The synthesis of a radar signal having nonlinear frequency modulation function

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Abstract

A method of signal synthesis is presented. After theoretical introduction an algorithm of frequency modulated (FM) signal synthesis is presented. Simulation results made in Matlab are presented in the last chapter.
Keywords: NLFM, signal synthesis, autocorrelation function.

1 Introduction

This paper presents the problem of the synthesis of signals modulated in frequency with an autocorrelation function that implements an optimal approximation to a given autocorrelation function.

The output signal of the matched filter is proportional to the autocorrelation function of the expected signal. Because of that one would want to use a signal whose autocorrelation function $R(\tau)$ would be "similar" to certain "perfect" $R_{opt}(\tau)$ in the sense of a criterion that would provide a desirable property or properties. In this case it is the square criterion of similarity

$$f = \int_{-\infty}^{\infty} \left[\left| R_{opt}(\tau) \right| - \left| R(\tau) \right| \right]^2 d\tau . \tag{1}$$

In addition we are assuming, that the energy spectrum of the signal $s(t)$,

$$G(\omega) = 0 \quad \text{for} \quad |\omega| > \Omega \tag{2}$$

is non zero in a finite range of frequencies.

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Realized autocorrelation function does not determine in explicit way signal $s(t)$, it can only determine the signals amplitude spectrum:

$$R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(\omega)|^2 e^{j\omega\tau} d\omega \,, \tag{3}$$

$$|G(\omega)|^2 = \int_{-\infty}^{\infty} R(\tau) e^{-j\omega\tau} d\tau \,, \tag{4}$$

where $|G(\omega)|$ - amplitude spectrum.

From the above one can see that any signal with a given amplitude spectrum and a random phase spectrum can have the desired autocorrelation function. In the next paragraph the signal synthesis algorithm will be presented.

2 Signal synthesis algorithm

The signals synthesis task is to determine a signal x, that is an element of a set of frequency modulated signals *X*, on the basis of the set of signals *Y* with desired a property or properties of the autocorrelation function. In other words one should synthesize a signal $x_{opt}(t)$, which is closest to the signal $y_{opt}(t)$ in the meaning of the square criterion. The synthesis of the optimum signal $x_{opt}(t)$ is equivalent to the determination of the shortest distance $d(x, y)$, in the sense of the square criterion, between the *X* and *Y* sets

$$d_{\min} = \min_{x \in X} d(x, y) \,. \tag{5}$$

The square criterion specifies a rule according to which to each pair of functions x and y distance $d(x, y)$ is assigned to each other. The distance $d(x, y)$ is often called a function space metric and should not be interpreted as the geometric distance. Signals belonging to this space are governed in terms of energy as mentioned earlier.

There are several ways to solve the problem of the signal synthesis i.e., finding a signal x with lowest value of distance $d(x, y)$. The problem was divided into two parts. The first part (first three blocks in fig. 1) was carried out only once during the initial stage of the algorithm and its purpose was to find the form of equation (iv) from fig. 1. The second part of the signal synthesis problem was solved, by building two separate algorithms (last two blocks in fig. 1) executed consecutively. In the first algorithm one chooses a facultative signal $x \in X$ and determines its best approximation on *Y* set of signals, [1, 2] (main program from fig. 1).

