

Reducing seismic stress on buildings through inertia limiters at floor level

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Abstract

A seismic stress control method based on the disconnection of non-structural floor masses during strong earthquakes is investigated in the present paper. Rigid-plastic devices (very stiff in the elastic range, in practice), which may plastically yield when a pre-set level of seismic load is reached, are assumed to connect structural and non-structural floor masses. Such rigid-plastic connectors act as inertia limiters so reducing seismic loads on building floors. In addition, they also act as seismic energy dissipaters due to plastic dissipation which is triggered at the connector level. The method may lead to significant seismic stress reductions. The price to be paid, however, might be large relative displacements of the disconnected mass. By referring to a single-storey frame and to recorded ground motions, a numerical investigation is performed in the present paper showing that the disconnected mass usually experiences rather small relative displacements (generally below 10 cm), even when high stress reductions are achieved. For practical purposes, a useful procedure to obtain a quick – though approximate – evaluation of the peak relative displacement of the disconnected mass from the earthquake rigid-plastic pseudo-spectrum is also provided in the paper.

Keywords: seismic stress control, rigid-plastic connectors, floor mass disconnection, inertia limiters.

1 Introduction

During a ground motion, inertia loads act on building floors. They are proportional to the ground acceleration as well as to the floor mass. In particular, the larger the floor mass is, the higher the inertia loads -and consequently the



stress on building members- become. Based on disconnecting a portion of active floor mass during strong earthquakes, a method of seismic stress control was proposed in [1] and investigated in [2]. In this method, rigid-plastic devices are interposed between structural and non-structural floor masses. Assumed to be very stiff in the elastic range, such connectors may plastically yield as soon as a pre-set level is reached by the horizontal loads acting on the floor masses. The rigid-plastic connectors, in fact, behave both as *inertia limiters* since they reduce the active mass and as *energy dissipaters* since plastic dissipation is involved when they yield. This dual action may reduce –even quite drastically- seismic stress, as also shown in [1, 2].

As a quid pro quo, however, high relative displacements of the disconnected mass might be required to achieve substantial reductions in stress, which would be a high price to pay. By referring to a single-story frame and to two strong recorded earthquakes, the present paper investigates this aspect by means of a wide non-linear numerical analysis. It is found that quite low relative displacements are actually experienced by the disconnected mass, even when one half of the floor mass is disconnected and stress reductions of more than 50% are achieved.

For the purpose of practical application of the method, a simple procedure is provided in the paper which leads to an estimation of the peak relative displacement of the disconnected mass under a given earthquake. This procedure is based on the earthquake rigid-plastic pseudo-spectrum, which is a single curve diagram introduced in [3], providing the peak displacements of rigid-plastic oscillators as a function of the ratio between yield load and mass of the oscillator, cf. [3–7].

It can be noted, finally, that other approaches can be found in the available literature which somehow isolate active mass for the purpose of seismic stress control, cf. e.g. [8–12]. The present method is, however, different from the above-mentioned. Its peculiarity lies in the fact that it reduces the active floor mass as the vibration level reaches certain limits, contemporarily making the disconnected mass work towards the plastic dissipation of seismic energy.

2 Rigid-plastic disconnection of non-structural floor mass in a single-story frame

A shear- type single-story frame of mass M and lateral stiffness k_l is considered in Figure 1(a) and referred to as *Frame A*. The maximum stress produced by a given ground motion $\ddot{u}_g(t)$ on pillars of this frame can be obtained by applying the following horizontal static load to mass M :

$$F_{SA} = k_l u_{E\max}(T, \xi) . \quad (1)$$

Here $u_{E\max} = u_{E\max}(T, \xi)$ denotes the peak elastic displacement of M under the given earthquake. It is a function both of the natural period T and of the

damping ratio ξ of the system. The value of $u_{E\max}(T, \xi)$ can be obtained by solving the motion linear equation:

$$\ddot{u} + \frac{4\pi\xi}{T} \dot{u} + \frac{4\pi^2}{T^2} u = -\ddot{u}_g(t). \tag{2}$$

In the above equation $u = u(t)$, $\dot{u} = \dot{u}(t)$ and $\ddot{u} = \ddot{u}(t)$ represent displacement, velocity and acceleration of M , respectively (all relative to the ground). Of course, when the earthquake response spectrum is available, the value of $u_{E\max}$ can be derived straightforwardly from it.

