WATER DEMAND SCALING LAWS AND SELF-SIMILARITY PROPERTIES OF WATER DISTRIBUTION NETWORKS

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ABSTRACT

The design of water distribution networks (WDNs) usually considers deterministic values of nodal water demand, calculated by multiplying the average water demand by an appropriate demand factor, which is the same for all nodes. Obviously, changes in the demand factor produce different, yet perfectly correlated, demand scenarios. Today's large availability of high-frequency water consumption monitoring allows describing water demand in statistical terms. The traditional deterministic approach, characterized by a perfect correlation between nodal demands, leads to an analytical dependency between the hydraulic heads in each of the nodes and the total flow entering the network. On the other hand, if we consider that the nodal demand is described by marginal probability distributions, differently correlated with each other, this result is still valid, but only for the mean. In this work, several scenarios have been generated through stratified random sampling (Latin hypercube sampling). The nodal water demand is described by Gamma probability distributions whose parameters are related to the type and number of users according to suitable scaling laws, derived from historical data sets. The results were obtained considering different types of users and different network topologies and highlighted the possibility of evaluating the mean function of the nodal hydraulic head vs the total entering flow based on the direct acyclic graph (DAG) of the network. Moreover, the dispersion of the data around the mean function was found to be dependent on the properties of the network: dimension and topological structure.

Keywords: scaling laws, water demand scenarios, self-similarity.

1 INTRODUCTION

water distribution network (WDN) sizing, calibration, and management largely depend on the water demand scenarios considered. When dealing with one of these issues, nodal demands are often not known beforehand: they must be measured or estimated. Measuring all nodal demands in an existing network is an unaffordable procedure, and not possible in the design phase.

Traditionally, deterministic water demand scenarios are estimated from annual average consumption measurements. These scenarios are properly scaled by a demand factor, depending on the size and socio-economic characteristics of the area covered by the WDN. Since they derive from a single multiplicative scaling factor, all the scenarios obtained from different demand factors are perfectly correlated with each other.

Different methods for generating stochastic demand scenarios have been proposed in the literature [1], [2], in order to overcome the conceptual limitations of this approach. In particular, the one proposed by Magini et al. [1], uses the observation of the scaling properties of the statistical moments of the demand probability distributions, deriving from the spatial aggregation of water users.

The hydraulic response of the network to a demand scenario is characterized by its values of pipe flow rates and nodal pressure heads. This highlights the mutual interaction between demand, network topology, and characteristics of the pipes. Consequently, this interaction allows predicting pressure at each node under given demand conditions. This allows

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envisaging possible critical issues in some parts of a WDN and accelerating numerical procedures in its hydraulic simulation.

This work presents some analytical evidence regarding the possibility to identifying the hydraulic behaviour of the WDN in stochastically perturbed scenarios with respect to an average demand scenario. In particular, the average scenario results to be adequate for the hydraulic description of the WDN. Finally, to verify the proposed approach, we propose an example on a WDN case from the literature, using synthetically generated stochastic demand scenarios.

2 DEMAND SCENARIOS

The total flow entering the network and, even more, nodal water demand, represent the most uncertain input parameters in the hydraulic modelling of WDNs. The estimation of these input parameters differs according to three different conditions: (1) a water-meter dataset is available, (2) a dataset measured from a similar system is used, or (3) no measured datasets are available. In the last case, the annual average demand scenario is usually considered [3], [4].

From this basic scenario, it is also possible to obtain peak demand scenarios, which are useful to determine production and distribution capacity and customers' metering in an existing network. Peak flow is a key factor for this purpose. In the absence of data, the usual approach to estimate the peak flow is to consider a peaking factor (PF). The PF is the ratio between the maximum water demand recorded in a given time interval and the average annual water demand. Increasing the average nodal flow rates by the same factor is equivalent to considering the common behaviours of network users and therefore the perfect correlation of their water demands. This approach does not allow the identification of the probabilistic dispersion of the nodal head that is consequent to the probabilistic dispersion of the water demand in the different scenarios.

However, internationally the use of the average daily water demand (ADD) and the PF is the most widespread method in professional practice. For example, to harmonize the European water supply standards, a peak flow probability method – originally used in Switzerland – was incorporated into EN 806-3. The American counties also propose this methodology, each adopting different peak factors [5]–[7].