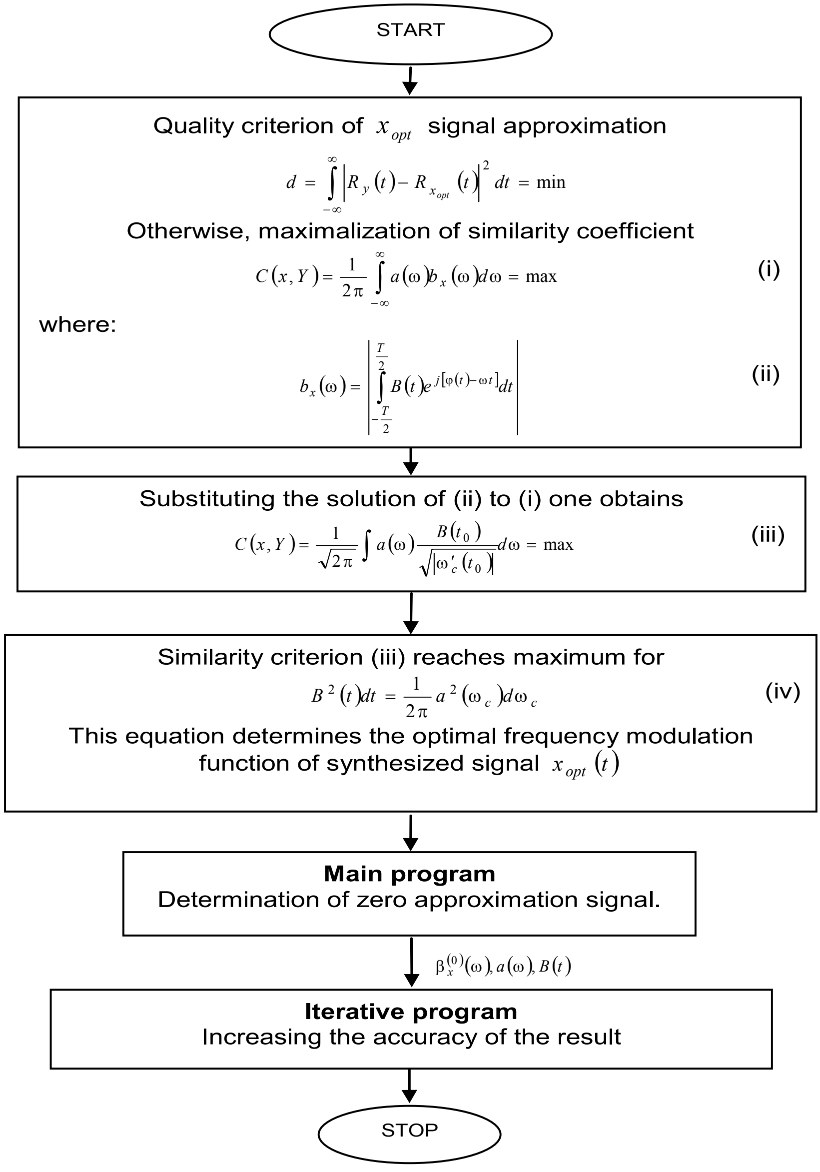

Figure 1: Signal synthesis algorithm.

The quality of approximation is characterized by a distance expressed by the following

$$d(x,Y) = \|x - y\|, \tag{6}$$

where $\|\cdot\|$ denotes the norm of the signal.

This distance corresponds to a specific signal y_0

$$y_0 = a(\omega)e^{-j\beta_{0x}} . \tag{7}$$

If the signal does not have the desired properties, the signal x is changed (moving inside X set), and the distance $d(x, y)$ is determined again, this task is carried out by iterative program from fig. 1. Successive distance values should form a decreasing sequence

$$d_1 > d_2 > d_3 > \ldots .$$

This operation is repeated until the minimum of eqn (6) is achieved. Both algorithms were implemented in Matlab using numerical methods.

The task of the signal synthesis is to synthesize the frequency modulation function $\omega_c(t)$ that implements the best approximation to the desired signal $y(t)$. Suppose that the elements of the set of possible signals X are given as

$$x(t) = B(t) \cdot e^{j\varphi(t)} \text{ for } |t| \le \frac{T}{2}, \tag{8}$$

where: T – duration of the pulse, $B(t)$ – signal envelope, $\varphi(t)$ – phase modulation function.

The spectrum of the signal $x(t)$ is given by

$$\widetilde{X}(\omega) = b_x(\omega) \cdot e^{-j\beta_x(\omega)}, \tag{9}$$

where: $b_x(\omega)$ – amplitude spectrum, $\beta_x(\omega)$ – phase spectrum.

In the process of the synthesis, it is assumed that the envelope of the signal ($B(t)$) is set, and the function $\varphi(t)$ is arbitrary. The frequency modulation function is expressed as follows

$$\omega_c(t) = \frac{d\varphi(t)}{dt} \tag{10}$$

Elements of Y have the following form

$$y(t) = A(t)e^{j\Phi(t)}, \tag{11}$$

and their spectra

$$\widetilde{y}(\omega) = a(\omega)e^{j\alpha(\omega)}, \tag{12}$$

where:

$A(t)$ - envelope of the signal $y(t)$,

$\Phi(t)$ - phase function of the signal $y(t)$,

$\alpha(\omega)$ - phase spectrum of s the signal $\widetilde{y}(\omega)$,

$a(\omega)$ - amplitude spectrum of the signal $\widetilde{y}(\omega)$.

