Experimental measurements for the control of a vortex shaft theoretical model

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Abstract

With reference to the hydrodynamic working of a dropshaft fitted with a vortex inlet, there are still two problems that are open to debate: the actual distribution of velocities at the inlet and the pressure distribution along the radius in the first cross section of the shaft. The majority of researchers assume that the velocity distribution is both irrotational and axially symmetrical, although some forty years ago Viparelli experimentally showed that such an assumption is actually incorrect, hypothesizing that the flow is symmetrical but not irrotational. Moreover, the determination of pressure distribution in the different proposed theories remains a debatable issue, with some claiming that the distribution of pressure is equal to zero while others maintain it is positive. In the present paper, two series of experimental measurements concerning the above-mentioned problems are analyzed. A first series of experimental tests performed with a Laser Doppler Anemometer confirms a different velocity distribution hypothesis: irrotational but not entirely symmetrical. A second series of experimental tests deals with pressure measurements made in the vertical shaft inlet. Contrary to what has been hypothesized by other researchers, these measurements indicate negative pressure values.
Keywords: dropshaft, vortex flow, mathematical model, experimental measurements, Laser-Doppler Anemometer.

1 Introduction

Inflow in a dropshaft with vortex inlet is a rather complex phenomenon which still presents certain issues that have not been fully clarified. In spite of this such device is of great interest because of its undeniable technical importance. Its

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inventor Drioli [1], and increasing numbers of authors thereafter (i.e. [2–9]) have investigated the vortex inlet with seemingly different procedures that can, however, be traced back to a single template. Nevertheless, the definition of the actual velocity distribution in the inlet chamber and the pressure distribution along the radius r in the first cross section of the shaft remain controversial issues. In particular the characteristic quantities of Drioli's inlet are as follows (see fig. 1):

- Δ: distance between the two axes of inflow channel and of vertical dropshaft;
- h: stream depth in the inflow channel;
- H: total energy (head) in the inflow channel;
- b: width of the inflow channel;
- δ: characteristic dimension of the inlet;
- R: radius of the shaft;
- r_o: internal radius of the vortex in section 0-0 (see fig. 1);
- $V_t(r)$: tangential velocity in the section 0-0;
- A or C: kinetic constant;
- p(r): pressure in the section 0-0;
- Q: flow rate discharged.

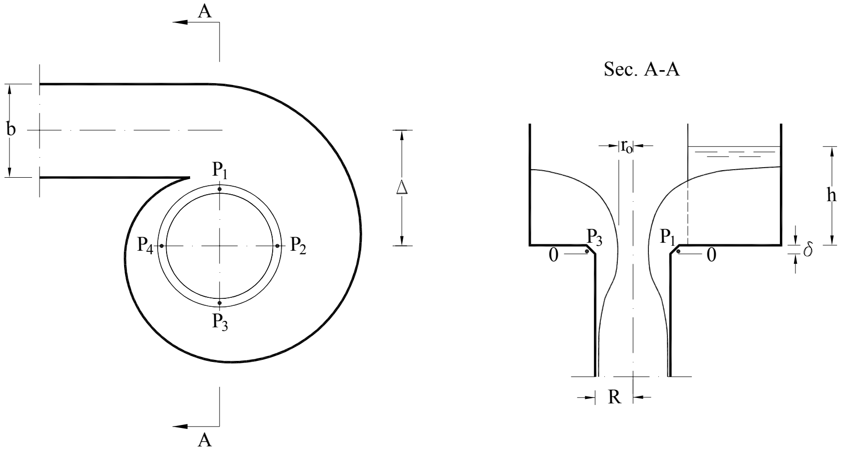

Figure 1: Typical Drioli vortex-flow inlet.