However, many recommendations in current design standards have been carried over from previous standards unquestioned and without revision. Furthermore, since the genesis of these measures is not well documented or understood, disagreement between regional methodologies increased.

In this paper, we also want to highlight that no scientific evidence confirms that the annual average scenario, even when appropriately amplified through the PF factor, is the most precautionary scenario for the WDN.

2.1 Stochastic demand scenarios

It's worth pointing out that nodal water demand can greatly influence the model accuracy [8]–[10]. The deterministic approach, despite (and due to) its simplicity of use, does not allow considering the stochastic component of demand. In fact, water demand consists of a deterministic component – linked to the household plumbing fixtures and appliances – and a stochastic component linked to the unpredictable behaviour of users. It is necessary that the cross-correlation between users' water demands is considered in the stochastic component, in order to estimate the real peak scenario. Therefore, real-time estimation of nodal water demand is a major task for the real-time modelling of WDNs. Thanks to the development of smart metering technology, a large amount of measured data has gradually become available in recent years.

The observation of the scaling properties related to spatial and temporal aggregation of water demand measures allowed defining a method for the generation of stochastic demand scenarios [1].

For this purpose, it is assumed that the statistical moments of nodal demand (mean, variance, and cross-correlation) depend on type and number of users according to suitable scaling laws [11]. The development of the scaling laws assumes that the demand can be described by a homogeneous and stationary process, which implies that the aggregated users are of the same type (residential, commercial, industrial, etc.), and that the statistical properties of demand are constant over time.

The water demand statistical features of the single users are obtained from historical datasets and, together with the number of users in each node, represent the main input data for scenario generation. The most appropriate probability distribution to represent the nodal marginals may depend on the number of aggregated users and derives from historical data. In the literature, the most used probability distributions to describe the variability of the demand for aggregated users are the Log-normal, Gamma, and Weibull.

The complete procedure for scenario generation adopted in the application is detailed in Magini et al. [1]. It is based on sampling from the marginal distributions using the Latin hypercube sampling (LHS) [12]. In order to respect cross-correlation between nodal demands a combination of the NORTA model [13], and the Iman–Conover method [14] is used. The LHS is a "stratified sampling" technique that produces a better description of the input probability distribution with fewer iterations compared with a simple random sampling. The NORTA model is a two-step process, first transforming a multivariate normal vector Z into a multivariate uniform vector U, then transforming the latter into the desired input vector. The joint distribution of U is a copula, and any joint distribution can be represented as a transformation of a copula. To improve compliance with the network-demand correlation structure, the restricted pairing Iman–Conover technique is applied to NORTA results. It induces rank correlation by shuffling finite-size samples obtained from NORTA. The appropriate shuffling is determined by ranking the input samples the same as in a reference sample with the desired rank correlation. The demand scenarios obtained represent an improvement of the commonly used deterministic ones. In fact, the proposed model can generate scenarios in which, given the peculiar characteristics of the users, nodal water demand contains the stochastic component. However, empirical analyses of the generated scenarios show that the stochastic component is small, compared to the values assumed by the average demand. These nodal water demand distributions can be defined as "stochastic scenarios".

3 THE WATER DISTRIBUTION NETWORK

A WDN is mainly composed by the pipes transporting water from an inlet with known hydraulic head towards the demand nodes. A looped WDN can be represented by a directed graph that schematizes its geometric structure, planimetrically and altimetrically, and the capacitive characteristics of pipes, i.e., diameters, roughness, and lengths. In accordance with the characteristics of the considered WDN, each water demand scenario gives rise to water flows in pipes along definite directions, which define the orientation of the arcs of the graph.

For any scenario and size of the network, the corresponding oriented graph has no loops, as the hydraulic system is governed by the variational principle of minimum energy dissipation [15] and respects the uniqueness property of the solution [16]. A structure with these properties is called a direct acyclic graph (DAG).

Based on these assumptions, it can be stated that each WDN is characterized by its own basic DAG, which is linked to the average demand scenario.

The direction of the flows in the pipes varies for each scenario that occurs on a network, therefore for each scenario a specific DAG is defined, which can be known solving the hydraulic model of the WDN.

