Sinusoidal regime analysis of heat transfer in microelectronic systems

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Abstract

The boundary element method has been used to evaluate the thermal impedances of electronic components in the micrometer range. All simulations gave rise to impedance plots composed of a few circular arcs. This conclusion has also been confirmed by experimental measurements and semi analytical calculations.
Keywords: thermal impedance, thermal conduction, AC, numerical calculation, boundary element method.

1 Introduction

All books devoted to heat transfer have mainly three chapters bearing the respective titles "conduction", "convection" and "radiation". The convective part is usually the most elaborated one for the obvious reason that this is the most important one in many industrial applications. If one is dealing with microelectronics, conduction turns out to be dominant. Indeed, the heat dissipated in the silicon semiconductor can only reach the cooling fins by thermal conduction through the silicon and the packaging materials. It often happens that the largest temperature drop occurs between the chip and the base of the cooling fin. The reason is obvious: the heat sources are so small that the immediate neighbourhood turns out to be the largest and hence most dominant thermal resistance.

In most books on heat transfer, the last part of "conduction" is devoted to time dependent problems (Bejan [1], Incopera and De Witt [2]). A section related to AC thermal conduction (phasor notation) is non-existent. Whereas in electricity and electronics the use of phasor notation ($j\omega$ instead of $\partial/\partial t$) is quite common, the application in the field of thermal analysis seems to be rather exceptional. In microelectronics however, an AC thermal analysis has a lot of importance. The

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electronic signal and hence the corresponding heat production are usually periodic functions of time. In order to account for the thermal feedback on the electrical behaviour a thermal AC analysis for both electric and thermal phenomena becomes more and more useful.

A limited number of papers has been devoted to the AC thermal behaviour (De Mey [3–5]). In this paper the emphasis will be on the calculation of the thermal impedance, i.e. the temperature difference between the heat source and the ambient divided by the power dissipation: $Z_{th} = \frac{T_{source}}{P}$, in K/W. Note that all quantities like temperature, power,etc., are in phasor notation, thus complex functions of the spatial coordinates x, y and z.

Some words should be said about the abbreviation AC. Strictly speaking it means Alternating Current. During the years this abbreviation has been used to denote any quantity which has a sinusoidal variation with respect to time. Even expressions like AC volts are used. In this contribution AC temperature simply means a temperature distribution varying sinusoidally with respect to time and certainly not Alternating Current Temperature.

2 Basic equations

The heat transfer equation for the temperature in an IC is given by:

$$k\nabla^2 T - j\omega C_v T = 0 \tag{1}$$

where k is the thermal conductivity in W/mK, $j = \sqrt{-1}$, $\omega = 2\pi f$ the angular frequency in rad/s and C_v the thermal capacity per volume unit in J/m^{3}K.

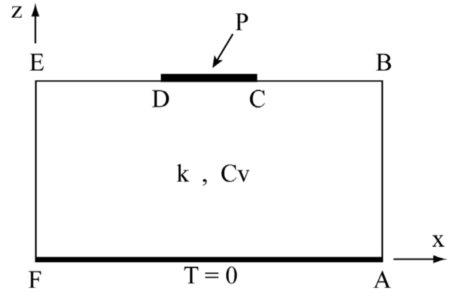

Figure 1: Cross section of a typical microelectronic structure.

The boundary conditions are explained with the help of a cross section being displayed in Fig. 1. On the bottom FA the ambient temperature $T = 0$ is applied. On the top a heat source dissipating P Watts is situated between the points D and C. Due to the small dimensions in microelectronic structures, the convective or

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radiative heat transfer through the upper part is negligible. Hence the power can only be transferred by conduction into the substrate material or:

$$\frac{P}{S} = k\frac{\partial T}{\partial z} \quad \text{(on CD)} \tag{2}$$

where S is the area of the heat source. A uniform power density inside the heat source has been assumed here.

Along the other sides, the adiabatic boundary condition is taken into account: $\frac{\partial T}{\partial x} = 0$ along AB and EF and $\frac{\partial T}{\partial z} = 0$ along ED and CB. Remark again that this paper deals with 3D problems and Fig. 1 is just an examplary 2D cross section of a 3D structure.

If two materials are in contact, the continuity of the temperature T and the normal heat flux $k\frac{\partial T}{\partial n}$ have to be taken into account.

