abstract

This work presents a description of partial wave reflection and transmission effects for rubble mound structures using time dependent depth averaged equations of the type used in a Boussinesq non linear wave model and in a hyperbolic linear wave model. Energy losses due to friction effects on the rough slope of a rubble mound structure can be represented using a friction area in front of a vertically faced homogeneous porous structure. The flow resistance within the structure is simulated in a similar way by incorporating an additional term in the momentum equation given by a Dupuit-Forchheimer relationship.

1. Introduction

Prediction of wave reflection and transmission for porous rubble mounds plays an important role in the estimation of the wave field inside a harbour. Rubble mounds are widely used in harbour engineering because of their ability to absorb significant amount of wave energy. This often helps to make access to the harbour easier in bad weather by avoiding confused seas at the entrance due to reflected wave energy and in some cases it can help to limit the disturbance inside the harbour.

The simple theoretical solutions for wave reflection and transmission for porous rubble mound breakwaters are not applicable in a complicated harbour geometry. On the other hand a numerical short wave model which is applied for the harbour design could be used. However, in order to account for wave dissipation, a full description of the flow both outside and inside the structure would require an extension of the wave models in the run-up region (incorporating breaking and using to very small space and time steps) as well as a coupling with a 2DV model for the internal flow (Hannoura and McCorquodale, [3]). A different way, proposed in the present work, is the representation of such structures in a wave model by incorporating new terms in the momentum equation. The method is based on a theory for the partial reflection and transmission of shallow water waves incident on rubble mound breakwaters which has been presented by Madsen Ole and White [5] (hereinafter referred to as MW).

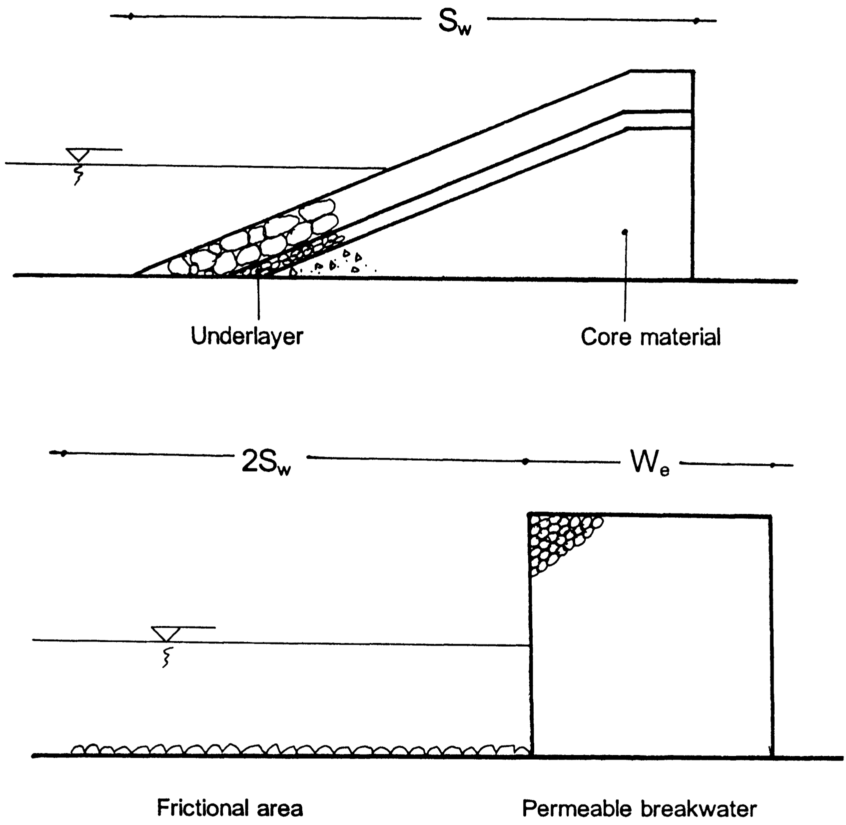

Figure 1. Realistic breakwater (top) and its equivalent structure with friction area in front (bottom).

