On a numerical model of a complete washing machine

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Abstract

We present a numerical model of a complete horizontal-axis washing machine and its validation. Our purpose is to develop an effective mathematical model able to predict the machine dynamics during steady-state spinning cycles, with a specific focus on the structural vibrations of the cabinet. The complete model couples a parametric linearized multi-body model under unbalanced-mass forcing for the drum unit and a structural finite element model for the cabinet. The drum unit and cabinet are connected by means of a suspension system composed of three extension springs and two dry-friction dampers. We present two different models to simulate the dampers' behaviour and we compute the response of the cabinet using modal superposition. Besides numerical modelling, we carried out an extensive experimental campaign in order to characterize every element of the washing machine, to determine every parameter of the model, and to collect a large database to validate the model. Experimental results are reported in Argentini et al. "Experimental characterization of the vibro-acoustic behaviour of a complete washing machine. *XXV IMAC, 2007*". A satisfying reproduction of the machine dynamics emerges from the comparison between numerical and experimental responses. The numerical model will be used to design and test new solutions to improve the dynamical response and to reduce structural vibrations.
Keywords: washing machine, numerical model, multi-body, FEM, friction dampers, vibrations.

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1 Introduction

In a market that is more and more oriented towards customer satisfaction, high performances together with high comfort levels provide the competitive edge to gain market share. In the case of washing machines, very high spin speeds (up to 1600 rpm) are offered to end-users. However, high performances often drop comfort levels: indeed, a drawback of having high spinning speeds is that the cabinet of the washer is forced at higher frequencies than it was designed for and it vibrates excessively, generating noise. Therefore, reducing the noise emitted during washing cycles, and specifically during spinning cycles, is a very attractive target, because it permits the matching of performance and comfort. Up to now, research and development of washing machines have been mainly carried on adapting existing machines by means of experimental and heuristic approaches, with high development costs and long research time. Within this scenario, considering that computational speed is continuously improving, numerical models are gaining ground as design tools that allow to improve products' time-to-market and to reduce design costs.

In this perspective, we present a numerical model of a horizontal-axis washing machine able to predict its dynamics. In particular, we focus our modelling on the dynamics during spinning cycles since, as we showed in [1], at these forcing frequencies the noise due to structural vibrations is particularly unpleasant for end-users.

To obtain the global equations of motion, we divided the model into two distinct parts. The drum unit is modelled with a parametrical linear multi-body approach, using lumped parameters to represent the inertial, elastic or dissipative contribution of each element. A parametrical approach, i.e. the possibility to modify one or more elements independently, is necessary to get a versatile design tool; a linear approach is adopted since we study the steady-state dynamics of the drum unit, that during spinning cycles is forced in its seismographic zone, and therefore we account for small displacements and linearized equations of motion. To model the cabinet we used a finite element approach [2], and we used a reduced modal superposition to evaluate the vibratory response of some significant points.

2 Drum unit model

The model we used to schematize the system is depicted in Figure 1. The drum unit is composed of four rigid bodies: a plastic tub (in which we include the inertia of the electrical motor), a rotating stainless steel basket, and two concrete ballasts constrained at the tub. The unit is linked to the cabinet by means of three springs, two dampers and a gasket. An unbalanced mass, constrained to the tub with a known eccentricity, forces the system. The model global reference system is placed in a corner of the cabinet: from it, we define the coordinate of the points of interest, such as the centre of gravity of each body and the hinges of springs and dampers, as shown in Fig. 1.

To write the equations of motion of the drum we consider six degrees of freedom (displacements and rotations of the tub) and the imposed angular velocity of the basket.

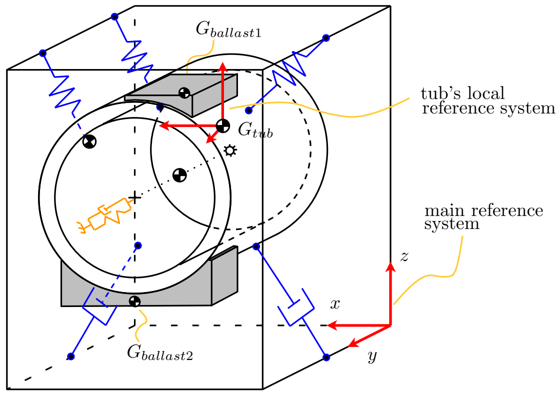

Figure 1: Idealization of the real washing machine: tub, gasket, suspensions, main and a local reference system.

