

A simplified shear strength evaluation model for reinforced concrete corbels

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Abstract

A shear strength model for reinforced concrete corbels is developed by modifying the softened strut-and-tie model. Using the concept of Lu and Wu, the constitutive equation is developed. Then, the constitutive equation is combined with the equilibrium equations of Hwang and Lee. The model is used to predict the shear strength of reinforced concrete corbels. The results of the model are compared with the experimental data and the results of ACI code.

Keywords: softened strut-and-tie model, reinforced concrete corbels, shear strength.

1 Introduction

Corbels are cantilevers having shear span-to-depth ratios not greater than unity, which tend to control by shear as a deep beam, rather than flexural members. Reinforced concrete corbels are becoming a common feature in building construction. Simple design procedures to produce safe and economic corbels are therefore required. Much research is carried into investigation about the shear strength of corbels.

In test, the most common modes of failure of corbels are crushing or splitting of the compression strut. ACI 318-95 [3] used the shear-friction method to calculate the shear capacities of corbels. It is shown that most of the shear capacities of corbels calculated by ACI code are underestimated [4–6]. ACI 318-02 [7] provides the strut-and-tie model as an alternate method to



calculate the shear capacities of corbels. A softened strut-and-tie model for predicting the shear strengths of reinforced concrete corbels is proposed by Hwang et al [4]. It is based on the strut-and-tie concept with satisfying equilibrium, compatibility and the constitutive law of concrete. Hwang and Lee [2] modified the softened strut-and-tie model to develop a simple predicting procedure. But, the procedure is still complicate.

To further simplify the design procedure, the present paper developed a shear strength model for reinforced concrete corbels. Using the concept of Lu and Wu [1], the constitutive equation is developed. Then, the constitutive equation is applied with the equilibrium equations of Hwang and Lee [2]. The results of the present model are compared with the results of ACI code and the experimental data from published papers [6, 8–11].

2 The ACI model

The design procedure for corbels given by section 11.9 in ACI-02 [7] code is based on the shear-friction method. The shear capacity of the corbel V_n is obtained by:

$$V_n = \mu A_{vf} f_y \quad (1)$$

with

$$V_n \leq 0.2 f'_c b_w d \quad (2)$$

and

$$V_n \leq 5.6 b_w d \quad (3)$$

where A_{vf} represents the area of shear-friction reinforcement; μ is coefficient of friction; b_w is the web width and d is the distance from extreme compression fiber to centroid of tension reinforcement; f_y represents the yield strength of reinforcement and f'_c represents the compressive strength of concrete.

3 The present model

In this paper, a simplified shear strength model for reinforced concrete corbels is developed by modifying the softened strut-and-tie model. The constitutive equation is developed by using the concept of Lu and Wu [1]. The diagonal compression stress is defined in terms of the yield strength of reinforced steel and the compression strength of concrete. Then, the constitutive equation is combined with the equilibrium equations of Hwang and Lee [2].



3.1 Equilibrium equations

The force equilibrium of a diagonal compression using a strut-and-tie perspective is shown in figure 1.

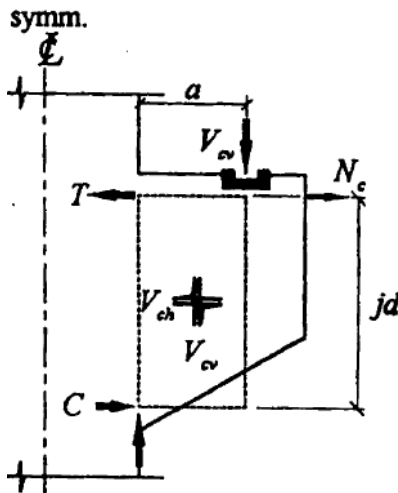
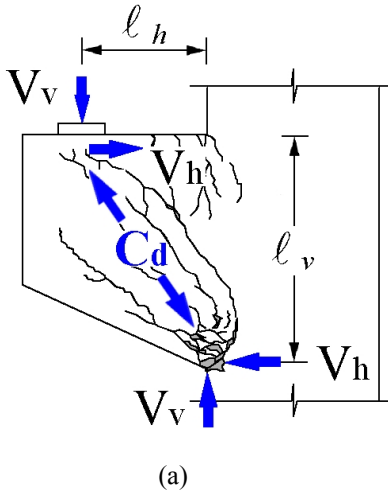


Figure 1: The softened strut-and-tie model of corbel.