Signals $x(t) \in X$ differ only in the form of the phase modulation function $\varphi(t)$, but have a given envelope $a(\omega)$. With given functions $a(\omega)$ and $B(t)$ one synthesizes the phase modulation function $\varphi(t)$ (or $\beta_x(\omega)$), which minimizes the distance between the X and Y sets [3]. This means that one must determine the similarity coefficient:

$$C(x,Y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} a(\omega) b_x(\omega) d\omega = \max , \tag{13}$$

$$b_x(\omega) = \left|\tilde{x}(\omega)\right| = \left|\int B(t) e^{j[\varphi(t)-\omega t]} dt\right| . \tag{14}$$

The optimal frequency modulation function, which maximizes the similarity coefficient (13), shall be determined on the basis of the differential equation

$$B^2(t)dt = \frac{1}{2\pi} a^2(\omega_c) d\omega_c . \tag{15}$$

After determining the frequency modulation function $\omega_c(t)$ from the above equation, in the next step the phase modulation function is determined as

$$\varphi(t) = \int \omega_c(t) dt . \tag{16}$$

In order to determine the phase spectrum of signal $x(t)$ one must resolve the integral

$$\tilde{X}(\omega) = \int_{-T/2}^{T/2} B(\omega) e^{j[\varphi(t) - \omega t]} , \tag{17}$$

Using the method of stationary phase [2, 4] one obtains a solution of the form

$$\beta_x(\omega) = -\left[\varphi(t) - \omega t_0 \pm \frac{\pi}{4}\right] , \tag{18}$$

where t_0 - point of stationary phase.

In the next step, from the above equation a phase spectrum $\beta_x(\omega)$ should be determined, which is then mapped to a specified amplitude spectrum $a(\omega)$, in order to obtain the signal $y(t)$ closest to the signal $x(t)$

$$\tilde{y}_0(\omega) = a(\omega) e^{j\beta_x(\omega)} . \tag{19}$$

The determined phase spectrum $\beta_x(\omega)$ is both the phase spectrum of the signal $x_{opt}(t)$ and the signal $y_0(t)$. The signal $y_0(t)$ is obtained by taking the inverse Fourier transform from the signal $\tilde{y}_0(\omega)$. At this stage, namely after determination of the signal $x_{opt}(t)$, the main algorithm ends. The results of the main program are affected by errors. These errors are coming from the stationary phase method and numerical methods used in the Matlab program that implements the algorithm. In order to reduce them an iterative method was build and the signal $\tilde{y}_0(\omega)$ was used as the input signal for this method.

In the next paragraph the simulation results of both programs, the main one and the iterative one, are presented.

3 Simulation results

With given functions $a(\omega)$ and $B(t)$ the phase function $\varphi(t)$ (or $\beta_x(\omega)$) of the optimal signal $x_{opt}(t)$ is synthesized. This function minimizes the distance between the sets X and Y. On the basis of the shape of $x_{opt}(t)$ signal, having the optimal phase function $\varphi(t)$, the resulting autocorrelation function $R_{opt}(t)$ is determined.

In order to verify the correctness of the program's performance, the synthesis of a signal with a nonlinear frequency modulation (NLFM) was made. The signal has a bell shaped amplitude spectrum, given by

$$a(\omega) = \frac{\sqrt{\frac{2}{\Omega}}}{\sqrt{1+\frac{\omega^2}{\Omega^2}}}, -\infty < \omega < +\infty . \tag{20}$$

The envelope of synthesized signal was rectangular

$$B(t) = \begin{cases} \frac{1}{\sqrt{T}} & \text{for } -\frac{T}{2} \leq \omega \leq \frac{T}{2} \\ 0 & \text{for } \quad |t| > \frac{T}{2} \end{cases} .$$

Signal parameters that were used during the simulation:

- $\Omega = 200$,
- $T = 4$.