By taking into account that $T = 2\pi\sqrt{M/k_1}$, the seismic load given by equation (1) can also be put in the form:

$$F_{SA} = \frac{4\pi^2}{T^2} M u_{E\max}, \tag{3}$$

which shows the dependence of F_{SA} on the floor mass M . In particular, eqn. (3) suggests that a reduction in the floor mass might be beneficial to reduce seismic load and, therefore, seismic stress on the frame. Yet, it should be considered that as M decreases the natural period T of the system decreases too and the value of $u_{E\max}$ might even increase. Therefore, the effectiveness of a reduction in the floor mass for the purpose of stress control needs to be carefully assessed.

To this end, a comparison between the seismic responses of the two frames depicted in Figure 1 may be helpful. The first one, referred to as *Frame A*, is a linear single-story shear-type frame. Under a given earthquake, its motion is governed by eqn. (2) and the maximum stress on its pillars may be evaluated by applying the static load, given by eqn. (1), to mass M .

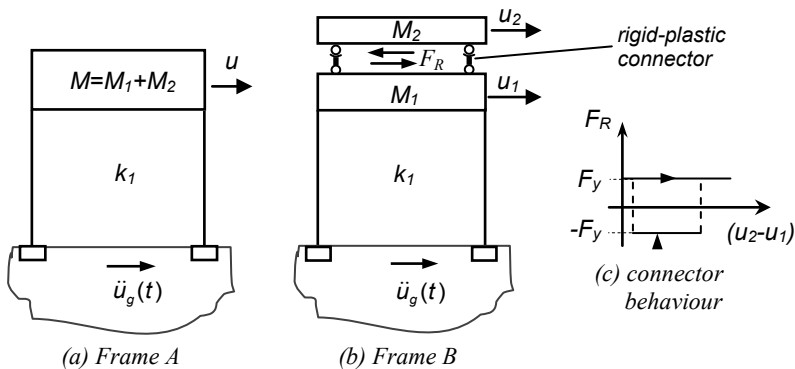


Figure 1: (a) Shear-type elastic frame; (b) non-linear frame with connectors; (c) rigid-plastic behavior of connectors.

On the contrary, the second frame in Figure 1, referred to as Frame B, is a non-linear system obtained from *Frame A* by sharing its total mass M into two portions, say M_1 and M_2 , and by inserting non-linear connectors between them. The connectors are assumed to be very stiff in the elastic range (practically rigid) and to plastically yield when a pre-set absolute value of force, say F_y , is reached. The behavior of such connectors is shown in Figure 1c.

Under a given ground acceleration $\ddot{u}_g(t)$ the motion equations of *Frame B* can be written as follows, cf. [2]:

elastic range

$$\begin{aligned} &\text{if } |\ddot{u}_2 + \ddot{u}_g| < \frac{F_y}{M_2} \text{ and } (\dot{u}_2 - \dot{u}_1) = 0 \\ \text{or if } |\ddot{u}_2 + \ddot{u}_g| = \frac{F_y}{M_2}, & (\dot{u}_2 - \dot{u}_1) = 0 \text{ and } \text{sign}(\dot{u}_2) \neq \text{sign}(u_2) \\ &\begin{cases} \ddot{u}_1 + \frac{4\pi\xi}{T}\dot{u}_1 + \frac{4\pi^2}{T^2}u_1 = -\ddot{u}_g(t) \\ u_2 = u_1 \end{cases} \end{aligned} \tag{4a}$$

plastic range:

$$\begin{aligned} &\text{if } |\ddot{u}_2 + \ddot{u}_g| = \frac{F_y}{M_2}, (\dot{u}_2 - \dot{u}_1) = 0 \text{ and } \text{sign}(\dot{u}_2) = \text{sign}(u_2) \text{ or if} \\ &(\dot{u}_2 - \dot{u}_1) \neq 0 \\ &\begin{cases} \ddot{u}_1 + \frac{4\pi\xi_1}{T_1}\dot{u}_1 + \frac{4\pi^2}{T_1^2}u_1 - \text{sgn}(u_2)\frac{F_y}{M_1} = -\ddot{u}_g(t) \\ \ddot{u}_2 + \text{sgn}(u_2)\frac{F_y}{M_2} = -\ddot{u}_g(t) \end{cases} \end{aligned} \tag{4b}$$