3.1 Water distribution network solvers

Numerous algorithms have been developed over time for solving the mixed set of linear and nonlinear equations governing the steady-state hydraulics of looped WDNs. The different approaches can be divided into local approaches (e.g., the Hardy Cross method [17]), which deal with one equation at a time, and global approaches, which solve simultaneously all the equations. For the second group, it is possible to make use of the Newton–Raphson (NR) linearization method or the linear theory (LT) successive approximation method to treat the system's nonlinear equations. One of the most important resolution methods for looped networks is the LT introduced by Wood and Charles [18]. Nevertheless, after more than thirty years, in the technical and scientific fields, the reference standard for the resolution of hydraulic networks is still the method introduced by Todini and Pilati [19], Todini [20] and Todini and Rossman [21]. Whichever the numerical solution is chosen, the nodal demand scenario represents a critical parameter, from which the response of the network drives.

3.2 The behaviour of WDNs in deterministic water demand scenarios

Given a network with l loops, p pipes and n demand nodes, without tanks inside, the set of resolutive equations can be expressed for the generic node i as follows:

$$Q_{tot} = \sum_{i=1}^{n} q_i , \tag{1}$$

$$\sum_{k=1}^{p} a_{i,j}\, Q_{k_{i,j}} + q_i = 0, \qquad i = 1,2,\dots,n, \tag{2}$$

$$\sum_{k=1}^{p} \beta_{j,k}\, r_k\, |Q_k|^{\alpha-1} Q_k = 0, \qquad j = 1,2,\dots,l. \tag{3}$$

This system of equations contains as unknowns only the pipe flow rates Q_k and it is determined. In the case of networks served by one or more reservoirs having the same hydraulic head H_0, the value of the hydraulic head H_i at each node i can be evaluated as follows:

$$H_i = H_0 - dH_i = \sum_{k=1}^{p} \gamma_{D_k}\, L_k\, Q_k |Q_k|^{\alpha-1}, \qquad i = 1,2,\dots,n. \tag{4}$$

Given a water demand scenario $D = (q_1, q_2, \dots, q_n)$, the total inflow Q_{tot} is automatically known, so it is possible to define the attenuation coefficient $ac_k = Q_k/Q_{tot}$ of the pipe flows and to link the head losses dH_i in eqn (4) to the total flow entering the WDN:

$$dH_i = \sum_{k=1}^{p} \gamma_{D_k} L_k \left(\frac{Q_k}{Q_{tot}} \cdot Q_{tot} \right)^{\alpha} = \sum_{k=1}^{p} \gamma_{D_k} L_k \left(ac_k \cdot Q_{tot} \right)^{\alpha}. \quad (5)$$

In a looped network, given a demand scenario D, the flow follows many different paths to reach the demand node along which the head loss dH_i is the same. For this reason, the parameters related to the flow rate can be enclosed by a single invariant parameter k at each demand node in the various scenarios:

$$dH_i = k_i \cdot {Q_{tot}}^{\alpha}. \quad (6)$$

As already mentioned, the reference scenario is the average scenario (subscript *m*), and in a deterministic approach all the other possible scenarios (subscript *s*) can be obtained by multiplying the former by a factor f. It follows that:

$$Q_{k,s} = f \cdot Q_{k,m}, \quad (7)$$

$$Q_{tot,s} = f \cdot Q_{tot,m}. \quad (8)$$

As a consequence, the head loss is scaled according to a power law:

$$dH_{i,s} = k_i \cdot f^{\alpha} \cdot {Q_{tot,m}}^{\alpha} = f^{\alpha} \cdot dH_{i,m}. \quad (9)$$

This equation shows that any multi-connected network has self-similarity properties between the total head losses in each node and an average total entering discharge. Then, for any deterministic scenario, the pressure head at each node is known from eqn (9) if the solution of the average demand scenario is known. Furthermore, eqn (7) shows that the deterministic scenarios are all represented by the reference DAG obtained for the average scenario.

3.3 Behaviour of WDNs in presence of stochastic water demand scenarios

The discussion just provided remains valid on average in the case that the demand scenarios are randomly perturbed with respect to the local average scenario of reference, respecting the scaling laws and the cross-correlation.

Each stochastic scenario can be compared with the average deterministic scenario obtained by considering the same total discharge value introduced into the network. These comparisons show that the differences in nodal demands between pairs of deterministic and stochastic scenarios are small, compared to the value of the demand.