The vertical shear force V_v can be obtained by:

$$V_v = C_d \sin \theta \quad (4)$$

where C_d represents the diagonal compression and the θ denotes the angle of inclination of the diagonal with respect to the compression horizontal axis. Also, the horizontal shear force V_h can be obtained by:

$$V_h = C_d \cos \theta \quad (5)$$

By using the equation (4) and (5), following relationship can be obtained by:

$$\frac{V_v}{V_h} = \frac{\ell_v}{\ell_h} = \tan \theta \quad (6)$$

Then, the angle of inclination θ can be obtained:

$$\theta = \tan^{-1} \left(\frac{\ell_v}{\ell_h} \right) \quad (7)$$

where ℓ_v and ℓ_h represent the internal level arms of the vertical and horizontal shear couples, respectively. According to linear bending theory, the ℓ_v and ℓ_h can be obtained as:

$$\ell_h = a \quad (8)$$

and

$$\ell_v = jd \quad (9)$$

with the lever arm jd can be estimated as:

$$jd = d - \frac{kd}{3} \quad (10)$$

where a presents the shear span and the kd represents the depth of the compression zone at the section with the coefficient k .



The coefficient k is chosen as 0.375 [12] in this paper. Equation (10) can be expressed as:

$$jd = 0.875d \quad (11)$$

Substituting equations (8) and (11) into equation (7), we obtain

$$\theta = \tan^{-1}\left(\frac{0.875d}{a}\right) \quad (12)$$

3.2 Constitutive laws

The constitutive equation of concrete used by Hwang and Lee [2] is:

$$C_d = K\zeta f'_c A_{str} \quad (13)$$

where K is the strut-and-tie index, ζ denotes softening coefficient, A_{str} is the effective area of the diagonal strut. Equation (13) is first suggested by Zhang and Jirsa [13].

The effective area of the diagonal strut A_{str} used by Hwang and Lee [2] is:

$$A_{str} = a_s b_s \quad (14)$$

with

$$a_s = kd \quad (15)$$

where a_s represents the depth of the diagonal strut, b_s represents the width of the diagonal strut which can be taken as width of the corbel.

The coefficient k is chosen as 0.375 [12] in this paper. Equation (15) can be expressed as:

$$a_s = 0.375d \quad (16)$$

Substituting equation (17) into equation (15), we obtain:

$$A_{str} = 0.375db_s \quad (17)$$

Instead of equation (13), the compression strength C_d is redefined in present paper as:

$$C_d = \sigma_d A_{str} \quad (18)$$



where σ_d represents the diagonal compression stress. In this paper, the diagonal compression stress σ_d is defined in terms of the yield strength of reinforced steel and the compression strength of concrete.

Based on the concept of Lu and Wu [1], the σ_d is defined by this paper as:

$$\sigma_d = f_1(\rho_h, f_{yh}) + f_2(f'_c) \quad (19)$$

where ρ_h represents the ratio of horizontal reinforced steel and f_{yh} represents the yield strength of horizontal reinforced steel. The first terms of equation (18), $f_1(\rho_h, f_{yh})$ present the effect of reinforced steel and the second term, $f_2(f'_c)$ presents the effect of the compressive strength of concrete.

In this paper, $f_1(\rho_h, f_{yh})$ and $f_2(f'_c)$ are determined by fitting the proposed model to the experimental data of Foster, Powell and Selim [9]. It is based on that the average of ratio of the calculated shear strength V_v to experimental result V_t is 1.00. There are:

$$f_1(\rho_h, f_{yh}) = 0.59 \rho_h f_{yh} \quad (20)$$

and

$$f_2(f'_c) = 0.53f'_c - 0.00143(f'_c)^2 \quad (21)$$

with f'_c in MPa. Then, equation (19) can be expressed as:

$$\sigma_d = 0.59 \rho_h f_{yh} + 0.53f'_c - 0.00143(f'_c)^2 \quad (22)$$

4 The procedure of numerical calculation

The procedure of determining the shear force corresponding to the reinforced concrete corbels is now further described. In this application, the a , b_s , d , ρ_v , ρ_h , f'_c , f_{yv} and f_{yh} are known. The procedure of the calculation for the shear force case is shown in figure 2 and described as follows:

Step 1. Calculate A_{STR} by use of equation (18), $A_{str} = 0.375db_s$

Step 2. Calculate θ by use of equation (12),

$$\theta = \tan^{-1} \left(\frac{0.875d}{a} \right)$$



Step 3. Calculate σ_d by use of equation (22),

$$\sigma_d = 0.59\rho_h f_{yh} + 0.53f'_c - 0.00143(f'_c)^2$$

Step 4. Calculate C_d by use of equation (14), $C_d = \sigma_d A_{str}$

Step 5. Calculate V_v by use of equation (4), $V_v = C_d \sin \theta$

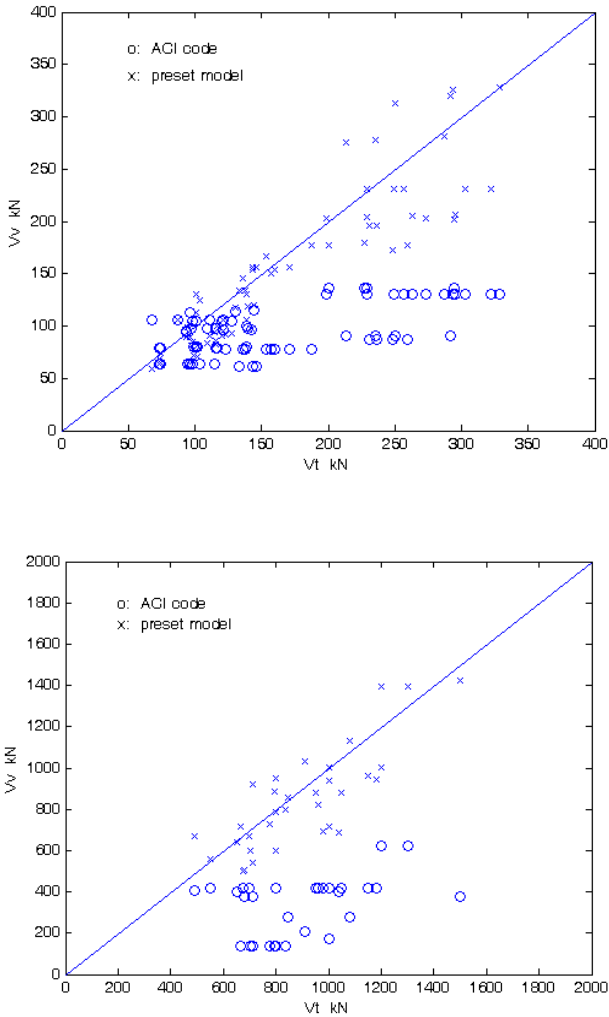


Figure 2: Comparison of the theoretical results with experimental data.



5 Comparison of theoretical and experimental results

A total of 105 experimental data from the literature are used to compare with the present model. The experimental data are from Foster et al [9], Lin [11], Fattuhi and Hughes [8], Yong and Balaguru [6], and Fattuhi [10]. The calculated shear strength of present model and ACI code V_v are compared with the experimental results V_t . Figure 2(a) shows the theoretical results and the experimental data of Lin [11], Fattuhi and Hughes [8], and Fattuhi [10]. Figure 2(b) shows the theoretical results and the experimental data of Foster et al [9] and Yong and Balaguru [6]. It is seen that the ACI code are under estimate and the present model is quite satisfactory.

Table 1: The theoretical results and experimental data.

Author	Number of specimen		$\frac{V_v}{V_t}$	
			ACI code	Present model
Foster et al [9]	22	AVG	0.52	1.00
		COV	0.31	0.23
Lin [11]	24	AVG	0.46	0.90
		COV	0.22	0.20
Ali and White [5]	11	AVG	0.48	1.02
		COV	0.12	0.07
Yong and Balaguru [6]	11	AVG	0.21	1.05
		COV	0.23	0.12
Fattuhi and Hughes [8]	37	AVG	0.84	0.87
		COV	0.20	0.18
Total	105	AVG	0.580	0.917
		COV	0.457	0.180

The average (AVG) and the coefficient of variation (COV) of the ratio of the calculated shear strength V_v to experimental result V_t is calculated and shown in table 1. It is noted that the AVG of the present model and ACI code, with respected to total experimental data, are 0.917 and 0.580, respectively. It is shown that the shear strength calculated from present model is closer to the experimental data than the shear strength calculated from ACI code. The COV of present model and ACI code are 0.180 and 0.457, respectively. It is shown that the calculated results from present model are more stable than the results from ACI code.

6 Concluding remarks

In this paper, a shear strength model for reinforced concrete corbels is developed by modifying the softened strut-and-tie model. Using the concept of Lu and Wu [1], the constitutive equation is developed. Then, the constitutive equation is combined with the equilibrium equations of Hwang, and Lee [2]. Present model are used to predict the shear strength of reinforced concrete corbel. The results of present model are compared with the experimental data and the results of ACI code. It is seen that present model are compares quite well.

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