In the above equations $\text{sign}(\cdot)$ denotes the sign of (\cdot) , while the meaning of other symbols can be taken from Figure 1(b). Parameters T_1 and ξ_1 appearing in the same equations are given by:

$$T_1 = 2\pi\sqrt{\frac{M_1}{k_1}} = T\sqrt{\frac{M - M_2}{M}} = T\sqrt{1 - \rho_M}, \tag{5}$$

$$\xi_1 = \frac{\xi}{\sqrt{\frac{M - M_2}{M}}} = \frac{\xi}{\sqrt{1 - \rho_M}}, \tag{6}$$



respectively. Here ρ_M denotes the ratio between the disconnected mass M_2 and the total mass M . In the following, it will be referred to as the *mass ratio*. It can be helpful to introduce also the *plastic ratio* ρ_y as defined by, cf. [2]:

$$\rho_y = \frac{F_y}{F_{E2}} = \frac{F_y}{M_2 a_{E\max}(T, \xi)}, \tag{7a}$$

where $a_{E\max} = a_{E\max}(T, \xi)$ is the peak elastic acceleration of an oscillator of period T and damping ratio ξ under the given ground motion. The quantity $F_{E2} = M_2 a_{E\max}$, in fact, represents the force that the earthquake would exert on mass M_2 should connection between masses M_1 and M_2 be perfectly rigid. Therefore, the *plastic ratio* ρ_y quantifies the plastic involvement of the rigid-plastic connectors during the considered earthquake. Since $a_{E\max} \cong \omega^2 u_{E\max}$, cf. [13], ω being the natural angular frequency given by $\omega = 2\pi/T$, the plastic ratio may also be expressed as:

$$\rho_y = \frac{F_y}{M_2} \frac{1}{\omega^2 u_{E\max}}. \tag{7b}$$

As a function of ρ_M and ρ_y , eqns. (4a)–(4b) may be re-written as:

elastic range

$$\begin{aligned} \text{if } |\ddot{u}_2 + \ddot{u}_g| < \rho_y \omega^2 u_{E\max} \text{ or } |\ddot{u}_2 + \ddot{u}_g| = \rho_y \omega^2 u_{E\max} \text{ and } \text{sign}(\dot{u}_2) \neq \text{sign}(u_2) \\ \begin{cases} \ddot{u}_1 + \omega \xi \dot{u}_1 + \omega^2 u_1 = -\ddot{u}_g(t) \\ u_2 = u_1 \end{cases} \end{aligned} \tag{8a}$$

plastic range:

$$\begin{aligned} \text{if } |\ddot{u}_2 + \ddot{u}_g| = \omega^2 \rho_y u_{E\max}, (\dot{u}_2 - \dot{u}_1) = 0 \text{ and } \text{sign}(\dot{u}_2) = \text{sign}(u_2) \text{ or if } \\ (\dot{u}_2 - \dot{u}_1) \neq 0 \end{aligned}$$

$$\begin{cases} \ddot{u}_1 + \frac{\omega \xi}{(1 - \rho_M)} \dot{u}_1 + \frac{\omega^2}{(1 - \rho_M)} u_1 - \text{sgn}(u_2 - u_1) \frac{\omega^2 \rho_M \rho_y}{(1 - \rho_M)} u_{E\max} = -\ddot{u}_g(t) \\ \ddot{u}_2 + \text{sgn}(u_2 - u_1) \omega^2 \rho_y u_{E\max} = -\ddot{u}_g(t) \end{cases} \tag{8b}$$

The above equations show that the response of *Frame B* to a given earthquake depends on four parameters, namely ρ_M , ρ_y , ξ and ω (or T). It should be considered, in fact, that for a given earthquake, $u_{E\max}(T, \xi)$ is a known quantity which can be derived from the earthquake response spectrum.



By solving eqns. (8a)–(8b), the maximum displacement u_{1max} reached by mass M_1 in *Frame B* during the given earthquake can be calculated. As a function of it, the maximum seismic stress on pillars of *Frame B* can finally be obtained by applying the following horizontal static load to mass M_1 :

$$F_{SB} = k_1 u_{1max} (T, \xi, \rho_M, \rho_y). \tag{9}$$

If u_{1max} is lower than $u_{E max}$, the seismic load F_{SB} , as given by eq. (9), will be lower than F_{SA} , as obtained from eqn. (1). When this happens the proposed method of plastic disconnection of floor mass becomes effective in reducing seismic stress on frame pillars. By referring to two strong earthquakes and to *Frame A* and *Frame B* depicted in Figure 1, a numerical investigation has been performed showing that condition $u_{1max} < u_{E max}$ is generally met. Some results of this investigation are given in Figure 2, where the peak displacements $u_{E max}$ and u_{1max} are plotted together against the natural period T of *Frame A*. For a more

