Behaviour model and experimental study for the torsion of reinforced concrete members

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Abstract

The typical behavioural curve of a reinforced concrete element with longitudinal bars and stirrups under torsion comprises two distinct regions; the elastic part until the first cracking and the part after cracking. The different character of the response in these regions reveals the different nature of the load resisting mechanism in each case. The present work addresses an approach that combines two different analytical models in order to predict the entire torsional behaviour. The prediction of the elastic part until the first cracking is achieved using a smeared crack analysis for plain concrete in torsion, whereas for the description of the post-cracking response the softened truss model is used. Further, the results of an experimental investigation on the behaviour of 15 reinforced concrete beams subjected to pure torsion are also presented in this paper. The reported results include the behavioural curves and the values of the initial torsional stiffness, the cracking torque moment and the ultimate torque capacity of the beams. Analyses for the torsional behaviour of the tested beams using the proposed approach were performed and analytical curves are produced and compared with the experimental ones. A good agreement between predicted and experimental results is observed.
Keywords: beams, reinforced concrete, smeared crack model, softened truss model, torsion, tests.

1 Introduction

Several theoretical and experimental researches have emphasized the differences between the pre-cracking and post-cracking behaviour of reinforced concrete members in pure torsion [1]. The first, pre-cracking part is characterized by the

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high value of torsional stiffness. The element behaves as a homogeneous one and the influence of reinforcement is of minor importance. The second, after cracking part is characterized by a further increase of the torque moment at a lower rate, depending on the volume of the transversal reinforcement. The consistently decreasing torsional stiffness reveals the different nature of the bearing mechanism. This justifies the fact that theories for the prediction of the ultimate torsional strength of reinforced concrete elements generally fail to describe the elastic response and to predict the torque moment at cracking.

Application of the two well-known space truss models to reinforced concrete elements under torsion exhibits very reliable results for the prediction of the post-cracking behaviour and the ultimate torque. The space truss model with spalling of concrete cover [2] and the space truss model with softening of concrete [3, 4] are experimentally verified and they can predict the ultimate torsional strength, the angle of twist, the steel and concrete strains throughout the post-cracking response of reinforced concrete beams, assuming that the section is cracked from the beginning. However, deficiency of the space truss models consists the fact that predictions of the elastic stiffness of analytical torque - twist curves lies considerable below the test curves in all the examined cases in the literature [3, 5–9].

The elastic torsional behaviour till the development of the first cracks of concrete of a reinforced concrete element is quite similar to a plain concrete member response. Thus, the ultimate torque moment of a plain concrete element is approximately equal to the torque moment at cracking of the same element with longitudinal bars and stirrups. The classical Saint Venant theory to the torsion problem of plain concrete members, although it properly describes the elastic behaviour, fails to predict the ultimate torsional strength even in the case of plain concrete [3, 10].

Recently, a new method for the analysis of plain concrete elements in torsion has been proposed by Karayannis [10]. This approach is using an efficient numerical scheme for the torsional analysis of concrete that although initially is based on the elastic theory, it utilizes a special numerical technique properly modified to include the smeared cracking approach. Extensive comparisons between analytical results yielded by this smeared crack analysis and experimental data derived from a broad range of parametrical studies established the validity of this analytical model [11–13].

This paper reports experimental results of tests on 15 beam specimens subjected to pure torsion and addresses the combination of two analytical models for the prediction of the entire torsional behaviour of reinforced concrete elements. The prediction of the elastic behaviour and the estimation of the torque moment at cracking are achieved using the smeared crack analysis for plain concrete in torsion [10], whereas for the description of the post-cracking response and the calculation of the ultimate torque moment the softened truss model developed by Hsu and Mo [3, 4] is used. Analyses for the prediction of the torsional behaviour of the tested beams using the proposed approach were performed. Predicted torsional behaviour curves are produced and compared with the experimental ones.

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2 Experimental program

The experimental program comprises 15 beams of rectangular cross-section sorted in five groups (R4, R6, RH4, RH6 and RH8) and tested under pure torsion action. The cross-section dimensions of the beams of the groups R4 and R6 were 10/20 cm (height to width ratio h/b = 2), whereas the beams of the groups RH4, RH6 and RH8 had cross-section dimensions 10/30 cm (h/b = 3). Each group comprises three specimens. The transversal reinforcement used in these three specimens was 8 mm diameter closed stirrups at a uniform spacing of 200 mm, 150 mm and 100 mm, respectively for each beam.