The goal of this simulation was to confirm the correctness and usefulness of the iterative procedure for the synthesis of the signal having a nonlinear frequency modulation function. On the basis of the given amplitude spectrum a signal with non-linear frequency modulation function was obtained (fig. 2).

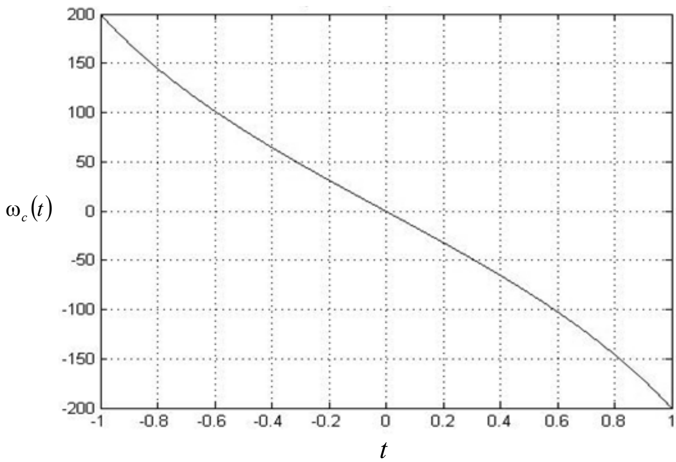

Figure 2: Frequency modulation function.

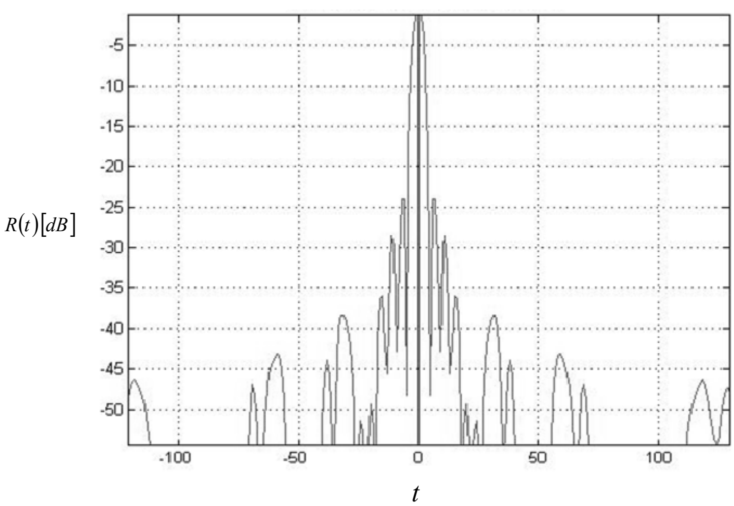

Figure 3: Autocorrelation function after one iteration $(\Omega = 200)$.

In fig. 3 the obtained spectrum and autocorrelation function is presented.

As can be seen in figures 3 and 4, the result of the synthesis is not satisfactory. Although the level of the first side lobe of the autocorrelation functions is -24 dB, the amplitude spectrum does not have the bell shape. The amplitude spectrum is almost rectangular. Because of that the result is passed to

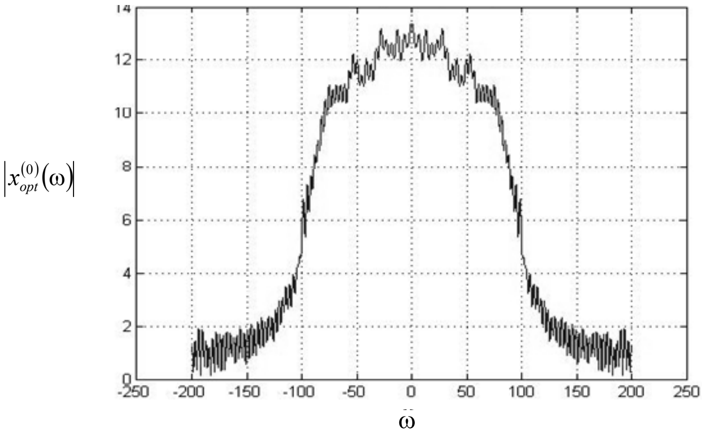

Figure 4: Amplitude spectrum after one iteration $(\Omega = 200)$.