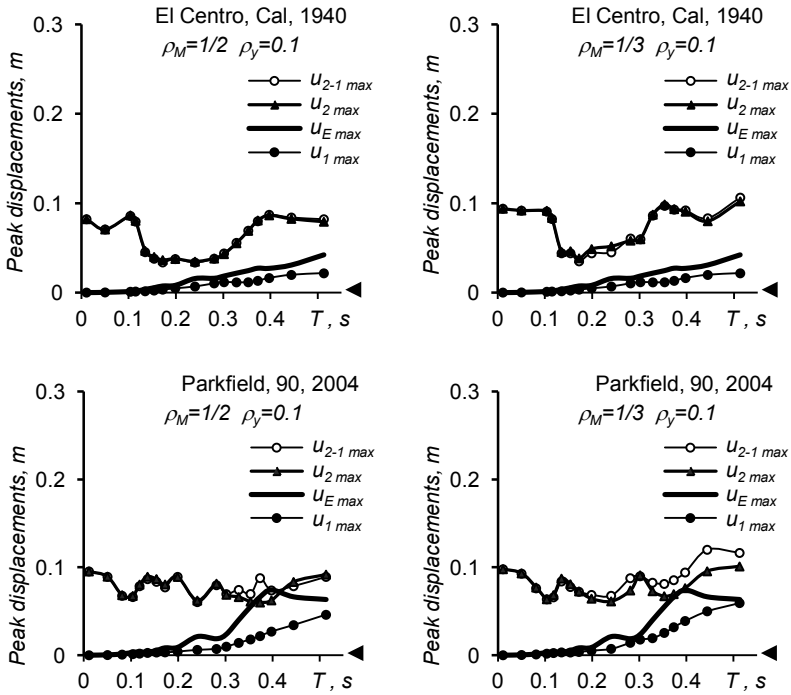


Figure 2: Peak displacements of masses of *Frame A* and *Frame B* under two different strong earthquakes, for $\rho_y = 0.1$ and two values of ρ_M .



comprehensive comparison, both the peak displacement of mass M_2 with respect to the ground, say $u_{2\max}$, and the peak relative displacement of M_2 with respect to M_1 , namely $u_{2-1\max} = \max|u_2(t) - u_1(t)|$, are also plotted in Fig. 2. It can be noted that the values of $u_{2\max}$ and $u_{2-1\max}$ are generally very close to each other.

3 Seismic stress reduction due to plastic mass disconnection

A comparison between eqn. (1) and eqn. (9) highlights that effective seismic stress control may be achieved as long as the maximum displacement $u_{1\max}$ of mass M_1 in *Frame B* is lower than the maximum displacement $u_{E\max}$ of mass M in *Frame A*. To assess the effectiveness of the proposed method, therefore, the following parameter can be helpful, cf. [2]:

$$\Delta_S = \frac{F_{SB} - F_{SA}}{F_{SA}} = \frac{u_{1\max} - u_{E\max}}{u_{E\max}} = \frac{u_{1\max}}{u_{E\max}} - 1. \tag{10}$$