The total kinetic energy of the drum is obtained through the sum of the kinetic energies of each body;

$$E_{k_{TOT}} = \sum_{i=1}^{N} E_{k_i} \text{ , with } E_{k_i} = \frac{1}{2}\dot{\underline{y}}_i^T \begin{bmatrix} [M] & [0] \\ [0] & [J] \end{bmatrix} \dot{\underline{y}}_i = \frac{1}{2}\dot{\underline{y}}_i^T \left[\tilde{M}_i\right] \dot{\underline{y}}_i \tag{1}$$

where $\underline{y}$ is a vector that represents the dofs of the centre of gravity of the i-th body, *[M]* is the local mass matrix, and *[J]* is the principal inertia tensor: these inertial and geometric properties can be easily retrieved from 3D solid models of new prototypes. The projection of the local principal references onto the global reference is made in two steps: first, we rotate it using a direction cosine matrix $[\Lambda_\theta]$ onto a reference system parallel to the global one and centred in G_i, i.e.: Second, we project the $[M_i^*]$ matrix onto the global reference system using a linear transformation matrix, $[\Lambda_T]$, which contains opportunely the three distances between global and local reference system.

$$E_{k_i} = \frac{1}{2}\dot{\underline{q}}_i^T \left[\Lambda_{T_i}\right]^T \left[\Lambda_{\vartheta_i}\right]^T \left[M_i^*\right]\left[\Lambda_{\vartheta_i}\right]\left[\Lambda_{T_i}\right]\dot{\underline{q}}_i = \frac{1}{2}\dot{\underline{q}}_i^T \left[\tilde{M}_i\right]\dot{\underline{q}}_i \tag{2}$$

To reproduce the effect of the unbalanced mass we consider it as an external force applied to the centre of mass of the tub. We write the equation of motion of the mass in the coordinate system of the basket, computing its kinetic energy and applying Lagrange's formalism. With reference to Figure 3, the velocity of the mass $v_{m_{ecc}}$ can be written as follows:

$$\begin{cases} v_{x_c} = \dot{x}_c + \dot{\theta}_{y_c} e\cos(\theta_{y_c}) - \dot{\theta}_{z_c} e_y \\ v_{y_c} = \dot{y}_c + \dot{\theta}_{z_c} e\sin(\theta_{y_c}) + \\ \qquad - \dot{\theta}_{x_c} e\cos(\theta_{y_c}) \\ v_{z_c} = \dot{z}_c - \dot{\theta}_{y_c} e\sin(\theta_{y_c}) - \dot{\theta}_{x_c} e_y \end{cases}$$

$$v_{m_{ecc}} = \sqrt{v_{x_c}^2 + v_{y_c}^2 + v_{z_c}^2} \tag{3}$$

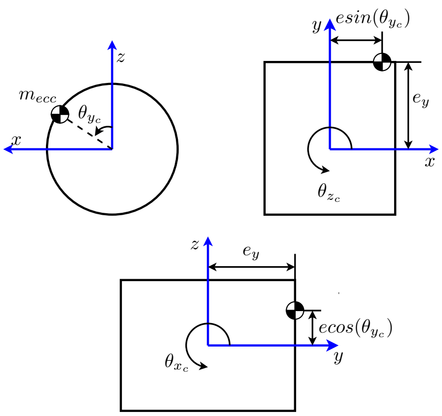

Figure 2: Position of the unbalanced mass in the basket local coordinates.