The basic approach in MW theory is to assume that energy is dissipated by two separate mechanisms: one is the frictional effect of having a rough slope and the second is the frictional effect on the flow within the porous structure. Based on the MW method the two above dissipation mechanisms are represented separately in the time dependent

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wave model (Boussinesq non linear model or 'mild slope' equation linear model). The approach adopted here is to represent the rough slope by a frictional area in front of the hydraulically equivalent structure while for the representation of the equivalent permeable structure a Dupuit-Forchheimer relationship is used. This representation is shown schematically in Figure 1.

In section 2 we show how to represent the frictional area in front of the structure while in section 3 the representation of the equivalent permeable structure is considered. Results of the method are represented in sections 4 and 5.

2. Representation of rough slopes in a time dependent intraperiod wave model.

The first point to make concerns the length of the frictional area in front of the equivalent structure. It is a simple matter to show that by making this length equal about twice the active slope length, there will be, to a reasonable approximation, an equivalent number of wavelengths within the frictional area to the number of wavelengths on the active slope length. This measure seems sensible in that a partially standing wave system will develop on the actual slope which in turn will affect the amount of energy dissipation. By attempting to reproduce a similar partially standing wave system in the equivalent frictional area, in the wave model, it should be possible to obtain better representation of energy losses. It is of interest to note that the length of the frictional area was varied outside this suggested length in runs of the wave model, but the suggested length generally gave the best results.

The second point to make is that based on experimental data reflection coefficients for rough impermeable slopes can be assumed, to a reasonable degree of accuracy to vary linearly with wave height. This suggests that a linear friction term can be used in the Boussinesq model to represent energy losses on the slope. Therefore, the one dimensional Boussinesq equations in the frictional area take the form (Karambas and Koutitas, [4]):

$$\frac{\partial \zeta}{\partial t} + \frac{\partial\left(U\left(d+\zeta\right)\right)}{\partial x} = 0$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + g\frac{\partial \zeta}{\partial x} - \frac{d}{2}\frac{\partial^3 (Ud)}{\partial x^2 \partial t} + \frac{d^2}{6}\frac{\partial^3 U}{\partial x^2 \partial t} - \gamma\frac{\partial^2 U}{\partial x^2} = 0 \qquad (1)$$

where ζ is the surface elevation, U is the depth averaged velocity, d is the water depth and γ is a coefficient given by:

$$\gamma = f_s \frac{d}{\tan^2 \alpha_s} (g d_m)^{1/2} \tag{2}$$

here f_s is a dimensionless linear friction coefficient, α_s is the slope angle and d_m is the average stone diameter.

In a similar way the linear time dependent 'mild slope' equations (for periodic waves as proposed by Copeland [2] are written:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial (Ud)}{\partial x} = 0$$

$$\frac{\partial U}{\partial t} + \frac{c^2}{d} \frac{\partial \zeta}{\partial x} - \gamma \frac{\partial^2 U}{\partial x^2} = 0 \tag{3}$$

in which c is the wave celerity given from the Airy theory.

All of the quantities in (2) are well defined apart from the friction coefficient f_s. But this coefficient can be defined once the reflection coefficient R_s for the rough (impermeable) slope has been determined from the MW theory.

The Boussinesq model can also be used with random waves in harbour applications and so it is necessary to decide which random wave parameters are to be used when obtaining reflection coefficients from the MW theory. Thus as a representative wave period the zero crossing period of the random waves can be adopted.