This allows evaluating the trend of the nodal head even in non-deterministic scenarios. Moreover, due to the exiguous residual value, the formula proposed by Todini and Pilati [19] can be linearized as follows:

$$H_i = [A_{21}(D_{11})^{-1}A_{12}]^{-1}\{A_{21}(D_{11})^{-1}[(D_{11} - A_{11})]Q - A_{10}H_0 + q_i\}. \quad (10)$$

For this purpose, a first-order truncated Taylor expansion is performed on eqn (10):

$$\Delta dH_{i,r} = [A_{21}\, D_{11}^{-1}\, A_{12}]^{-1} \cdot \Delta q_{i,r}. \quad (11)$$

Consequently, the head loss of node i is given by the sum of the result obtained considering the related deterministic scenarios and the residual value of the head loss due to the stochastic scenario, obtained from the previous linearization.

$$dH_{i,r} = dH_{i,s} + \Delta dH_{i,r} = f^{\alpha} \cdot dH_{i,s} + \Delta dH_{i,r}\,. \tag{12}$$

The residuals obtained from the comparison between the local mean demands $q_{i,s}$ and the relative perturbed demand $q_{i,r}$ can be considered a random variable of assigned mean μ_{q_i} and variance $\sigma^2_{q_i}$. This random variable Δq is linked to the residual of the nodal head $\Delta dH_{i,r}$ by eqn (11). For the theory of random variables, the linear combination of random variables provides an expression of the expected value and variance for the derived variable. It is therefore possible to obtain the statistical parameters of the residual nodal heads by the demands':

$$E[\Delta dH_i] = [A_{21}\, D_{11}^{-1}\, A_{12}]^{-1} \cdot E[\Delta q_i]\,. \tag{13}$$

$$\begin{aligned}\sigma^2{}_{\Delta dHi} = [A_{21}\, D_{11}^{-1}\, A_{12}]^{-1} \cdot \{diag[\sigma^2{}_{\Delta dq_i}] + cov[\Delta Q_i, \Delta Q_j]\} \cdot \\ \cdot \{[A_{21}\, D_{11}^{-1}\, A_{12}]^{-1}\}^T\end{aligned} \tag{14}$$

Considering eqns (13) and (14), it is possible to define the approximate values of the nodal head losses and of the pipe flow due to the random realization of a stochastic scenario by assigning only a total inflow. For stochastic water demand scenarios, the equations proposed by Todini and Pilati [19] results as:

$$\begin{aligned}H_{i,r} = H_0 - dH_{i,s} + [A_{21}(D_{11})^{-1}A_{12}]^{-1} \cdot E[\Delta q_i] + \{[A_{21}(D_{11})^{-1}A_{12}]^{-1}\}^2 \cdot \\ \cdot \sigma^2{}_{\Delta dHi} + 2 \cdot [A_{21}(D_{11})^{-1}A_{12}]^{-1} \cdot cov[\Delta Q_i, \Delta Q_j].\end{aligned} \tag{15}$$

$$Q_{k,r} = Q_{k,m} - \left(D_{11,m}\right)^{-1}\left(A_{11,m} \cdot Q_{k,m} + A_{12} \cdot H_{i,r} + A_{10} \cdot H_0\right). \tag{16}$$

It can happen that in WDN with a non-redundant size the network DAG can undergo local reversals in some arcs. However, it is empirically demonstrated that the reference DAG remains the predominant DAG in analysed scenarios. Hence, the average scenario confirms its importance in defining the nodal behaviour, pointing out the filter effect of the network.

4 THEORICAL RESULT VALIDATION

The above theoretical analysis is verified in the follows through simulations on the WDN of Fossolo, a suburban area of Bologna, Italy (Fig. 1). The topology of this network was proposed by Bragalli et al. [22] and shared by the University of Exeter. The original WDN size has been modified so as to have a higher redundancy, as it occurs in real networks.

Water users are residential, and the annual average nodal demand is known. The number of users and the average peak hour demand in each node have been estimated using demand data by Bragalli et al. [22]. The sizing, the average annual demand and the estimated number of users are available in the appendix.

The demand factor and the correlation coefficient have been estimated through a preliminary analysis on historical data of a similar real residential dataset subjected to smart metering observation [23]. Hence, the statistical parameters for the generation of the stochastic scenarios were opportunely calibrated to comply with these values. The characteristics of the generated stochastic scenarios are presented in Table 1 and in Fig. 2.