Furthermore, the equations considered by the various authors are as follows:
1) Bernoulli's equation for the definition of total head H;
2) the equation of flow rate:

$$Q = 2\pi\sqrt{2g}\int_{r_o}^{R}\sqrt{H-(p/\gamma)-\left(V_t^2/2g\right)}\cdot r\cdot dr\,; \qquad (1)$$

in which V_t and the integral are determined in the section 0-0 while r is the generic radius in that section;

3) the equation needed for the calculation of r_o determined either by means of experimental data or by means of the principle maximum flow rate which provides (i.e. Knapp [5]):

$$r_o = A / \sqrt{2gH} \; ; \tag{2}$$

4) the equation of continuity or the equation of momentum; the latter leads (i.e. Viparelli [3]) to the determination of the following formula:

$$A = \frac{Q\Delta}{bh} \; ; \tag{3}$$

5) the distribution of tangential velocity provided either by:

$$V_t(r) = C \; ; \tag{4}$$

which represents a constant distribution, or by:

$$V_t(r) = A/r \; ; \tag{5}$$

which represents an irrotational distribution;
6) distribution of pressure p(r) along the radius r_o in the section 0-0 provided either by:

$$p(r)/\gamma = 0 \; ; \tag{6}$$

or by:

$$p(r)/\gamma = \left(A^2/2g\right)\cdot\left(1/r_o^2 - 1/r^2\right). \tag{7}$$

As far as the velocity distribution is concerned, the overwhelming majority of authors hypothesizes that motion is irrotatational throughout the field and therefore assumes the characteristic expression of irrotatational flow shown in eqn. (5). Adopting eqn. (4) would contradict the hypothesis of irrotational flow. In actual fact, the velocity distribution in the curve of type V_t = constant mentioned by Ramponi [10] refers to secondary motion arising in the curve when the velocity before the curve is no longer constant in the straight section as a result of friction with the channel walls and, therefore, the motion is already no longer irrotataional. Viparelli [3] bases his reasoning both on eqn. (4) and on eqn. (5): even if Viparelli assumes eqn. (4) nevertheless continues to refer to the validity of Bernoulli's equation which requires motion to be irrotational. Furthermore, the tangential velocity distribution of eqn. (5) combined with the equation of momentum (3) provides Viparelli with values of A that are so high as to make the flow rate expression imaginary, forcing Viparelli to arbitrarily reduce the value of A (thus effectively negating the validity of the momentum equation). Nor it is possible to share Knapp's hypothesis [5] which introduces Δ+b/2 instead of Δ into eqn. (3). Given the uncertainty of the hypothesis, Pica [7] suggests using eqn. (4) as it makes the process simpler to deal with.

As regards the pressure distribution along the radius in the first cross section of the shaft, the authors investigating this issue fall into two broad groups. On the basis of experimental measurements, Viparelli [3], Knapp [5] and Pica [7] hold eqn. (6) to be valid, theoretically justifying this position with the hypothesis of a balancing of the outward centrifugal forces (caused by the curvature of the trajectories in the horizontal plane 0-0, reported in fig.1) with the inward centrifugal forces (caused by the curvature of the trajectories in vertical planes). Binnie and Hookings [2], Ackers and Crump [4] and Adami [6], on the other hand, believe that only the curvature in the horizontal plane should be taken into account and, therefore, although different schemes are considered, they hypothesize a pressure distribution reported in eqn. (7).

2 Velocity distribution analysis

Almost all the models assume motion to be irrotataional and symmetrical to the vertical axis of the vortex shaft. Only Viparelli [3] and Pica [7] assume there is symmetry with respect to the so-called core of the vortex (the free space in the proximity of the shaft axis through which a continual supply of air is known to pass, thus preventing the closure of the vortex) whose axis turns out in their tests not to perfectly match the vertical axis of the shaft. The hypothesized symmetry generally requires the values of the three velocity components V_r, V_θ and V_z (in cylindrical coordinates r, θ and z, where the z axis is shared with the vertical axis of the vortex) to be constant along circumferences lying on horizontal planes or on arcs of these circumferences up to the point where they meet the solid wall: this makes it possible to define a symmetrical behaviour also for a system that is geometrically non-symmetrical. The three components of the rotation vector Ω along the three axes z, r, and t (the latter normal to z and r in the considered point) can be written as follows:

$$\Omega_r = \frac{1}{r}\frac{\partial V_z}{\partial \theta} - \frac{\partial V_t}{\partial z}; \tag{8}$$

$$\Omega_t = -\frac{\partial V_z}{\partial r} + \frac{\partial V_r}{\partial z}; \tag{9}$$

$$\Omega_z = \frac{\partial V_t}{\partial r} - \frac{1}{r}\frac{\partial V_r}{\partial \theta} + \frac{V_t}{r}. \tag{10}$$