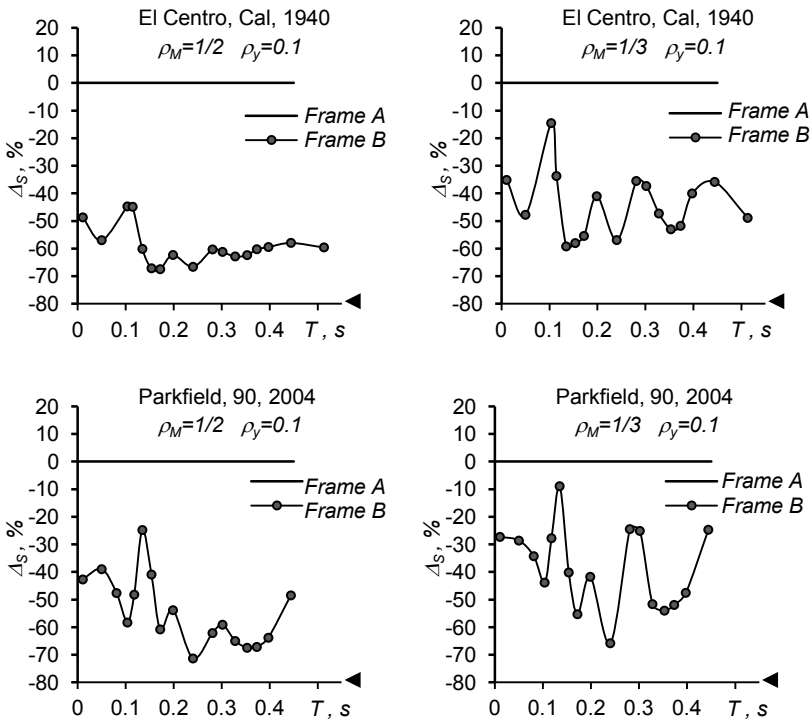


Figure 3: Stress reduction factor versus the natural period T of *Frame A*. The diagrams refer to the same instances considered in Figure 2.



It gives the reduction in seismic loads. Due to proportionality between loads and stress, it also provides the seismic stress reduction and can be referred to as the *stress reduction factor*. Should the displacement ratio $u_{1\max}/u_{E\max}$ be lower than one, the stress factor Δ_F becomes negative, so quantifying, in fact, the stress reduction percentage. Relevant to the same instances presented in Figure 2, the stress reduction factor Δ_S is plotted in Fig. 3.

Diagrams in Figure 3 show that a stress reduction of up to 55% may be reached when one third of the floor mass is disconnected ($\rho_M = 1/3$) and a low enough plastic limit is set, namely $\rho_y = 0.1$. Should one half of the mass be disconnected ($\rho_M = 1/2$) the seismic stress reduction may even rise to 70%. A more comprehensive collection of results, relevant to other values of ρ_y and ρ_M , may be found in [2].

4 Peak displacements of the disconnected mass

Results given in Figure 3 highlight that, generally, the greater the disconnected mass the lower the seismic stress on pillars of *Frame B* will be. However, as the peak displacement $u_{1\max}$ of mass M_1 of *Frame B* becomes lower with respect to displacement $u_{E\max}$ of mass M in *Frame A* -so reducing seismic stress on pillars- the peak displacement of mass M_2 is expected, on the contrary, to rise higher and higher. Of course, too high relative displacements of M_2 would make it inconvenient to apply the present method in practice.

Nonetheless, the results given in Figure 2, together with those given in Figures 5–6, show that the peak relative displacements of mass M_2 are generally small (lower than 10 centimeters in the considered instances), even when strong earthquakes are considered and high stress reductions are achieved (as can also be inferred by comparing Figure 2 and Figure 3).

5 Estimating the peak displacements of the disconnected mass from the earthquake rigid-plastic pseudo-spectrum

To assess in advance whether the present method of rigid-plastic disconnection of floor mass can be advantageously applied in practice, a quick estimate of both the stress reduction which can be achieved and the maximum displacement which will be attained by the disconnected mass M_2 can be useful. A simple procedure to obtain an estimate of the stress reduction from the earthquake elastic response spectrum is provided in [2]. While, a swift way of estimating the peak relative displacement of mass M_2 from the rigid-plastic pseudo spectrum of the earthquake will be provided in this section.



Under a given ground acceleration $\ddot{u}_g(t)$, the peak displacement of a rigid-perfectly-plastic oscillator of mass M_2 and yield load F_y (absolute value) can be obtained by solving the following motion equations:

$$\text{if } |\ddot{u}_g| < \frac{F_y}{M_2} \text{ and } \dot{u}_2 = 0 \quad \text{no motion} \quad (11)$$

$$\text{if } |\ddot{u}_g| \geq \frac{F_y}{M_2} \text{ and } \dot{u}_2 = 0 \quad \text{or if } \dot{u}_2 \neq 0 \quad \ddot{u}_2 - \text{sgn}(\ddot{u}_g) \frac{F_y}{M_2} = -\ddot{u}_g(t) \quad (12)$$

Here, $u_2 = u_2(t)$, $\dot{u}_2 = \dot{u}_2(t)$ and $\ddot{u}_2 = \ddot{u}_2(t)$ are relative displacement, velocity and acceleration of mass M_2 . The above equations show that, under a given earthquake, the rigid-plastic motion depends only on the ratio $a_y = F_y / M_2$ which can be referred to as the oscillator *yield acceleration*, cf. [3–6]. As a function of a_y , the peak displacements of the rigid-plastic oscillator, say u_{\max}^{RP} , may be collected in a diagram called *rigid-plastic pseudo-spectrum*, cf. [3–7]. The rigid-plastic pseudo-spectra of the two earthquakes considered in the present investigation are given in Fig. 4 (where g denotes the gravity acceleration).