Without loss of generality we can assume constant depth. We also assume that the frictional area lies between $0 \le x \le 2S_w$ with a perfectly reflecting boundary at $x=2S_w$. A plane wave of amplitude a_i and radian frequency ω is assumed to be incident on the frictional area from the left in Figure 1 (for $x<0$) with a reflected wave of amplitude a_r travelling back from the area. Thus, outside the friction area ($\gamma=0$) we have:

$$\zeta = a_i e^{i(\omega t - kx)} + a_r e^{i(\omega t + kx)}$$

where, for the Boussinesq model:

$$k^2 = \frac{\omega^2}{gd - \omega^2 \frac{d^2}{3}}$$

and for the ‘mild-slope’ model:

$\omega^2 = gk \tanh kd$

(which is the dispersion relationship form Airy linear theory).

Inside the friction area we let

$$\zeta = a_1 e^{i(\omega t - Kx)} + a_2 e^{i(\omega t + Kx)}$$

where K is now complex and satisfies (for the Boussinesq model):

$$\omega^2\left(1 + K^2 \frac{d^2}{3}\right) - i\omega\gamma K^2 = gdK^2$$

and for the ‘mild-slope’ model:

$$\omega^2 - i\omega\gamma K^2 = c^2K^2$$

We can solve for the reflection coefficient $R_s = |a_i/a_r|$ using the boundary conditions that surface elevation and horizontal velocity be continuous at x=0 and that the horizontal velocity vanish on the reflecting boundary at $x=2S_w$. This gives,

$$R_s = \left| \frac{\frac{K}{k}\left(1 + e^{-4iKS_w}\right) - \left(1 - e^{-4iKS_w}\right)}{\frac{K}{k}\left(1 + e^{-4iKS_w}\right) + \left(1 - e^{-4iKS_w}\right)} \right| \qquad (4)$$

The complex wave number K is a function of the friction coefficient f_s so by solving the above equation by iteration, for a given value of the reflection coefficient R_s from the MW theory, we can define the friction factor to use in the Boussinesq model and the ‘mild-slope’ model.

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3. Representation of permeable structures in the wave models

This aspect is relatively straight forward to represent. The MW theory already reduces a quite general multi-layered rubble mound structure to an equivalent vertically faced homogeneous porous structure. It is then an easy matter to represent this structure directly in a depth averaged model like that based on the Boussinesq or the 'mild-slope' equations. The only difference here is that the shallow water wave equations for a porous medium used in the MW theory are generalised to the equivalent Boussinesq or 'mild-slope' equations for a porous medium. Thus, adopting a Dupuit-Forchheimer relationship. the Boussinesq equations (1) are rewritten:

$$n\frac{\partial\zeta}{\partial t}+\frac{\partial\left(U(d+\zeta)\right)}{\partial x}=0$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+ng\frac{\partial\zeta}{\partial x}-\frac{d}{2}\frac{\partial^3(Ud)}{\partial x^2\partial t}+\frac{d^2}{6}\frac{\partial^3 U}{\partial x^2\partial t}+nU\left(\alpha+\beta|U|\right)=0 \tag{5}$$

and the 'mild-slope' equations (3):

$$n\frac{\partial\zeta}{\partial t}+\frac{\partial(Ud)}{\partial x}=0$$

$$\frac{\partial U}{\partial t}+n\frac{c^2}{d}\frac{\partial\zeta}{\partial x}+nU\left(\alpha+\beta|U|\right)=0 \tag{6}$$

Here, n is the porosity of the equivalent structure and α and β coefficients of laminar and turbulent flow resistance given by:

$$\alpha=\alpha_o\frac{(1-n)^3}{n^2}\frac{\nu}{d_m^2}$$

$$\beta=\beta_o\frac{(1-n)}{n^3}\frac{1}{d_m}$$

in which ν is the kinematic coefficient of viscosity and α_o =2700, β_o=2.7 as suggested by Madsen and White when representing physical models. Clearly, when representing real structures more realistic parameters can be used in the numerical models.

As the Boussinesq model is non-linear, we can include the full non-linear form of the frictional term in the momentum equation (5). The same non linear term is also incorporated in the linear equation (6) without affecting its numerical stability.