The longitudinal reinforcement used in the beams of groups R4 and RH4 was four longitudinal bars of diameter 8 mm (4∅8) located at the corners of the closed stirrups. The longitudinal reinforcement of beams of groups R6 and RH6 comprised 6∅8; four of them located at the corners of the stirrups and the other two bars located at the midheight of the closed stirrups (total longitudinal reinforcement 6∅8 uniformly distributed). Finally, the beams of group RH8 had eight bars of diameter 8 mm (8∅8); four of them located at the corners of the stirrups and the other four bars were uniformly distributed at the height of stirrups sides. Steel yield strength was 518 MPa for the high bond longitudinal steel bars and 365 MPa for the mild plain steel stirrups. Reinforcement arrangement for the tested beams is also presented in Table 1.

Table 1: Reinforcement and concrete strength of tested beams.

Group	Beam code name	Longitudinal bars	Stirrups	f_c (MPa)	f_{sp} (MPa)
R4	R4-20	4∅8	∅8/20 cm	20.96	2.89
	R4-15		∅8/15 cm		
	R4-10		∅8/10 cm		
R6	R6-20	6∅8	∅8/20 cm	24.59	3.33
	R6-15		∅8/15 cm		
	R6-10		∅8/10 cm		
RH4	RH4-20	4∅8	∅8/20 cm	26.56	3.04
	RH4-15		∅8/15 cm		
	RH4-10		∅8/10 cm		
RH6	RH6-20	6∅8	∅8/20 cm	24.90	3.42
	RH6-15		∅8/15 cm		
	RH6-10		∅8/10 cm		
RH8	RH8-20	8∅8	∅8/20 cm	27.39	3.09
	RH8-15		∅8/15 cm		
	RH8-10		∅8/10 cm		

The cement used in this experimental work was a locally manufactured general purpose ordinary Portland type cement. Sand with a high fineness modulus and coarse aggregates with a maximum size of 9.5 mm were used. The concrete mixture was made using cement, sand and crushed aggregates in a proportion 1:2.8:1.2, respectively, and water to cement ratio equal to 0.43. Also included in Table 1 are the concrete compressive and tensile strength values as deduced from supplementary compression and splitting tests, respectively.

The total length of the beams was common for all specimens equal to 1.60 m. Experimental setup and instrumentation are shown in Figure 1. All specimens were tested monotonically under pure torsion [14]. They were supported on two roller supports that allowed beam to twist and to elongate longitudinally at both ends. The load was applied on the ends of two steel arms fixed at the end parts of each tested beam, through a steel spreader. The end parts of the beams were properly over-reinforced so that they can bear without cracking the imposed loading. Cracking and, finally, failure localized at the middle part of each beam.

The load was imposed consistently in low rate and was measured by a load cell. The average angle of twist per unit length of the tested beams was estimated using two linear variable differential transducers with high accuracy. These two LVDTs measured the opposite deformations of each specimen as it rotates.

Initial elastic torsional stiffness, torque moment at cracking and ultimate torque moment, as measured from the tests, are presented in Table 2. Further, the entire torsional behaviour of the beams of groups R4, R6, RH4, RH6 and RH8 is also presented in Figures 2, 3, 4, 5 and 6, respectively, in terms of torque moment versus angle of twist per unit length experimental curves.

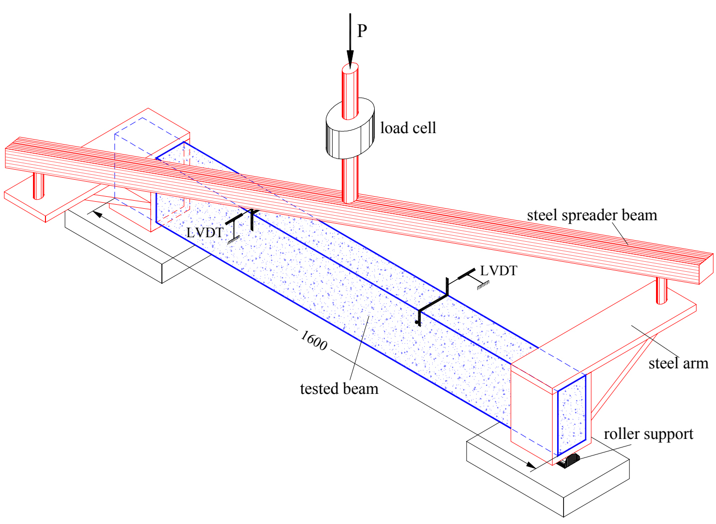

Figure 1: Test setup.