If the hypotheses of symmetry and irrotationality are assumed to be simultaneously verified, then it must be true that:

$$\frac{\partial V_t}{\partial z} = 0\,; \tag{11}$$

$$\frac{\partial V_z}{\partial r} = \frac{\partial V_r}{\partial z}\,; \tag{12}$$

$$\frac{\partial V_t}{\partial r} = -\frac{V_t}{r}\,. \tag{13}$$

The first of these relations tells us that V_t must be constant along every vertical. Since, by symmetry, V_t must be constant along every circumference (r=constant, z=constant) it follows that V_t must be constant along every cylinder of axis z. Furthermore, the third relation makes it possible to determine the law of velocity distribution of eqn. (5) by means of simple integration. However, the experimental measurements of Viparelli (who moreover did not take precise velocity measurements) do not verify this velocity distribution. From his observations, Viparelli therefore has inferred that a law of eqn. (5) could not be verified and Viparelli has completed his model after abandoning the hypothesis of irrotationality. Relinquishing the hypothesis of irrotational flow moreover seems to be a solution requiring suitable analyses as no other author (except, as already mentioned, Pica) has gone down this road. In actual fact, the other authors have assumed a velocity distribution of eqn. (5) leaving the contradiction of experimental data highlighted by Viparelli still unsolved. In their studies directed to formulate a new mathematical model on the hydraulic working of vortex shaft, Ciaravino et al. [8] have examined a different velocity distribution based on the renunciation to the symmetry hypothesis rather than on the renunciation to the irrotationality hypothesis. If the symmetry hypothesis is abandoned also the following position is abandoned:

$$\frac{\partial V_r}{\partial \theta} = 0\,. \tag{14}$$

Therefore a relation of the following type can be hypothesized in the whole flow field:

$$\frac{\partial V_r}{\partial \theta} = k \cdot V_A = \cos t \neq 0\,; \tag{15}$$

where V_A is the velocity in the inlet channel. Starting from the irrotational flow condition, eqn. (13) becomes:

$$\frac{\partial V_t}{\partial r} = -\frac{V_t}{r} + \frac{1}{r}\frac{\partial V_r}{\partial \theta}\,. \tag{16}$$

Taking eqn. (15) into account, after the integration it is simple process to obtain:

$$V_t = \frac{A}{r} + k \cdot V_A = \frac{A}{r} + B. \tag{17}$$

It is interesting to observe that eqn. (17) is also obtained by Viparelli, not starting from eqn. (15) but by assuming non-irrotatational flow. In actual fact, Viparelli sees the term kV_A as representing the motion rotation value rather than the contribution of V_r which contrasts the rotationality generated by V_t. Moreover, it is possible to hypothesize the existence of a second type of asymmetry on the basis of the fact that the inlet chamber wall gradually draws closer to the shaft axis: this determines ever smaller sections for the passage of the annular flow. This appears to bring about rises in the free surface (measured experimentally) and an increase in velocity, even when there is a decrease in the flow rate determined by the centripetal flow entering the shaft. The assumed increase in tangential velocity can be taken into account (in a first approximation) by adding a linear term in θ to the velocity distribution shown in eqn. (17), so that the tangential velocity V_t is expressed by:

$$V_t = \frac{A}{r} + B + k_\theta \cdot \theta. \tag{18}$$

The decision to assume either the tangential velocity distribution of eqn. (5) or that of eqn. (18) requires considerable differences also in the definition of the distribution of radial velocity V_r. Thus introducing eqn. (5) or eqn. (18) into eqn. (17) yields, in the two different cases, by integrating with respect to V_r:

$$V_r = V_{ro}(r); \tag{19}$$

$$V_r = V_{ro}(r) + B \cdot \theta + k_\theta \frac{\theta^2}{2}. \tag{20}$$

In the first case a symmetrical radial velocity distribution is verified (the θ terms are missing) dependent on r; in the second case, in addition to the dependence on r there are two extra θ terms, one linear and the other quadratic, which introduce the presumed asymmetry of motion.