In order to set up an equivalent boundary integral equation for (1), the Green's function of (1) has to be used:

$$G(\bar{r}|\bar{r}') = \frac{1}{4\pi k|\bar{r}-\bar{r}'|}\exp\left(-\sqrt{\frac{j\omega C_v}{k}}|\bar{r}-\bar{r}'|\right) \tag{3}$$

where $\bar{r}$ is the so called field point and $\bar{r}'$ the so called source point. Further details about the integral equation, the discretisation procedure,etc., will be omitted here.

For the impedance calculations, one cannot take the heat source temperature, because this is generally a non uniform function. Therefore the average temperature over the heat source has been used to evaluate the impedances.

3 Results

First of all some experimental results will be shown (Kawka [6]). A typical result is displayed in Fig. 2.

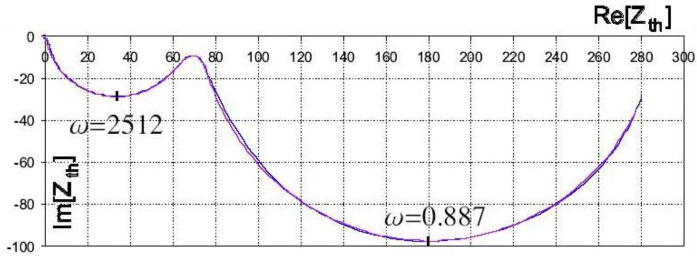

Figure 2: Experimentally measured impedance plot.

Because a pure alternating heat source does not exist, measurements have been done using a step input power. The temperature has been recorded a function of time, and by suitable Fourier techniques these data could be converted into an impedance plot as shown in Fig. 2. One observes very clearly that the plot is composed of two distinct circular arcs.

The question now is whether these almost perfect circular shapes are just a pure coincidence or do they have a more general validity. The only way to answer this question is to investigate several structures analytically or numerically.

For a rectangular shaped heat source on the surface of a half infinite thermal conducting medium, the impedance can be calculated directly by a numerical integration as shown by Vermeersch and De Mey [7]. The results for a square shaped heat source ($200\,\mu\text{m} \times 200\,\mu\text{m}$) on a half infinite silicon substrate ($k = 160\,\text{W/mK}$, $C_v = 1.784 \times 10^6\,\text{J/m}^3\text{K}$) are presented in Fig. 3.

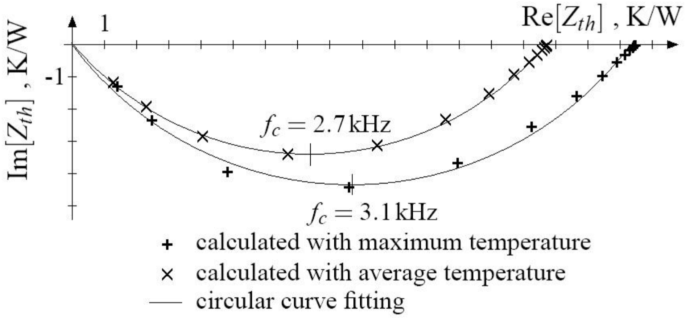

Figure 3: Semi-analytically calculated impedance plot.

For the sake of comparison, the impedance has also been evaluated using the maximum instead of the average temperature inside the heat source. For both plots the so called central frequency f_c, for which $\text{Im}[Z_{th}]$ reaches a minimum, is indicated.

Using the BEM, more general 3-D structures can be analyzed. A first example is shown in Fig. 4.

The top of the cube has a uniform heat source whereas the bottom plate is at $T = 0$ reference temperature. The four other sides are assumed adiabatic. For this particular geometry, an analytical solution can be easily found because the temperature only depends on the vertical coordinate z. The agreement between the numerical and the analytical results is extremely good even for various frequencies ranging from $f = 10\,\text{Hz}$ up to $f = 100\,\text{kHz}$.

A second example is shown in Fig. 5. For clarity, the values for the DC impedance $Z_0 = R_{th}$ are indicated in the graphs.