It can be noticed that the velocity of the unbalanced mass has a non-linear dependence upon the rotation and on the angular velocity of the basket. Deriving these terms according to the Lagrange's formalism, their dependence on the six degrees of freedom of the system is highlighted. Organizing the terms according to their dependence upon the six dofs of the tub and their derivatives, we can write the contribute of the unbalanced mass as follows:

$$\underline{Q_c} = \left[M_{ecc}(t)\right]\underline{\ddot{q}_c} + \left[R_{ecc}(t)\right]\underline{\dot{q}_c} + \underline{f_{ecc}}(t) \tag{4}$$

The terms *[$M_{ecc}(t)$]* and *[$R_{ecc}(t)$]* contain the dependence of these elements on the accelerations and velocities of the tub, the external force $f_{ecc}(t)$ contains the terms that have no dependence upon the chosen degrees of freedom.

The elastic potential is due to the three springs. The general formulation of the potential energy for each spring is:

$$V_s = \frac{1}{2}\underline{\Delta l}^T\left[K\right]\underline{\Delta l} = \frac{1}{2}\begin{Bmatrix}\Delta l_x \\ \Delta l_y \\ \Delta l_z\end{Bmatrix}^T \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix} \begin{Bmatrix}\Delta l_x \\ \Delta l_y \\ \Delta l_z\end{Bmatrix} \tag{5}$$

where $\underline{\Delta l}$ is a vector containing the dynamical extension of the spring projected onto the local orthogonal coordinate system of the spring, shown in Figure 3. The stiffness k_x is the value found experimentally [1], whereas k_y and k_z are the stiffness values due to the fact that the spring is statically pre-loaded [5]. These values can be computed as:

$$\begin{aligned} k_y &= k_x \Delta l_0 \left.\frac{\partial^2 \Delta l}{\partial y_A^2}\right|_{y_A=0} = k_x \frac{l_{def} - l_0}{l_{def}} \\ k_z &= k_x \Delta l_0 \left.\frac{\partial^2 \Delta l}{\partial z_A^2}\right|_{z_A=0} = k_x \frac{l_{def} - l_0}{l_{def}} \end{aligned} \tag{6}$$

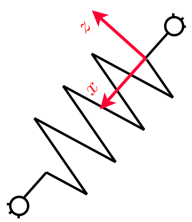

Figure 3: Spring local coordinate system (2D view).

where y_A and z_A are the displacement of the hinge on the drum in the spring local reference, while l_{def} and l_0 are respectively the pre-loaded and unloaded length of the spring.

A big part of the dissipating contribution to the system is due to the gasket, which also preserves elastic properties. This element is not easy to model (with non-linear dependence on the amplitude and the frequency of motion) but, at the effects of our work, it is sufficient to consider a system of linear elastic and dissipative elements, placed in an equivalent point (see Fig.1). This point is the center of the porthole and, knowing its coordinates, we can write the potential and dissipating terms of the gasket. We consider as well the linearized gyroscopic effect due to the spin speed of the basket [5].

3 Damper models

Beside the gasket, most part of the dissipative energy is due to the two dampers placed under the washing machine tub (see Fig.1). During the experimental campaign, we evidenced the non-linear characteristic of these elements: we now propose two different models to reproduce this behavior.

The first approach uses a formula that fits the hysteresis loop of the damper [3]: we will call it the *closed form* formulation, in witch the damper force can be expressed as:

$$f_{damper}(t) = f_{ab} + \left[\frac{x_v(t) - x_b}{x_s \left(1 - 2 \frac{f_{ab}}{\Delta f}\right) \pm \left(x_v(t) - x_b\right)} \right] \left(\frac{\Delta f}{2} \mp f_{ab} \right) \tag{7}$$

where $x_v(t)$ represents the motion of the connection point of the damper with the tub, x_b and f_{ab} are the last value of $x_v(t)$ and $f_{damper}(t)$ before the reversal of the motion, Δf is the amplitude of the hysteresis loop, and x_s a parameter proportional to the slope of the hysteresis loop (see Fig. 4). The operators $\pm$ and $\mp$ change their value if the damper is in a compression or traction phase.