The hydraulic simulation of the WDN allows obtaining the relation between nodal head loss dH_i and total inflow Q_{tot} (eqn (6)) in each node, both for the deterministic and stochastic scenarios. Fig. 3(a) reports the results for node (ID9): the red line refers to deterministic

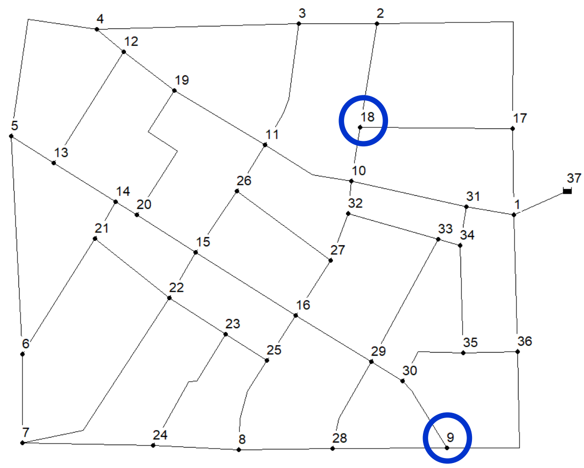

Figure 1: Fossolo network and node IDs.

Table 1: Statistical parameters used to generate stochastic scenarios.

Mean demand	Standard deviation	Cross-correlation
l/min	l/min	–
0.680	3.000	0.0016

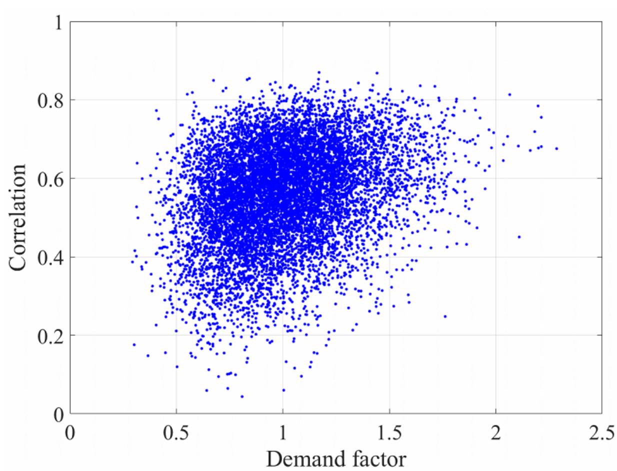

Figure 2: Demand factor and correlation of the generated stochastic scenarios.

scenarios, blue dots to stochastic scenarios. The red line of the deterministic scenarios is the mean curve of the stochastic scenarios. Fig. 3(b) reports the histogram of the residuals of the nodal head losses from the local mean value.

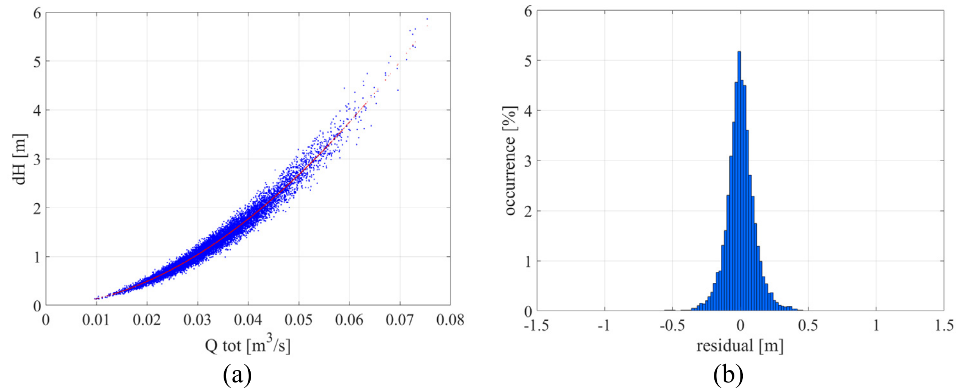

Figure 3: (a) Nodal head losses of one node (ID9) of Fossolo WDN in case of deterministic scenarios (red line) and stochastic scenarios (blue points); and (b) histogram of the residuals of the nodal head losses from the local mean value (deterministic trend).

Fig. 4(a) and (b) reports comparable results for node (ID18), which is characterized by higher variability (highlighted by the two histograms in Fig. 3(b) and Fig. 4(b)). However, the residuals from the deterministic curve keep a normal type of probability distribution, even if the variance seems higher.