4. Results for rough impermeable slopes

Starting with the Madsen and White experiments for an impermeable rubble mound slope with a stone size of 25mm and a water depth at the toe of the slope of 0.3m, we use the technique described in Section 2 to represent partial reflections in the wave models. The values of the various quantities appearing in the momentum equations (of 1 and 3) are listed below along with the Madsen and White experimental reflection coefficients, together with the reflection coefficients actually obtained in running the Boussinesq and the 'mild slope' model. The grid size used in the models was 0.125m.

Slope	Wave period (s)	Reflection coefficient		
		Boussinesq model	'Mild-slope' model	Experi-ments
1 in 1.5	1.6	0.75	0.75	0,77
	1.8	0.81	0.79	0,79
	2.0	0.86	0.84	0,83
1 in 2	1.6	0.70	0.69	0,62
	1.8	0.75	0,73	0,74
	2.0	0.80	0,77	0,78
1 in 3	1.6	0.38	0,39	0,46
	1.8	0.45	0,47	0,48
	2.0	0.53	0,52	0,54

In applying the technique described in Section 2 it is necessary to choose a representative wave period for the set of experimental results. This was taken to be the middle period T of 1.8s. It meant that the friction factor f_s was determined for each slope using equation (4) with the reflection coefficient R_s put equal to the experimental value for T=1.8s. This same friction factor was then used for the other two periods. The other point to note is that the value of tan α_s in equation (2) is put to 0.5 for the 1 in 1.5 slope. This just helps to make the friction factor f_s reasonably independent of slope angle.

On the whole, the reflection coefficients finally obtained in the wave models produce a reasonable match to the experimental values measured by Madsen and White. The fact that such a match can be obtained for a range of wave periods, with just one friction factor for each slope, suggests that the technique should work satisfactorily for the more realistic case of a random sea where energy is spread over a number of wave period components.

5. Results for permeable structures

The example chosen for application of the numerical models is the permeable rough slope shown at the top of Figure 1 consisting of a single armour layer with a stone size of 49mm on a slope of 1 in 2. The water depth is 0.38m. Using MW theory we can calculate the width W of the equivalent homogeneous vertically faced breakwater with a stone size of 25mm. This defines the dimensions of the area in the wave models over which equations (5) and (6) are used. The values of porosity (n) and the friction coefficients (α) and (β) correspond with those given in Section 3 i.e.:

$\alpha=4.94$

$\beta=741.32$

By carrying out tests with just the friction area represented in the wave models it was established that dissipation of energy on the rough slope was being well represented. The length of the frictional area in front of the equivalent structure ($2S_w$ in Figure 1) was taken to correspond to twice the active slope length i.e. approximately 1.5m. And, in running the Boussinesq and the 'mild-slope' model, reflection coefficients of 0.82 and 0.79, respectively, were obtained for the 2.2s wave period case. This compares with an expected reflection coefficient of about 0.8 for the slope alone.

Having checked the frictional area on its own it was combined with the equivalent porous structure. In this case the example chosen was one that had been flume tested at HR Wallingford (Bowers and Budin, [1]). The following results were obtained for single period waves and with a grid size of 0.2m in the wave models.

Wave period (s)	Wave height (m)	W (m)	Reflection coefficient		
			Boussinesq model	'Mild-slope' model	Experiments
1.4	0.06	0.8	0.3	0.27	0.28
	0.16	0.8	0.28	0.28	0.29
2.2	0.06	1.0	0.56	0.54	0.53

As in the previous section, the agreement between the wave model reflection coefficients and experimental values is very encouraging. The

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experimental results are, in fact, average reflection coefficients obtained with random sea conditions with zero crossing periods of 1.4s and 2.2s.

Conclusions

The representation of rubble mound structures in the wave model has been checked against physical model flume results for partial reflection and transmission coefficients using single period waves. Encouraging agreement was obtained with experimental results, showing that the proposed technique provides a robust method of representing partial reflection and transmission in Boussinesq and in linear 'mild-slope' models.

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