3 Behavioural model and comparisons with test results

In this study, for the prediction of the entire torsional behaviour of reinforced concrete elements, the combination of two different theories is adopted. The elastic till the first cracking part is described by a smeared crack analysis for plain concrete in torsion [10] and the post-cracking part is described by the well-known softened truss model [3, 4].

It is justified that for the elastic till the first cracking part the percentage of steel has a minor effect on the torsional response and reinforced concrete elements behave as plain concrete members [1]. Therefore, the analytical smeared crack model for plain concrete in torsion proposed by Karayannis [10] is applicable to reinforced concrete beams for the prediction of the first elastic part till the developing of concrete cracking. This approach is using an efficient numerical scheme for the torsional analysis of concrete that is initially based on the elastic theory and utilizes a special numerical technique properly modified to include the smeared cracking approach. The model is based on an analytical technique that employs constitutive relations expressed in terms of normal stress and crack width, for the behaviour of the crack process zones. Detailed derivation of the equations and the solution technique of this theory can be found in reference 10.

Table 2: Test results and analytical predictions of the tested beams.

Beam code name	Initial stiffness ($\times10^{-3}$ kNcm^2/rad)			Cracking torque (kNcm)			Ultimate torque (kNcm)		
	exp.	calc.	exp./calc.	exp.	calc.	exp./calc.	exp.	calc.	exp./calc.
R4-20	3771	3346	1.13	217.05	204.65	1.06	238.49	263.26	0.91
R4-15	3084		0.92	201.25		0.98	264.87	277.52	0.95
R4-10	3229		0.97	200.79		0.98	325.40	292.30	1.11
R6-20	3452	3624	0.95	240.12	235.89	1.02	287.28	288.75	0.99
R6-15	3557		0.98	259.63		1.10	318.38	322.81	0.99
R6-10	3442		0.95	264.60		1.12	374.22	346.65	1.08
RH4-20	5913	6500	0.91	322.00	337.82	0.95	394.76	439.99	0.90
RH4-15	6689		1.03	364.54		1.08	501.27	500.89	1.00
RH4-10	6186		0.95	418.78		1.24	583.42	525.01	1.11
RH6-20	6494	6292	1.03	378.70	379.97	1.00	481.14	444.21	1.08
RH6-15	7215		1.15	355.10		0.93	586.91	509.88	1.15
RH6-10	6755		1.07	372.10		0.98	661.63	541.61	1.22
RH8-20	7744	6596	1.17	345.27	343.39	1.01	503.69	462.20	1.09
RH8-15	7191		1.09	330.37		0.96	611.95	551.14	1.11
RH8-10	6658		1.01	368.30		1.07	694.97	603.72	1.15

Among the various theories available in the literature, researchers have a common consensus that the space truss models (the spalled and the softened truss model) are the most rational and powerful models for dealing with torsional problems. In the present study, the theory of the softened truss model is used [3], [4]. This method relies on solving three equilibrium and three compatibility equations along with the constitutive laws of an element taken from a member subjected to pure torsion. The equations of equilibrium and compatibility are based on the assumption that the material is continuous. Especially for the concrete in compression, it is considered that concrete struts strength is greatly reduced by the diagonal cracking caused by tension in the perpendicular direction (concrete softening). For further details refer to references 3 and 4.

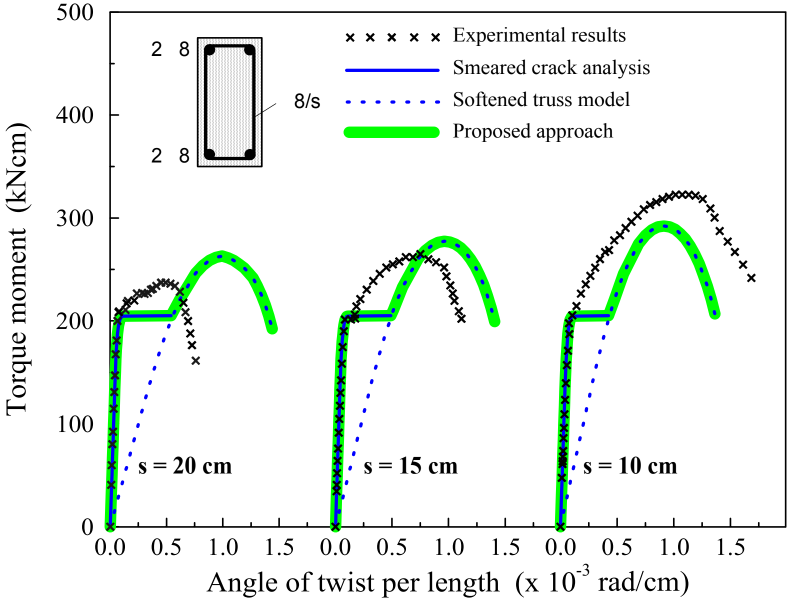

Figure 2: Experimental and calculated curves of the beams of group R4.