Two media with different thermal conductivities are involved here. The impedance plots clearly shows two circular arcs. By changing the thermal conductivity k_2 of the bottom layer, the relative magnitude of the high frequency arc with respect to the low frequency arc is modified. One should however keep in mind that the

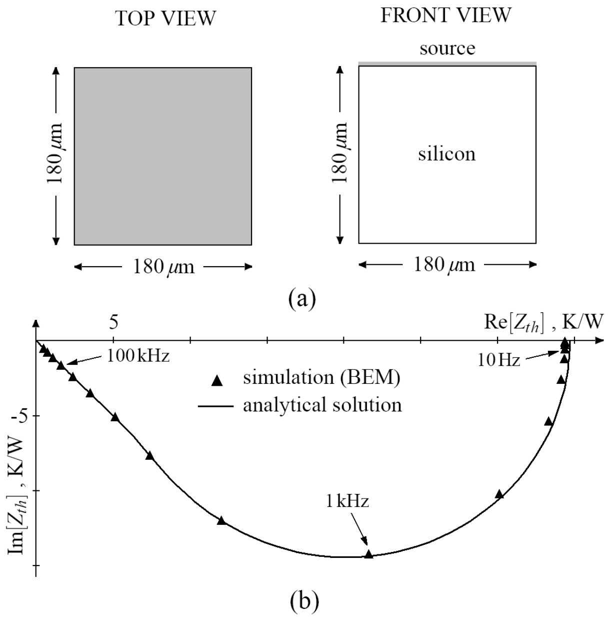

Figure 4: Example 1 – cubic shaped silicon substrate completely covered by a uniform heat source: (a) geometry, (b) calculated impedance plot.

impedance curves are plotted on different scales. A closer look in Fig. 5 reveals that only the lower frequency arc is influenced, while the other remains unchanged throughout the graphs. This can easily be explained by taking into account that for AC thermal conduction, the size of the influenced region in the material depends on the source frequency. For high frequencies only the upper part of the silicon is heated, hence k_2 does not play a role in the behaviour of the thermal impedance.

4 Conclusion

It has been proved, with the help of the boundary element method, that the thermal AC behaviour of a microelectronic structure can be modelled. Comparisons have been made with experimental and analytical results.

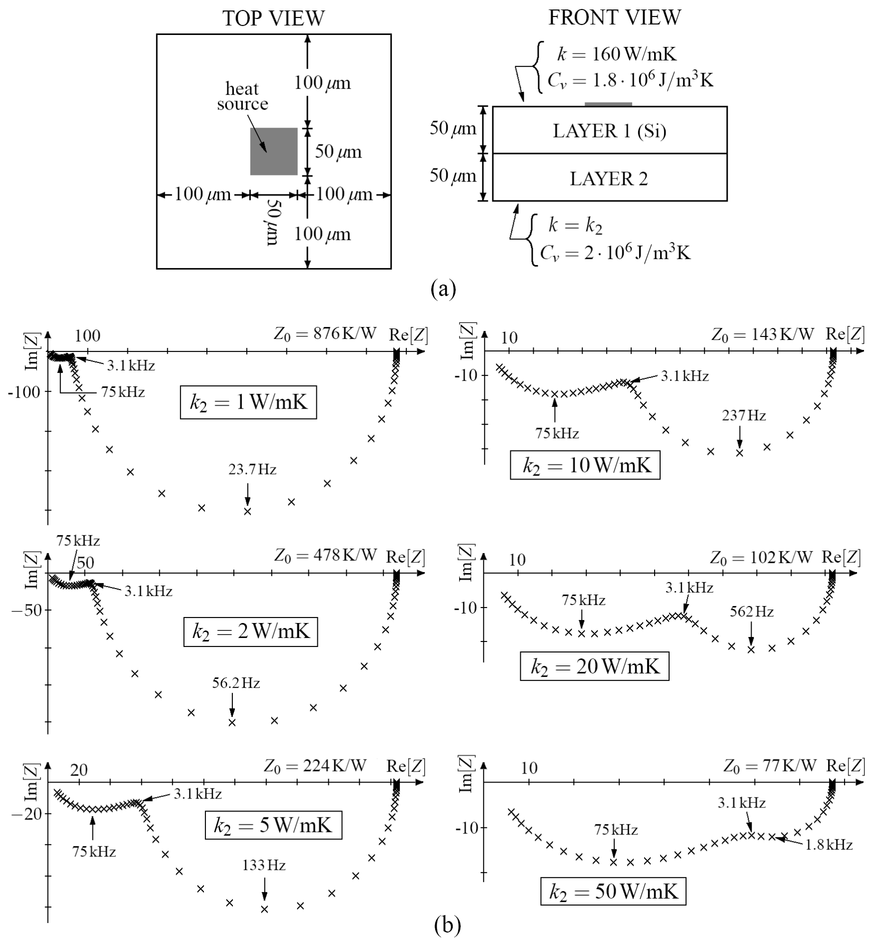

Figure 5: Example 2 – silicon substrate mounted on packing material: (a) geometry, (b) calculated impedance plots for various thermal conductivities of the bottom layer.

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