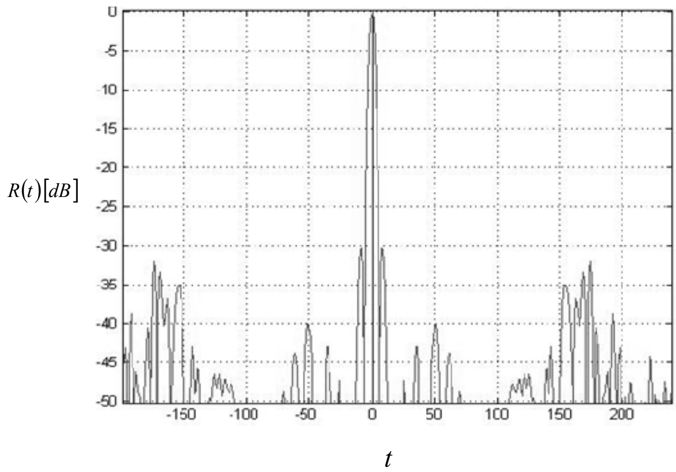

Figure 5: Autocorrelation function after thirty iterations $(\Omega = 200)$.

the second (iterative) program. After the execution of thirty iterations significant improvement of the properties of the autocorrelation function can be seen (fig. 5 and fig. 6).

In fig. 5 and fig. 6 one can see the effectiveness of the iterative method. After thirty iterations the level of the first side lobe decreased by 7 dB (to –31 dB). In fig. 6 one can see how the shape of the power spectral density (PSD) function PSD of thirtieth iteration rise much slower than after the first iteration and in the same time both PSD-s overlap in $\omega = -50 \ldots 50$ region.

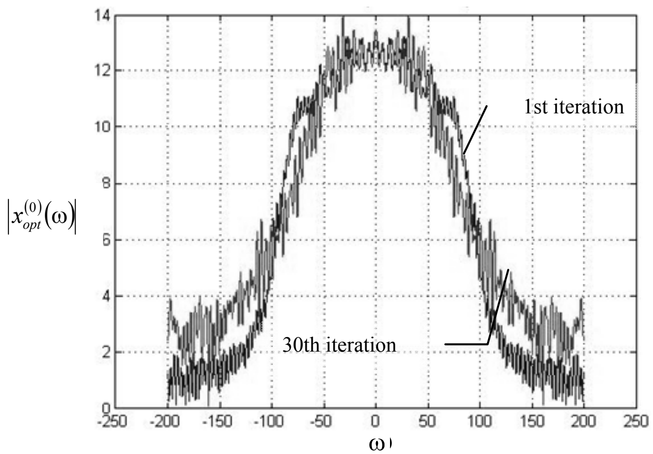

Figure 6: Power spectral density after first iteration and after thirty iterations ($\Omega = 200$).

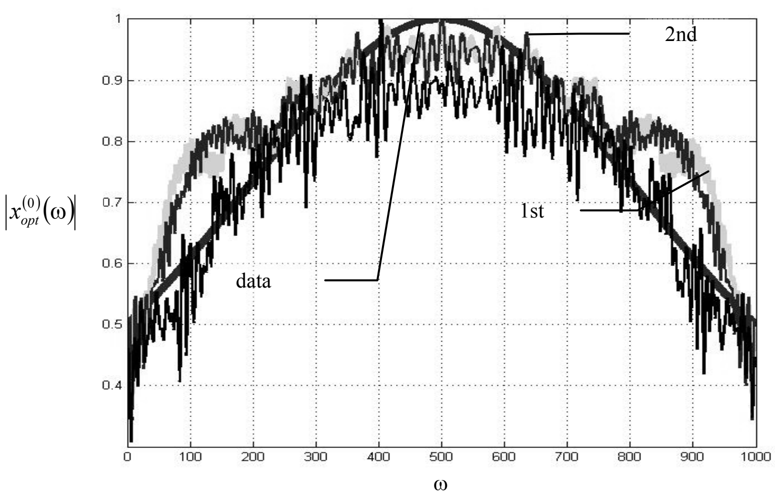

Figure 7: Comparison of PSD-s after the first, second and thirtieth iterations.

Fig. 7 shows how the iterative method changes the PSD function in a chosen number of iterations. The curve of the thirtieth iteration is filtered to improve the clarity of the figure.

The final autocorrelation function, after 150 iterations, and the PSD are shown in fig. 8 and fig. 9 respectively.