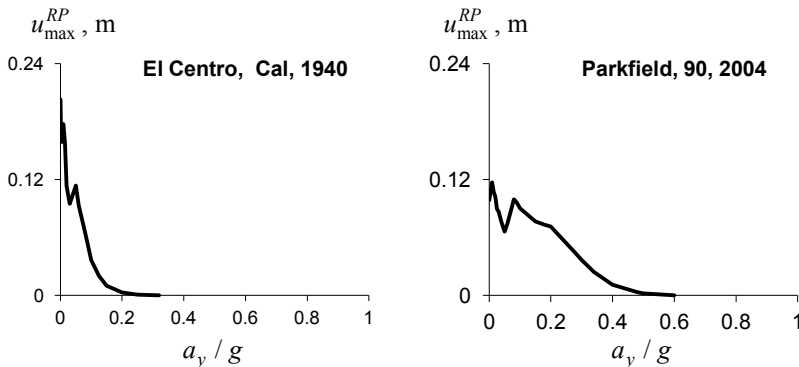


Figure 4: Rigid-plastic pseudo-spectra of the considered earthquakes.

The peak displacement u_{\max}^{RP} as taken from the rigid-plastic pseudo-spectrum for a given a_y , can be exploited to estimate the peak relative displacement $u_{2-1\max} = u_{2-1\max}(\rho_y, \rho_M, T, \xi)$ of mass M_2 of *Frame B*. In view of eq. (7b), in fact, a_y can also be put as a function of ρ_y, T and $u_{E\max}(T, \xi)$:

$$a_y = \rho_y \frac{4\pi^2}{T^2} u_{E\max}(T, \xi). \quad (13)$$



For each given value of a_y , the value of u_{2-1max} may be estimated as follows:

$$\text{for } T < 0.3s \quad u_{2-1max} = u_{MAX}^{RP}(a_y) \quad (14a)$$

$$\text{for } T \geq 0.3s \quad u_{2-1max} = u_{MAX}^{RP}(a_y/2) \quad (14b)$$

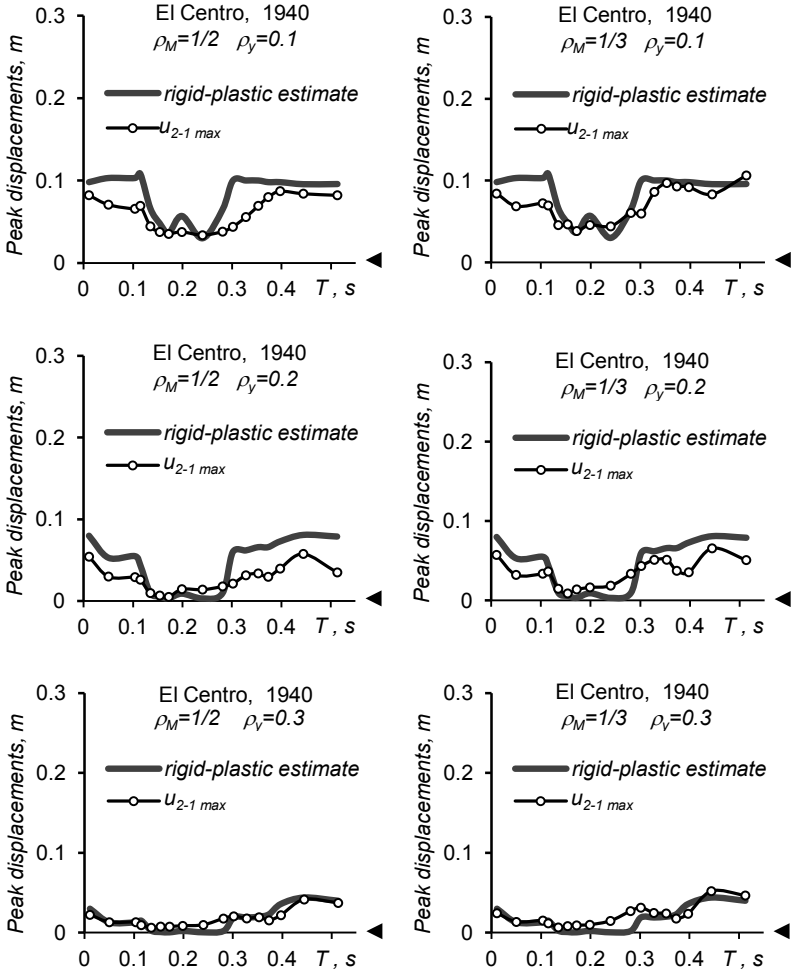


Figure 5: Calculated and estimated peak relative displacements of mass M_2 under the 1940 El Centro earthquake. The estimated values are obtained from the rigid-plastic spectrum, by applying eqns (14a)–(14b).

Figures 5–6 show that the rigid-plastic estimate, as obtained from the earthquake rigid-plastic pseudo-spectrum by means of relations (14a)–(14b), is rather good, whatever the earthquake, the value of ρ_y , ρ_M or T .

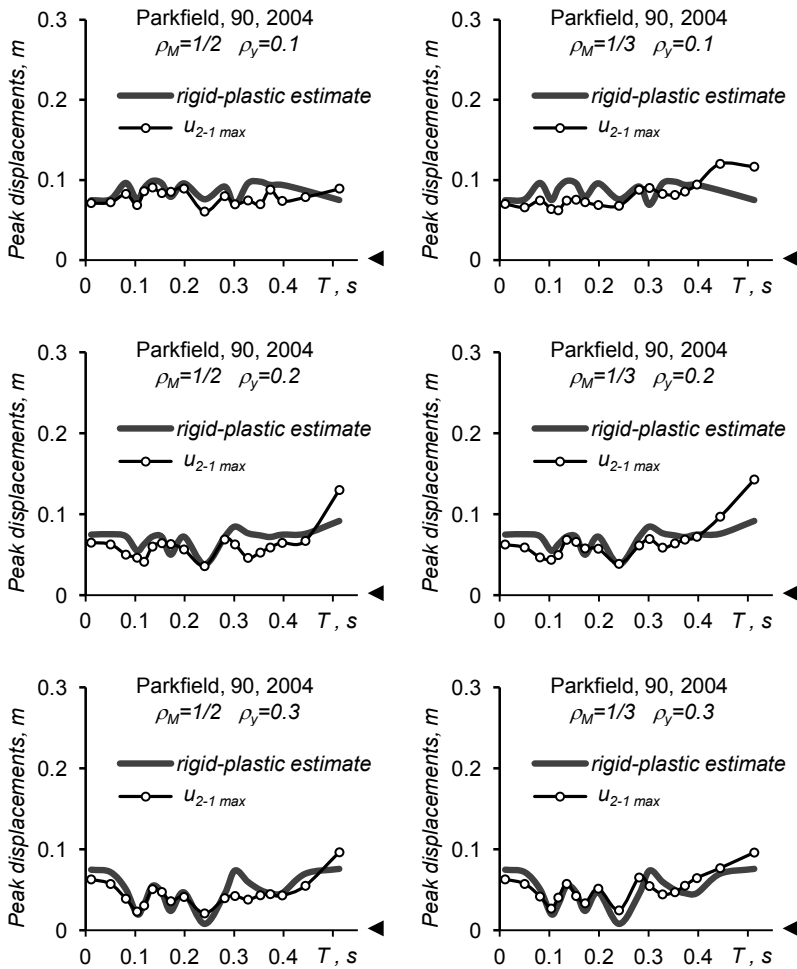


Figure 6: Diagrams analogous to those in Figure 4, but relevant to the Parkfield 2004 earthquake.

6 Conclusions

A seismic stress control method based on the rigid-plastic disconnection of building non-structural floor masses is assessed in the paper. By means of a non-linear numerical analysis, the displacement demand on the disconnected mass of a single-story frame is assessed for different earthquakes. The results show that,



despite the drastic stress reduction which may be achieved, rather small relative displacements are experienced by the disconnected mass (generally lower than 10 cm in the considered instances), whatever the period of vibration of the frame, the plastic limit of the rigid-plastic connectors, the percent of mass disconnected or the violence of the earthquake. For practical purposes, a simple way of obtaining a good prediction of the peak displacement of the disconnected mass from the earthquake rigid-plastic pseudo-spectrum is provided in the paper.

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