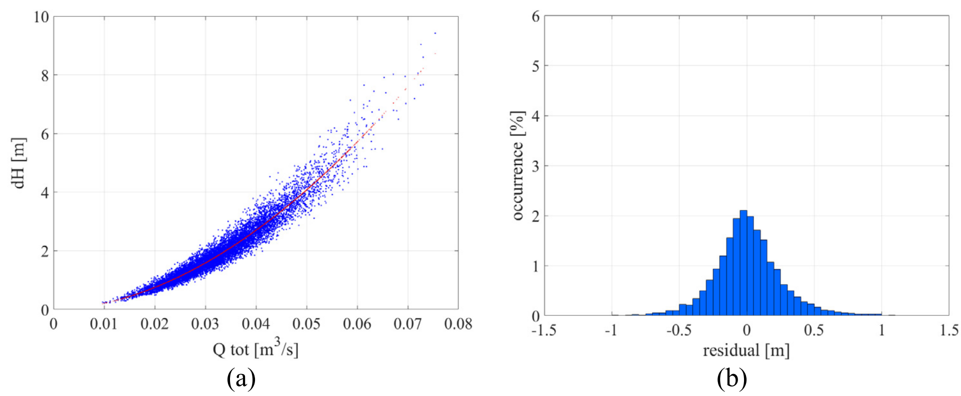

Figure 4: (a) Nodal head losses of one node (ID18) of the Fossolo WDN in case of deterministic scenarios (red line) and stochastic scenarios (green points); and (b) histogram of the residuals of the nodal head losses from the local mean value (deterministic trend).

The residual values of the head losses of the single node allow calculating the eqn (14) and obtaining the estimated value of the variance of the residuals head losses with respect to the residual nodal demands.

Fig. 5 shows a comparison between the value assumed from the variance evaluated on the residuals obtained from the simulation of ten thousand stochastic scenarios, and the one estimated by solving the eqn (14). This figure shows the relation between the two variances with respect to each node, hence proving that the linearization provides with a sufficient degree of accuracy the differences within nodal head losses from their expected deterministic value.

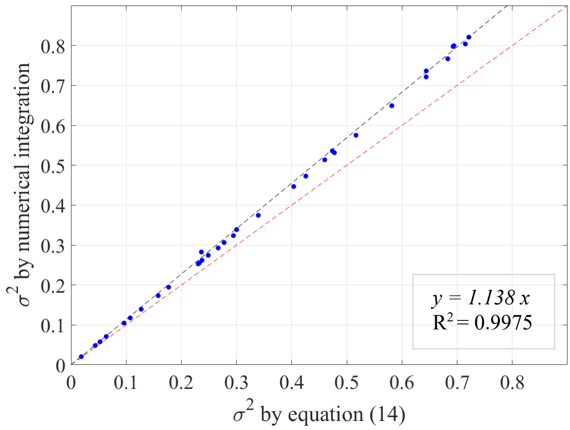

Figure 5: Comparison between the nodal values of variance, that is the ones from linearization and those calculated for the 10,000 stochastic scenarios.

Fig. 5 shows a slight underestimation (13.8%) of the theoretical values compared to the empirical ones. This underestimation is due to the effect of linearization.

5 CONCLUSION

In this paper the hydraulic behaviour of the WDNs is presented as a function of the average demand scenario and network topology.

The theoretical formulation was validated considering both deterministic scenarios and stochastically generated stochastic scenarios.

The first finding is that considering any deterministic scenario, the pressure at each node can be evaluated if the pressure of an average demand scenario is known. Furthermore, all the deterministic scenarios can be represented by a reference DAG obtained considering the average scenario. This conclusion is also generally valid for randomly perturbed demand scenarios, as long as they respect the scaling laws and the cross-correlation, with respect to the reference average scenario.

By comparing deterministic and stochastic scenarios having the same total inletting discharge, we showed that the relative differences in nodal demands are small. This result allows evaluating the trend of the nodal head even in non-deterministic scenarios.

Furthermore, estimating pipe flow rates and nodal head losses can be considered as an optimal starting point for the resolution of the equation proposed by Todini and Pilati to increase the speed of convergence in the numerical solution of the hydraulic model.

From a practical point of view, the results achieved make it possible to provide an objective criterion for evaluating the pressures detected in each node based only on the overall flow rate value. This allows operational decisions to be made in real time.