In order to verify which of these two velocity distributions is closer to reality, a series of preliminary experimental measurements of velocity have been taken inside the vortex (measurements which, as already mentioned, Viparelli did not have). Velocity measurements have been taken using a Laser Doppler Anemometer (LDA) on the experimental installation available at the Department of Hydraulic and Environmental Engineering of the University of Naples Federico II [9]. These tests have been conducted with a flow rate of 0.049 m^3/s and with two alignments (orthogonal to the axis of the inlet channel) placed in a horizontal plane at a height of 0.06 m from the bottom of the shaft. This position has made it possible to achieve measurement points with conditions of movement that have been disturbance free both on the bottom and on the free surface. The distance from the shaft axis of the two alignments has been 0.115 m (for the inner one) and 0.150m (for the outer one). The first of these two distances has allowed the first alignment to be taken as close as possible to the shaft axis without interfering with the free surface. The second distance has allowed the second alignment to be tangential to the vertical cylinder that is the ideal continuation of the vertical shaft receiving the flow. Therefore, velocity measurements have been taken in different points of each of the two alignments. Under these conditions the measured velocity component is a combination of the V_r and V_t as a function of the considered alignment point. The measured component $V(\theta)$ (indicated by the value θ of the angular coordinate) is orthogonal at that point to the alignment and therefore:

$$V(\theta) = V_t \sin\theta - V_r \cos\theta . \tag{21}$$

For negative values of the angle θ it is alternatively possible to write:

$$V(-\theta) = -V_t \sin\theta - V_r \cos\theta . \tag{22}$$

Consistently with Viparelli's observations [3], the simultaneous hypotheses of symmetry and irrotationality have given rise to incongruences in the interpretation of the results obtained from the experimental velocity measurements. In actual fact, introducing the expressions of V_t and V_r derived from eqn. (5) and eqn. (19) (valid for the simultaneous hypotheses of symmetry and irrotationality) into eqn. (21) and eqn. (22) yields:

$$V(\theta) = \frac{A}{r}\sin\theta - V_{ro}(r)\cos\theta ; \tag{23}$$

$$V(-\theta) = -\frac{A}{r}\sin\theta - V_{ro}(r)\cos\theta . \tag{24}$$

Adding these relations member by member, it is a simple process to obtain:

$$V_{ro}(r) = -\frac{V(\theta) + V(-\theta)}{2\cos\theta} . \tag{25}$$

The latter expression makes it possible to determine the distribution of the radial velocity component orthogonally to the considered alignment.

The results of the calculations performed with eqn. (25) turn out to be incongruent as – in the field where the two adopted alignments overlap – they return substantially different and non-matching radial velocity values that are such as to put in crisis the hypothesis of simultaneous symmetry and irrotationality. If, on the other hand, the validity is assumed of eqns (18) and (20), which abandon the symmetry hypothesis, then the same methodology used above provides:

$$V(\theta)=\left(\frac{A}{r}+B+k_{\theta}\cdot\theta\right)\sin\theta-\left(V_{ro}(r)+B\cdot\theta+k_{\theta}\frac{\theta^{2}}{2}\right)\cos\theta\,; \qquad (26)$$

$$V(-\theta)=-\left(\frac{A}{r}+B-k_{\theta}\cdot\theta\right)\sin\theta-\left(V_{ro}(r)-B\cdot\theta+k_{\theta}\frac{\theta^{2}}{2}\right)\cos\theta\,; \qquad (27)$$

$$V_{ro}(r)=-\frac{V(\theta)+V(-\theta)}{2\cos\theta}+k\cdot\left(\theta\cdot tg\theta-\frac{\theta^{2}}{2}\right). \qquad (28)$$

The results of the calculations made using eqn. (28), with k = 0.236, point to a good coincidence of the radial velocity component values calculated in the field where the two chosen alignments overlap. At this level of investigation it therefore seems reasonable to assume a velocity distribution represented by eqns (18) and (20).

3 Pressure distribution analysis

Viparelli's experimental measurements [3] indicate zero pressure values in the section 0-0 (fig.1) thus justifying eqn. (6) (as Knapp [5] and Pica [7] do), while the measurements of Binnie and Hookings [2] return positive values, thus justifying eqn. (7) (as Ackers and Crump [4] do).