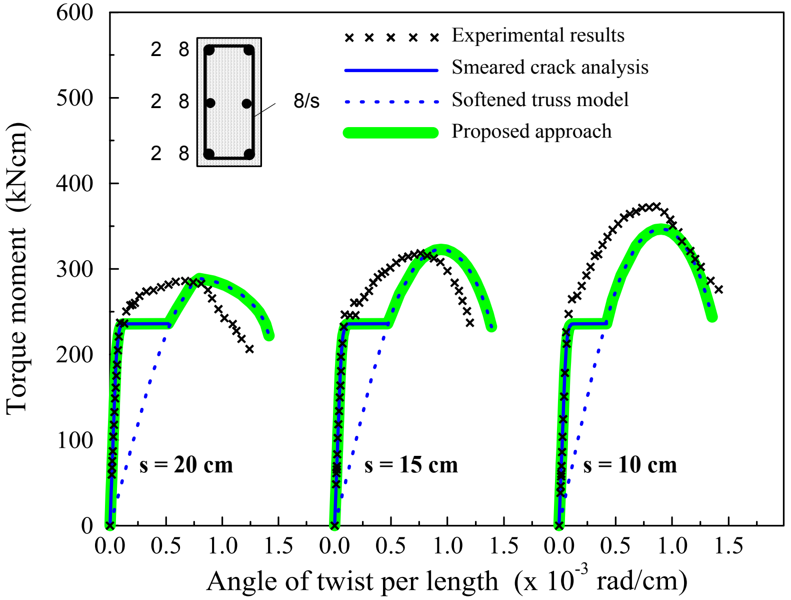

Figure 3: Experimental and calculated curves of the beams of group R6.

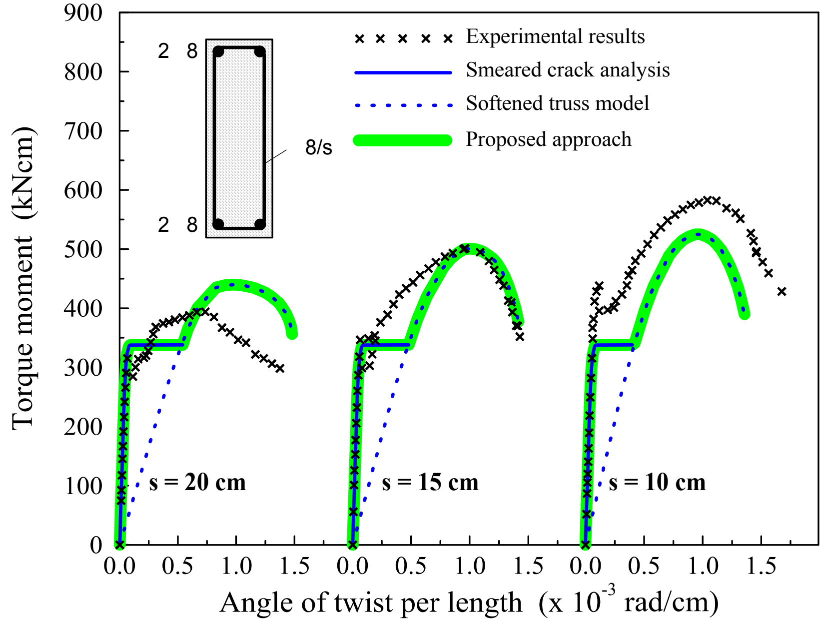

Figure 4: Experimental and calculated curves of the beams of group RH4.