NOTATION

The following symbols are used in the paper:

A_{10} = $[p, n_0]$ incidence matrix relating pipes to known head nodes;
A_{11} = $[p, p]$ diagonal matrix, whose generic term is $A_{11}(k, k) = r_k|Q_k|^{\alpha-1}$;
A_{12} = $A^T{}_{21} = [p, n]$ incidence matrix relating pipes to unknown head nodes;
$a_{i,j}$ = generic element of A_{12};
α = exponent. Its value is 1.852 when using the Hazen–Williams equation;
B = [l, p] incidence matrix relating pipes to loops;
$\beta_{l,k}$ = generic element of B;
D_{11} = $[p, p]$ diagonal matrix of derivatives of A_{11} with respect to Q;
H_0 = $[n_0]$ length vector of known fixed head nodes;
i, j, k = generic index;
γ_{D_k} = $[p]$ vector of coefficient that depends by diameter and roughness of pipes;
l = number of independent loops;
L_k = $[p]$ vector of length of pipes;
n_i = number of pipes connected to node i;
n = number of nodes with unknown head;
n_0 = number of nodes with known head;
p = number of pipes;
Q = $[p]$ vector of pipe flows;
q = $[n]$ vector of nodal demands;
q_i = known nodal demand at node i;
$Q_{k_{i,j}}$ = flow in the generic pipe $k_{i,j}$ connected to node i;
Q_{tot} = total instantaneous discharge;
r_k = $[p]$ vector of coefficients that depends on the dimensions used on the pipe like diameter, roughness and length.

APPENDIX

Table A1: Average demand at peak hour in each node and relative number of users.

Node's ID	Peak nodal demand (l/s)	Number of users	Node's ID	Peak nodal demand (l/s)	Number of users	Node's ID	Peak nodal demand (l/s)	Number of users
1	0.49	126	13	1.16	297	25	0.77	198
2	1.04	267	14	0.54	141	26	1.69	435
3	1.02	261	15	1.10	282	27	1.42	366
4	0.81	210	16	1.21	312	28	0.30	78
5	0.63	162	17	1.27	327	29	0.62	159
6	0.79	204	18	2.02	519	30	0.54	141
7	0.26	69	19	1.88	483	31	0.90	231
8	0.58	150	20	0.93	240	32	1.03	264
9	0.54	141	21	0.96	246	33	0.77	198
10	1.11	285	22	0.97	249	34	0.74	192
11	1.75	450	23	0.86	222	35	1.16	297
12	0.91	234	24	0.67	174	36	0.47	123

Table A2: Pipes' size: Internal diameter (D) (mm), Length (L) (m), Roughness (C) ($mm^{1/3} \cdot s^{-1}$).

ID	D	L	C	ID	D	L	C	ID	D	L	C
1	144.8	132.8	130	21	113	84.0	140	41	45.2	203.8	150
2	67.8	374.7	130	22	126.6	49.8	130	42	45.2	248.1	150
3	45.2	119.7	140	23	144.8	78.5	130	43	45.2	65.2	145
4	67.8	312.7	140	24	81.4	99.3	140	44	57	210.1	145
5	45.2	289.1	150	25	99.4	82.3	140	45	67.8	147.6	145
6	67.8	336.3	145	26	57	147.5	140	46	67.8	103.8	140
7	67.8	135.8	145	27	67.8	197.3	140	47	45.2	211.0	140
8	57	201.3	145	28	113	83.3	140	48	81.4	75.1	140
9	57	132.5	145	29	45.2	113.8	140	49	113	180.3	150
10	45.2	144.7	145	30	81.4	80.8	140	50	57	149.1	140
11	45.2	175.7	145	31	45.2	341.0	140	51	57	215.1	130
12	81.4	112.2	145	32	57	77.4	150	52	81.4	144.4	130
13	99.4	210.7	130	33	45.2	112.4	150	53	99.4	34.7	130
14	162.8	75.4	130	34	45.2	37.3	145	54	126.6	59.9	130
15	99.4	181.4	130	35	45.2	108.9	145	55	67.8	165.7	130
16	67.8	147.0	150	36	81.4	182.8	150	56	99.4	120.0	130
17	81.4	162.7	150	37	113	136.0	140	57	67.8	83.2	130
18	45.2	99.6	150	38	99.4	56.7	140	58	203.4	1.0	130
19	57	53.0	150	39	81.4	124.1	130				
20	67.8	163.0	140	40	144.8	234.6	130				

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