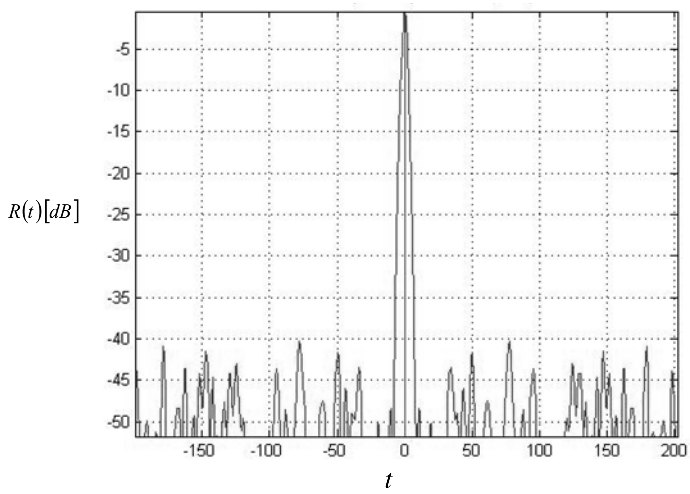

Figure 8: Autocorrelation function after 150 iterations ($\Omega = 200$).

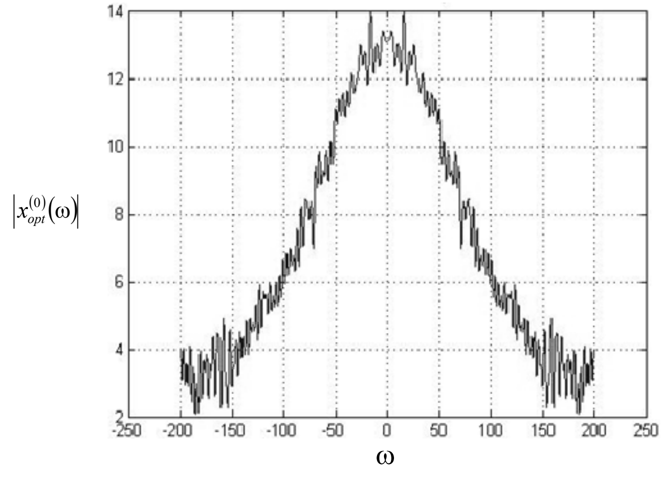

Figure 9: Power spectral density after 150 iterations ($\Omega = 200$).

In fig. 8 the resulting autocorrelation function was plotted. The side lobes are on equal level of -40 dB. The first side lobe is barely noticeable and its level is – 48 dB. That is an improvement of 24 dB in comparison to the first iteration. Also the PSD plotted in fig. 8 is almost the ideal bell shaped. This proves the effectiveness of the iterative method. The noise that can be seen in fig. 9 comes from used numerical methods.

4 Conclusion

This paper discusses the key aspects of the signal synthesis needed for the selection of signals with a desired autocorrelation function, for example in radar technology. The results of previous theoretical studies and numerical results confirm the usefulness of the method discussed in the article from the standpoint of the signal optimisation. The iterative method reduces the errors introduced by the method of the stationary phase and the errors that are coming from used numerical methods. The presented method of the signal synthesis is very useful for cases where the desired autocorrelation function and the subsequent results of the calculations cannot be represented in a strict analytical form. Another key advantage of the presented algorithm is that the used numerical methods allow us to find the optimal solution, having at the entrance, only a discrete set of desired signals PSD points without any prior knowledge about the signal itself.

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References

[1] Sołowicz, J., Institute of Radioelectronics Technical Report, Warsaw, 2008, (in Polish).
[2] Vakman, D.E. & Sedleckij, R.M., Voprosy sinteza radiolokacionnych signalov, Sovietskoye Radio: Mosva, 1973 (in Russian).
[3] Kawalec, A., Lesnik, C., Solowicz, J., Sedek, E. & Luszczyk, M., Wybrane problemy kompresji i syntezy sygnałów radarowych. *Elektronika,* **3**, pp. 76-83, 2009 (in Polish).
[4] Cook, C.E. & Bernfeld, M., *Radar Signals: An Introduction to Theory and Application.* Artech House: Boston and London, 1993.