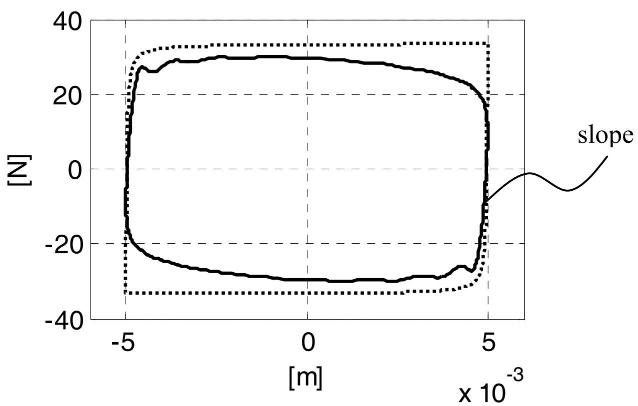

Figure 4: Hysteresis loops: closed formulation (dotted) and rheologic model (solid).

The second model implemented is a *rheologic* one. This model can be schematized with a one degree of freedom damper-spring-mass system that slides with friction (see Fig. 5).

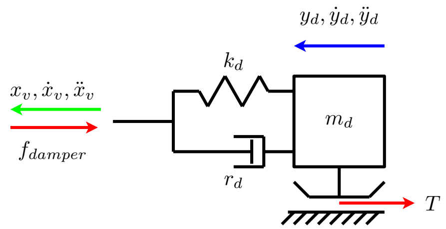

Figure 5: Rheologic model scheme.

The damper force (f_{damper}) can be easily computed by force balance, considering that k_d is a linear stiffness, r_d a viscous damping and T is a friction force whose amplitude is proportional to the mass and whose sign is function of its velocity. A typical hysteresis cycle is represented in Fig. 4.

For both models the determination of the parameters comes from the minimization of the error between the numerical and the experimental dissipated energy. The main advantage of the closed formulation model is its simplicity and the little computational time that it requires; unfortunately this method does not contain any information about the excitation frequency. Experimentally, we evidenced that this is acceptable for the real damper in the range over 5 Hz, therefore this model is very useful especially for spinning cycles simulations. On the other hand, the peculiarity of the rheologic model is its frequency dependent response and the consequent easiness of performing a sensitivity analysis since each parameter has a physical meaning. Its major drawback is the substantial increase in the integration time due to the reduced time step necessary to simulate the friction force and to the addition of one dof for each damper.

To test both models we made a comparison with the real damper, using the results of the experimental campaign. In particular, we focus our attention on the total dissipated energy per cycle and on the frequency content of the force. In Fig. 6 we show the hysteresis loop in a test at 10 Hz with 5 mm amplitude of sinusoidal imposed motion.

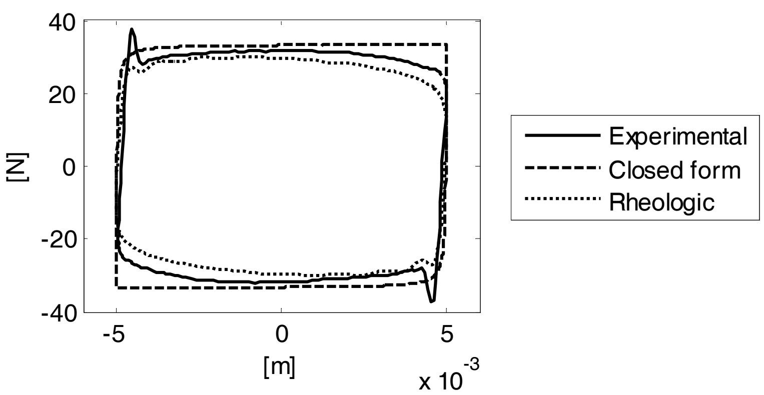

Figure 6: Hysteresis loop comparison between experimental and numerical dampers (5 mm, 10 Hz).

It can be seen that the non linearity of the experimental force (red line) is better represented by the rheologic damper (dotted line), but its dependence on the frequency and the motion amplitude is distant from the experimental one (solid line). Yet the closed-form damper (dashed line) has always the same behavior in the frequency domain and it best fits the area of the loop, even if it introduce more energy at higher frequencies (see Fig. 7). It is possible for the closed-form model to fit better the high frequency forces using a low-pass filter. Coupling these systems to the drum unit model, we evaluated that the closed form damper (especially during spinning cycles) guarantees the best agreement between experimental and numerical results for all the drum angular velocities from 600 to 1600 rpm.