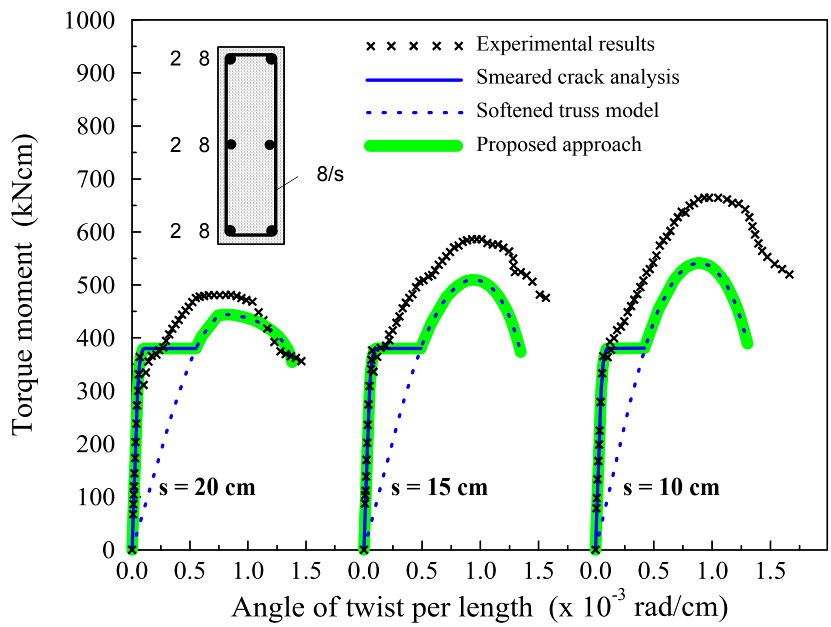

Figure 5: Experimental and calculated curves of the beams of group RH6.

Analyses for the prediction of the torsional behaviour of the tested beams using the proposed combined approach were performed. Smeared crack analysis was performed in order to determine the elastic response and the torque moment at cracking, whereas the softened truss model was performed for the prediction of the post-cracking response and the ultimate torque moment. The predicted values of the initial torsional stiffness, the torque moment at cracking and the ultimate torque moment are presented and compared with the measured ones in Table 2. Furthermore, analytical torque curves for the torsional behaviour of the beams of groups R4, R6, RH4, RH6 and RH8 are presented and compared with the experimental ones in Figures 2, 3, 4, 5 and 6, respectively.

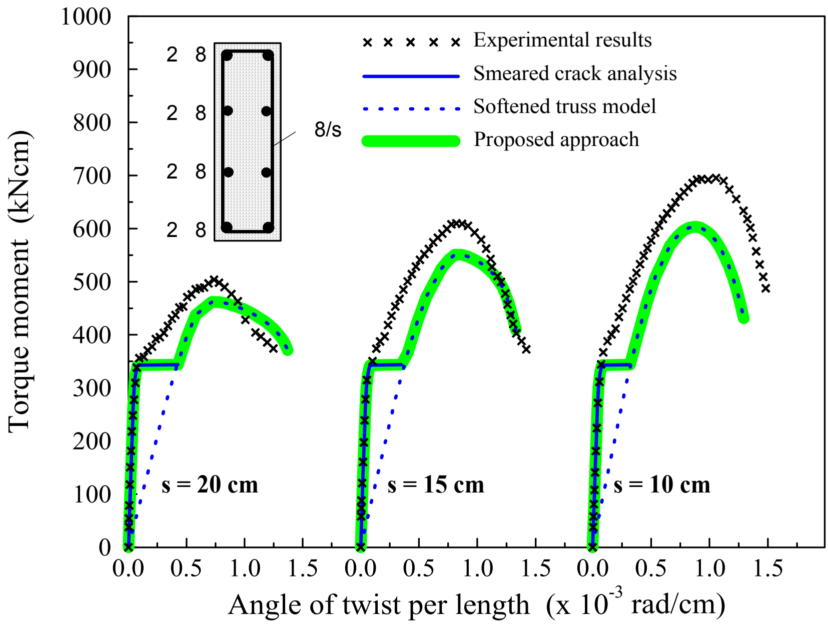

Figure 6: Experimental and calculated curves of the beams of group RH8.

4 Conclusions

The following concluding remarks are drawn from the results reported herein. Volume of the transversal and the longitudinal reinforcement significantly affects the torsional behaviour of the tested beams, as it was expected. Two distinct regions can be observed in a typical experimental torque – twist curve of the tested reinforced concrete beams. The different character of the response in these regions reveals the different nature of the load resisting mechanism in each case. In order to describe the entire torsional behaviour of the tested beams and based on the above observation, the combination of two different theories is adopted. For the estimation of the elastic till the first cracking response, a smeared crack analysis for plain concrete in torsion is used, whereas the prediction of the post-cracking behaviour is achieved using the softened truss model. Comparisons between the predicted torsional behaviour and the measured one showed that this combined approach yields realistic torsional curves for the entire response of the element and is capable to predict with satisfactory accuracy the initial torsional stiffness, the torque moment at cracking and the ultimate torsional strength.

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