In order to account for this difference, it can be assumed that Viparelli's section 0-0 does not match that of Binnie and Hookings. Viparelli's section is probably very close to the edge of the inlet with the result that the curvatures in the vertical planes acquire greater value while the pressure can be evaluated using eqn. (6) only in that part of the shaft in which the radius of the vortex core no longer changes, i.e. just below the inlet channel (moreover, in this case the value of H to be inserted into the equations should be slightly higher than the one supplied by eqn. (1)).

Moreover, as far as Adami [6], the distinction between eqn. (6) and eqn. (7) refers only to the theory for flow rates beyond critical value.

The criteria adopted with eqn. (6) (i.e. Knapp [5]) result in the calculated flow rate values being systematically and significantly lower than the experimental ones. On the other hand, the adoption of eqn. (7) leads to even smaller flow rate values which are therefore even further from the experimental data. Then it has been considered that the section 0-0 (in which the edge effect must be felt) has a negative pressure distribution. Such pressure distribution results in even higher velocity values. Therefore, if the values of the tangential velocity components V_t

are fixed in the computation model, the increase in the vertical components V_z results in an increase in the calculated flow rate. Adequate negative pressure values can therefore account for the flow values measured experimentally. In particular, it has been decided to use a pressure distribution law which, when inserted into eqn. (2), facilitates its integration. In effect the trajectories are curved both in the plane of section 0-0 and in vertical planes: the two curvatures determine centrifugal forces which are opposing and, therefore, they have an opposite effect on the pressure value in the section 0-0.

The experimental data (table 1) can be interpreted to show that the influence of curvatures in vertical planes can be prevalent and, therefore, negative pressure can be verified (even if not along the entire perimeter with the higher flow values). Consequently it has been assumed, in alternative to eqn. (6) or eqn. (7), that:

$$\frac{p}{\gamma} = \frac{B^2}{2g}\left[\frac{1}{r^2} - \frac{1}{r_o^2}\right]; \tag{29}$$

where B is a constant value which has to be determined. In actual fact, if $r = r_o$, eqn. (29) yields that p/γ is zero; if $r > r_o$, p/γ is negative.

In conclusion, it can also be noted that the pressure measurements (taken at the four points P indicated in fig. 1 and reported in table 1 in water column) constantly show a tendency towards an eminently asymmetrical distribution.

Table 1: Experimental tests.

Q (m^3/s)	P_1 (m)	P_2 (m)	P_3 (m)	P_4 (m)
0.0010	-0.0030	- 0.0003	-0.0015	-0.0008
0.0052	-0-0028	- 0.0012	-0.0006	-0.0003
0.0160	-0.0027	- 0.0013	-0.0015	0.0000
0.0260	-0.0027	- 0.0014	0.0005	0.0002
0.0380	-0.0025	- 0.0015	0.0012	0.0004
0.0490	-0.0020	- 0.0017	0.0021	0.0005
0.0580	-0.0016	- 0.0018	0.0027	0.0010
0.0690	-0.0010	- 0.0024	0.0041	0.0015
0.0790	-0.0005	- 0.0025	0.0048	0.0018

4 Conclusions

The theoretical analysis, performed using experimental tests, shows how the flow in a vortex shaft, contrary to the hypotheses made by all the other authors, is asymmetrical and has negative pressures in the inlet section. Above all in the initial part of the vortex flow (which is the one examined with the reported preliminary tests), the tangential velocities are accelerated, a characteristic which is reflected in particular trends of radial velocities which are once again non-symmetrical. The asymmetry of the vortex motion and the negative pressure in the inlet section are able to justify certain experimental incongruences reported by some of the cited authors and make it possible to formulate a theory of hydraulic working (on the basis only of the geometric data of the outlet device and the water depth in the inflow channel) which leads to determine flow rates and discharge coefficients that are consistent and in good agreement with the experimental measurements. The results obtained through the analysis of a limited number of experimental tests certainly need to be verified by extending the measurements to the entire flow field and to a larger number of measurements regarding the pressure distribution in the inlet section. It can nevertheless be concluded that an increased accuracy in the quantitative assessment of current theoretical models on the hydraulic working of vortex shafts can only be assured by taking the asymmetry hypothesis into account.

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