4 Cabinet model

A model of the cabinet is necessary to integrate the drum unit model in order to get a complete washing machine dynamical model. The cabinet, whose solid geometric model was available from its designers, can be modeled using a FEM approach [2]. The resulting FE model leads to very large structural matrices ($n = 817218$ degrees of freedom, in our case), consequently solving directly the equation of motion is computationally expensive and time consuming.

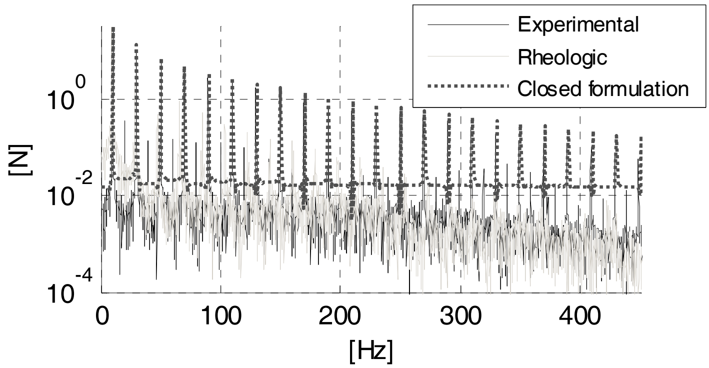

Figure 7: Force spectrum comparison between experimental and numerical dampers (5 mm, 10 Hz).

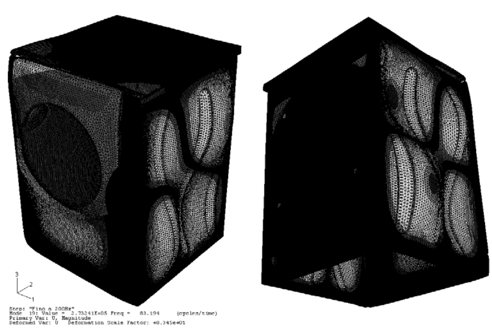

Figure 8: Example of FEM analysis result, 18° mode at 81Hz.

Therefore, to predict the behaviour of the cabinet we developed a model using a reduced modal superposition approach using the results of a FEM eigenvalue analysis (see Fig. 8). We approximate the modal response using k eigenmodes of the n we arrange, considering a defined bandwidth, in our case 200 Hz. For this reason we need to solve a system of k decoupled single-dof equations in modal coordinates q_i. Each equation has the following form:

$$\ddot{q}_i + 2h_i\omega_{0i}\dot{q}_i + \omega_{0i}^2 q_i = Q_i(t) \qquad i = 1,\dots,k \tag{8}$$

where i in the mode number, h_i the modal damping, ω_{0i}^2 the eigenvalue associated with the i-th mode, and Q_i the force vector projected onto the i-th eigenvector. Forces in Q_i are the forces transmitted by the three springs and the two dampers.

5 Model validation

It is important to verify that the hypothesis made during its development is justified by a good correspondence between numerical and experimental results. First we validate the tub motion, then we proceed with the cabinet motion.

One of the model results, at an imposed angular velocity excitation, is the time history of the tub's centre of mass, while experimentally we can evaluate the acceleration of six points on its surface [1]. Through an appropriate transformation matrix the output of the model has been made consistent with the experimental one. In Fig.9 we show a comparison between steady-state responses at different excitation frequencies: the washing machine is tested with the two dampers disconnected, and with an unbalanced mass of 190g.

The model represents the response of the vertical accelerometers (*z-front, z-back*), but, while in the experimental results the vertical mode is coupled to a yaw one, in the numerical response this phenomenon does not appear. The rear and lateral accelerometers responses (*y-right, y-left*, *x-front, x-back*) are overestimated at 1.5-3 Hz, while they are underestimated in the range 4-5 Hz.

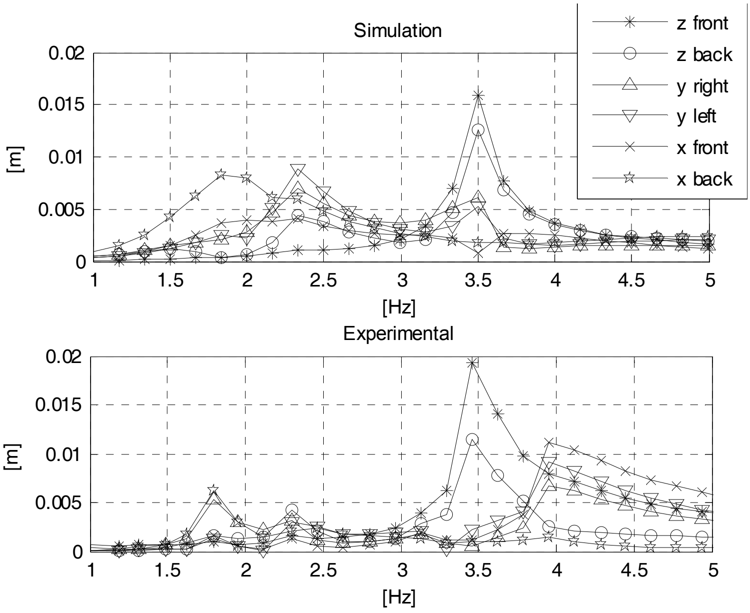

Figure 9: Drum unit motion: numerical vs. experimental.

These discrepancies are due to the simplifications introduced in the gasket modeling as it has been evidenced by a comparison done excluding it. Since our objective is the simulation of the vibratory response of the cabinet at high rpms, it is more meaningful to see directly a comparison between the experimental and numerical responses of the cabinet for a high frequency excitation. The cabinet motion is measured with a number of accelerometers placed on the five panels. As an example, in Figure 9 we show the response of one point the cabinet during a spinning cycle with a 500g unbalanced mass at 1100rpm. As visible, the odd harmonics, which are the most relevant, are very close to the experimental one, whereas the even harmonics, whose contribution to the frequency content is minor, reflect the characteristics of the proposed models for the dampers.

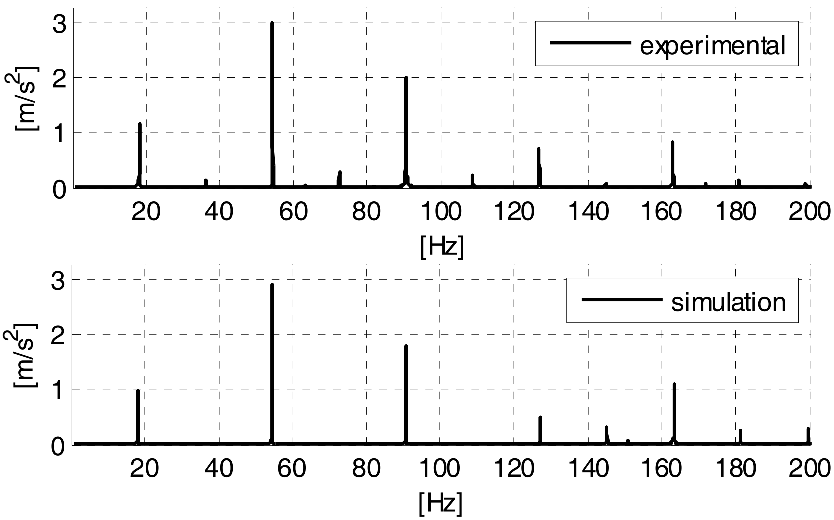

Figure 10: Spectra of the accelerometer placed in the centre of the cabinet left panel.

6 Conclusions

We summarized the passages taken to realize a numerical model of a complete washing machine. The linear approach adopted is justified by our initial hypothesis and by the results presented that validate the model for the simulation of the vibratory response of the cabinet during spinning cycles. The limits of the model are as well evidenced. This tool allows the prediction of the dynamic behaviour of the machine when it is modified by either moving the position of the concrete ballasts or adding other springs to the system. It is therefore a versatile design tool to test very easily many different solutions able to improve the system response.

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